Log or Linear? Distinct Intuitions of the Number Scale in Western and Amazonian Indigene Cultures

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Log or Linear? Distinct Intuitions of the Number Scale in Western and Amazonian Indigene Cultures

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The mapping of numbers onto space is fundamental to measurement and to mathematics. Is this mapping a cultural invention or a universal intuition shared by all humans regardless of culture and education? We probed number-space mappings in the Mundurucu, an Amazonian indigene group with a reduced numerical lexicon and little or no formal education. At all ages, the Mundurucu mapped symbolic and nonsymbolic numbers onto a logarithmic scale, whereas Western adults used linear mapping with small or symbolic numbers and logarithmic mapping when numbers were presented nonsymbolically under conditions that discouraged counting. This indicates that the mapping of numbers onto space is a universal intuition and that this initial intuition of number is logarithmic. The concept of a linear number line appears to be a cultural invention that fails to develop in the absence of formal education.

What then is mathematics if it is not a unique, rigorous, logical structure? It is a series of great intuitions carefully sifted, and organized by the logic men are willing and able to apply at any time.

– Morris Kline, *Mathematics: The Loss of Certainty* [(*1*), p. 312]

The mapping of numbers onto space plays an essential role in mathematics, from measurement and geometry to the study of irrational numbers, Cartesian coordinates, the real number line, and the complex plane (*1*, *2*). How does the human mind gain access to such abstract mathematical concepts? Constructivist theories view mathematics as a set of cultural inventions that are progressively refined in the history of mathematics and are slowly acquired during childhood and adolescence (*3*). However, the mental construction of mathematics may have deeper foundations. Mathematical objects may find their ultimate origin in basic intuitions of space, time, and number that have been internalized through millions of years of evolution in a structured environment and that emerge early in ontogeny, independently of education (*2*, *4*). Here we present evidence that reconciles these two points of view: Our results suggest that all humans share the intuition that numbers map onto space, but that culture-specific experiences alter the form of this mapping.

Previous psychological and neuroimaging research supports the view that a sense of number is present in humans and many other species at an early age and has a reproducible substrate in the bilateral intraparietal sulcus (*5*–*8*). This region is remarkably close to or even overlapping with areas engaged in the coding of spatial dimensions such as size, location, and gaze direction (*9*–*11*). Interactions between numerical and spatial codes in the parietal cortex may therefore occur at this level. Indeed, in human adults, the mere presentation of an Arabic numeral automatically elicits a spatial bias in both motor responding and attention orienting (*11*–*13*). Brain-lesioned patients show corresponding impairments in comparing and bisecting line segments and numbers (*14*), and some people even report a vivid experience of seeing numbers at fixed locations on an idiosyncratic spatially contiguous "number form" (*15*, *16*).

Recent experiments document a remarkable shift in the child's conception of how numbers map onto space (*17*–*19*). When asked to point toward the correct location for a spoken number word onto a line segment labeled with 0 at left and 100 at right, even kindergarteners understand the task and behave nonrandomly, systematically placing smaller numbers at left and larger numbers at right. They do not distribute the numbers evenly, however, and instead devote more space to small numbers, imposing a compressed logarithmic mapping. For instance, they might place number 10 near the middle of the 0-to-100 segment. This compressive response fits nicely with animal and infant studies that demonstrate that numerical perception obeys Weber's law, a ubiquitous psychophysical law whereby increasingly larger quantities are represented with proportionally greater imprecision, compatible with a logarithmic internal representation with fixed noise (*7*, *20*, *21*). A shift from logarithmic to linear mapping occurs later in development, between first and fourth grade, depending on experience and the range of numbers tested (*17*–*19*).

All of these observations, however, were made in Western children who all had access to mathematical education and culture at an early age. Before formal schooling, Western children may acquire the number-line concept from Arabic numerals seen on elevators, rulers, books, etc. Thus, existing studies do not reveal which aspects of the number-space mapping constitute a basic intuition that would continue to exist in the absence of a structured mathematical language and education. In particular, we do not know whether the log-to-linear shift would occur spontaneously in the course of brain maturation or whether it requires exposure to critical educational material or culture-specific devices such as rulers or graphs.

To address these issues, we gathered evidence from psychological experimentation in the Mundurucu, an Amazonian indigene culture with little access to education (*22*, *23*). Previous research has established that, although their lexicon of number words is reduced and they have little or no access to rulers, measurement devices, graphs, or maps, the Mundurucu entertain sophisticated concepts of both number and space, albeit in an approximate and nonverbal manner (*22*, *23*). We therefore asked whether they conceive of these two domains as being related by a systematic mapping and, if so, what form this number-space mapping takes.

A total of 33 Mundurucu adults and children were tested individually in a number-space task (Fig. 1) (*24*). In each trial, a line segment was displayed on a computer screen, with 1 dot at left and 10 dots at right (or, in a separate block, 10 and 100 dots, respectively). Then other numbers were presented in random order, in various forms (sets of dots, sequences of tones, spoken Mundurucu words, or spoken Portuguese words). For each number, the participant pointed to a screen location and this response was recorded by a mouse click, without feedback. Only two training trials were presented, with sets of dots whose numerosity corresponded to the ends of the scale (1 and 10). The participants were told that these two stimuli belonged to their respective ends, but that other stimuli could be placed at any location. Because training did not involve intermediate numbers, performance on all subsequent trials served to reveal whether the participants would spontaneously use systematic mapping, and if so, whether their mapping would be compressive or linear.

The Mundurucu's mean responses revealed that they understood the task. Although some participants tended to use only the end points of the scale (*24*), most used the full response continuum and adopted a consistent strategy of mapping consecutive numbers onto consecutive locations (Fig. 2). There was a significant positive correlation between stimulus number and mean response location, regardless of the modality in which the numbers were presented. The task was easy when the stimuli were sets of dots similar to the reference labels placed at the end points [numbers 1 to 10, correlation coefficient (r^2) = 92.6%, 8 df; numbers 10 to 100, r^2 = 91.9%, 8 df]. However, the Mundurucu continued to use systematic number-space mapping with stimuli they

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had not been trained with, which shared with the reference labels only an abstract concept of number: sequences of tones 1 to 10 (r^2 = 92.5%, 8 df), spoken Mundurucu number words (r^2 = 91.8, 6 df), and Portuguese number words (r^2 = 91.1%, 8 df), although a small proportion of random responses tended to slightly flatten the curves. The Mundurucu stimuli included complex expressions that are very rarely uttered, such as "pũg põgbi ebadipdip bodi" [approximate translation: "one handful (and) four on the side"]. The results suggest that the Mundurucu partially understand the quantity to which these expressions refer.

Crucially, however, linear regression did not provide the best model of the Mundurucus' responses. Rather, for all modalities of presentation, the curves were negatively accelerated. A multiple regression procedure evaluated the contribution of a logarithmic regressor, over and above the linear regressor. The logarithmic compression effect was significant for all stimulus modalities, although it was only marginal with Portuguese words (one-tailed test, $P = 0.04$; see significance levels and regression weights in Fig. 2). Additional analyses allowed us to exclude interpretations in terms of linear responding with different slopes for small and large numbers, parallax error, experimenter bias, or bimodal responding (*24*). The Mundurucu seem to hold intuitions of numbers as a log scale where the middle of the interval 1 through 10 is 3 or 4, not 5 or 6.

Previous number-space mapping experiments with Western participants included only symbolic numerals, whereas the present experiment included nonsymbolic visual and auditory numerosities. Thus, it was important to verify whether these novel stimuli were rated linearly or logarithmically in educated Western participants. As shown in Fig. 2, American adults rated linearly all numerals presented in English and in Spanish, as well as the sets of 1 to 10 dots, which could easily be counted. However, they exhibited a significant logarithmic component with sets of 10 to 100 dots and with sequences of tones. When the two groups of participants were compared directly, the Mundurucu showed a greater compressive nonlinearity than the American participants only with sets of 1 to 10 dots ($P = 0.003$) and with numerals in the first language (Mundurucu versus English numerals, $P = 0.033$). This finding concurs with previous data suggesting that Western people estimate large numerosities in an approximate and compressive manner (*25*, *26*). Their judgments are linear only when the numbers are presented in a symbolic manner or as small sets whose numerosity can be precisely assessed.

The Mundurucu population is heterogeneous, and some of our participants, particularly the children, had received a little education. To examine the impact of this variable, we calculated, for each participant, an index of nonlinearity in the number-space mapping: the weight of the log regressor in a multiple regression of the data on linear and log regressors. For this analysis, we pooled the trials with 1 to 10 dots and number words, but excluded those with 10 to 100 dots and with tones, for which Western participants showed some nonlinearity. The index confirmed a highly significant nonlinearity in Mundurucu participants (Student's t test = 6.20, 34 df, $P < 10^{-6}$). In American participants, performance did not deviate from linearity ($P = 0.08$) and differed markedly from that of the Mundurucu (Welch's t test = 4.37, 48.6 df, $P < 0.0001$). Crucially, the Mundurucu's nonlinearity remained significant even when the analysis was restricted to adults ($t = 4.34$, 23 df, $P = 0.0002$), to monolingual speakers ($t = 5.36$, 29 df, $P < 10^{-5}$), or to uneducated participants ($t = 2.60$, 7 df, $P = 0.035$; see figs. S7 to S10 for a graphic depiction of subgroup performance) (*24*). t tests and linear and rank-order regression analyses showed no effect of gender, age, education, or bilingualism. There was only a trend toward reduced nonlinearity as a function of age (Kendall tau = –0.23, $P = 0.055$). Although this observation suggests that older Mundurucu may evolve a greater understanding of the linear number line, it should be noted that in Western children, the mapping becomes linear over the range from 10 to 100 by the first or second grade (*17*–*19*), whereas in our data, even the oldest Mundurucu adults (those over 40) continued to show highly significant nonlinearity over the range from 1 to 10 ($t = 3.36$, 11 df, $P = 0.006$).

Finally, we analyzed the special case of Portuguese numerals. Although overall performance was logarithmic, subdivision by education level indicated that logarithmic responding held for participants with 1 to 2 years of education ($t = 3.15$, 16 df, $P = 0.006$; fig. S9) but not for those with no education at all or with more education. In uneducated participants, performance with Portuguese numerals was highly variable and weakly correlated with number (r^2 = 39.0, $P = 0.053$; fig. S8), suggesting that many of these participants simply did not know the meaning of Portuguese numerals. For the most educated group, on the other hand, performance was strictly linear (r^2 = 94.5%, $P < 10^{-5}$; fig. S10). Excluding participants with no education, we found that greater education significantly changed the responses to Portuguese from logarithmic to linear ($t = 2.48$, 16.6 df, $P = 0.024$) but left responses to Mundurucu numerals and dot patterns unchanged ($P > 0.5$), thus yielding a significant interaction ($P = 0.008$). Strikingly, within the more educated group, performance varied significantly with number notation ($t = 3.12$, 9 df, $P = 0.012$), because it was linear for Portuguese numerals but logarithmic for Mundurucu numerals and dot patterns from 1 to 10.

Overall, these results reveal both universal and culture-dependent facets of the sense of number. After a minimal instruction period, even members of a remote culture with reduced vocabulary and education readily understand that number can be mapped onto a spatial scale. The exact form of this mapping switches dramatically from logarithmic to linear, however, depending on the ages

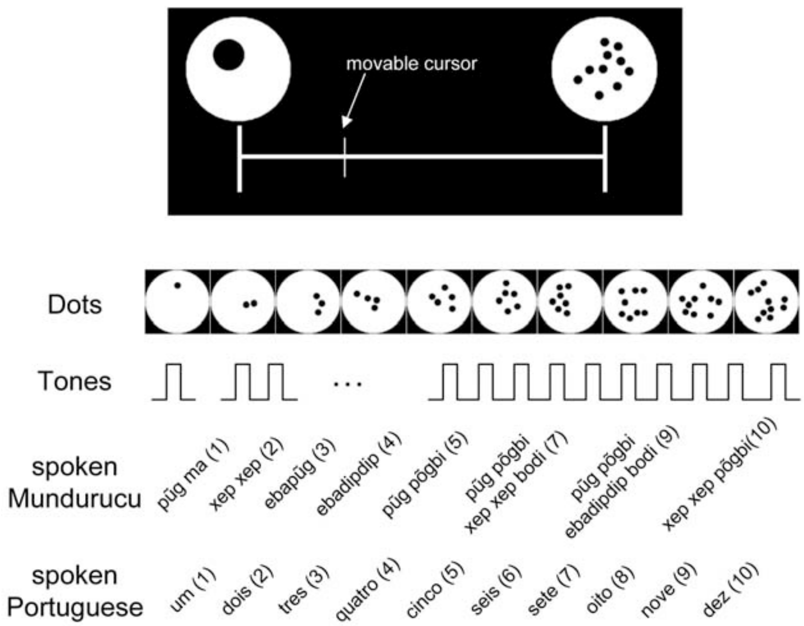

Fig. 1. Number mapping task with numbers from 1 to 10. A horizontal segment, labeled with a set of 1 dot on the left and a set of 10 dots on the right, was constantly present on screen. Numbers were presented visually as sets of dots or auditorily as sequences of tones (*24*), Mundurucu numerals, or Portuguese numerals. For Mundurucu numerals, a rough translation into Arabic numerals is provided (for example, "pũg põgbi xex xep bodi" ≈ "one handful (and) two on the side" ≈ 7; "xex xep põgbi" ≈ "two handfuls" ≈ 10). For each stimulus, participants pointed to a place on the line, and the experimenter clicked it with the computer mouse, which made a small bar appear.

at which people are tested, the education they have received, and the format in which numbers are presented.

In light of the performance of Amazonian adults, it is clear that the mental revolution in Western children's number line does not result from a simple maturation process: Logarithmic thinking persists into adulthood for the Mundurucu, even for very small numbers in the range from 1 to 10, whether presented as dots, tones, or spoken Mundurucu words. What are the sources of this universal logarithmic mapping? Research on the brain mechanisms of numerosity perception have revealed a compressed numerosity code, whereby individual neurons in the parietal and prefrontal cortex exhibit a Gaussian tuning curve on a logarithmic axis of number (*27*). As first noted by Gustav Fechner, such a constant imprecision on a logarithmic scale can explain Weber's law—the fact that larger numbers require a proportional larger difference in order to remain equally discriminable. Indeed, a recent model suggests that the tuning properties of number neurons can account for many details of elementary mental arithmetic in humans and animals (*21*). In the final analysis, the logarithmic code may have been selected during evolution for its compactness: Like an engineer's slide rule, a log scale provides a compact neural representation of several orders of magnitude with fixed relative precision.

It is not yet known which critical educational or cultural experience turns this initial representation into a linear scale. When a cultural difference in conceptual representation is observed in a remote population, Whorf's hypothesis is often invoked (*28*), according to which language determines the organization of thought. In the present case, however, the Whorfian explanation fails, because neither linguistic competence per se (present in all Mundurucu), nor numerical vocabulary and verbal counting [present in bilingual Mundurucu and in young children (*24*)] suffice to induce the log-to-linear shift (*17*–*19*). Speculatively, two factors underlying the shift may be experience with measurement, whereby a fixed numerical unit is applied to different spatial locations; and experience with addition and subtraction, ultimately yielding the intuition that all consecutive numbers are separated by the same interval + 1. The most educated Mundurucu eventually understand that linear scaling, which allows measurement and invariance over addition and subtraction, is central to the Portuguese number word system. At the same time, they still do not extend this principle to the Mundurucu number words, where perceptual similarity between quantities is still seen as the most relevant property of numbers. The system of Mundurucu number words may be a cultural device that does not emphasize measurement or invariance by addition and subtraction as defining features of number, contrary to Western numeral systems.

The simultaneous presence of linear and compressed mental representations of numbers is probably not unique to the Mundurucu. In American children, logarithmic mapping does not disappear all at once, but vanishes first for small numbers and much later for larger numbers from 1 to 1000 (up to fourth or sixth grade in some children) (*17*–*19*). In fact, a logarithmic representation may

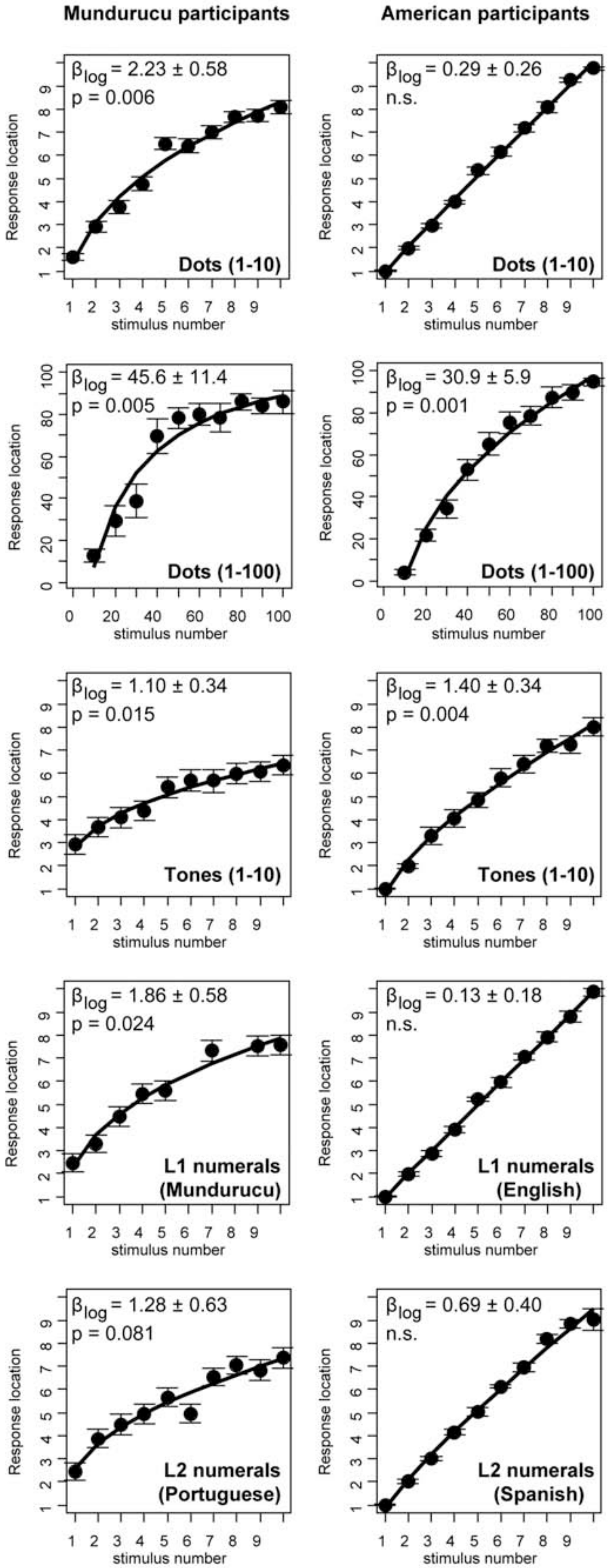

Fig. 2. Average location of numbers on the horizontal segment, separately for Mundurucu participants (left column) and for American participants (right column). β_{log}, weight of the logarithmic regressor in a multiple regression analysis with linear and logarithmic predictors. Data are mean ± SE of the mean. Graphs of performance broken down by age group and education are available as supporting online material (*24*). L1, first language; L2, second language.

remain dormant in all of us for very large numbers or whenever we approximate numbers (*29*), including prices (*30*). Thus, log and linear scales may be deeply embedded in all of our mental activities.

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Supporting Online Material

www.sciencemag.org/cgi/content/full/320/5880/1217/DC1
Methods
SOM Text
Figs. S1 to S10
References

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Coordination of Early Protective Immunity to Viral Infection by Regulatory T Cells

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Suppression of immune responses by regulatory T cells (Tregs) is thought to limit late stages of pathogen-specific immunity as a means of minimizing associated tissue damage. We examined a role for Tregs during mucosal herpes simplex virus infection in mice, and observed an accelerated fatal infection with increased viral loads in the mucosa and central nervous system after ablation of Tregs. Although augmented interferon production was detected in the draining lymph nodes (dLNs) in Treg-deprived mice, it was profoundly reduced at the infection site. This was associated with a delay in the arrival of natural killer cells, dendritic cells, and T cells to the site of infection and a sharp increase in proinflammatory chemokine levels in the dLNs. Our results suggest that Tregs facilitate early protective responses to local viral infection by allowing a timely entry of immune cells into infected tissue.

Regulatory T cells (Tregs) expressing the transcription factor Foxp3 play an essential role in controlling immune response–mediated inflammation. Their importance is emphasized by the fact that deficiency in Tregs results in a fatal autoimmune syndrome affecting multiple organs (*1*, *2*). Theoretically, the potent suppressor function of Tregs might present a serious obstacle to establishing robust protective immunity toward pathogens, and recent studies addressing a role for Tregs during infection have suggested several mutually exclusive scenarios. Some studies have suggested that by limiting late immune responses to an infectious agent, Tregs minimize associated tissue damage while at the same time preventing or diminishing pathogen clearance (*3*–*5*). Alternatively, it has been proposed that during viral infection, Tregs lose their suppressor capacity in response to engagement of virus-sensing mechanisms such as Toll-like receptor (TLR) signaling (*6*). Another study suggests that effector T cells responding to infection might become resistant to Treg-mediated suppression as a result of exposure to proinflammatory cytokines and increased costimulatory signals (*7*). Thus, with several scenarios proposed, the role for Tregs during infection remains unclear.

We examined a role for Tregs in mucosal herpes simplex virus (HSV) infection by taking advantage of $Foxp3^{gfp}$ and $Foxp3^{DTR}$ knock-in mice harboring Treg subsets tagged with green fluorescent protein (GFP) or a human diphtheria toxin receptor (DTR), respectively. This allowed us to track and isolate Tregs and to efficiently ablate these cells upon in vivo DT treatment (*8*, *9*). In these studies we used a well-established model of genital HSV-2 infection via a natural route (*10*). In HSV-2–infected mice, initial viral replication is limited to the vaginal mucosa (*11*), followed by spread into the central nervous system (CNS) upon retrograde transport of virions into the sacral ganglia, resulting in a fatal paralysis. The adaptive immune response to HSV-2 is dominated by virus-specific T helper 1 (T_H1) cells essential for limiting HSV-2 replication (*12*, *13*).

We first examined whether Tregs respond to viral infection by monitoring the dynamics of the GFP-tagged Treg subset relative to "effector" T cells, as defined by the absence of Foxp3, in infected $Foxp3^{gfp}$ mice. After genital infection with HSV, the total numbers of both non-Tregs and Tregs drastically increased in the draining lymph nodes (dLNs) and at the site of infection with essentially identical kinetics, and both subsets displayed an increased proportion of cells expressing activation markers (Fig. 1, A to D). Furthermore, using continuous in vivo 5-bromo-2′-deoxyuridine (BrdU) labeling, we observed that $CD4^+Foxp3^-$ and $Foxp3^+$ Tregs that had undergone cell division could be found in both the dLNs and at the infection site within 4 days of viral challenge (Fig. 1, E and F). Finally, Tregs isolated from the dLNs of infected mice had a measurably greater potency in suppressing the virus-specific proliferative response of CD4 T cells relative to Tregs isolated from uninfected mice (Fig. 1G). Taken together, these

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