Core foundations of abstract geometry

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Human adults from diverse cultures share intuitions about the points, lines, and figures of Euclidean geometry. Do children develop these intuitions by drawing on phylogenetically ancient and developmentally precocious geometric representations that guide their navigation and their analysis of object shape? In what way might these early-arising representations support later-developing Euclidean intuitions? To approach these questions, we investigated the relations among young children's use of geometry in tasks assessing: navigation; visual form analysis; and the interpretation of symbolic, purely geometric maps. Children's navigation depended on the distance and directional relations of the surface layout and predicted their use of a symbolic map with targets designated by surface distances. In contrast, children's analysis of visual forms depended on the size-invariant shape relations of objects and predicted their use of the same map but with targets designated by corner angles. Even though the two map tasks used identical instructions and map displays, children's performance on these tasks showed no evidence of integrated representations of distance and angle. Instead, young children flexibly recruited geometric representations of either navigable layouts or objects to interpret the same spatial symbols. These findings reveal a link between the early-arising geometric representations that humans share with diverse animals and the flexible geometric intuitions that give rise to human knowledge at its highest reaches. Although young children do not appear to integrate core geometric representations, children's use of the abstract geometry in spatial symbols such as maps may provide the earliest clues to the later construction of Euclidean geometry.

spatial cognition | mathematical cognition | map reading

Abstract concepts of formal geometry underlie a wide range of human achievements, but their source has been debated for millennia (1). Human abilities to navigate the environment and to recognize objects develop early and are shared across diverse animal species. In recent years, intensive study at levels from neurons to cognition (2–5) has illuminated the geometric information guiding these abilities in animals from insects to vertebrates (6–8) and in humans from infants to adults (9–14). When navigating, humans and animals represent their position by encoding the distances and directions of extended surfaces in the terrain rather than the angles at which surfaces meet (15, 16). In contrast, humans and animals represent objects by encoding the angles and relative lengths defining 3D part structures or 2D shapes rather than their absolute sizes or the directional relations that distinguish a form from its mirror image (17, 18). Despite the pervasiveness and power of these core geometric representations, neither in isolation is adequate to support abstract geometric intuitions, which require an integrated representation of distance and angle (13, 19, 20). Still, these two sets of core representations together may provide a foundation for abstract geometry.

By the age of 4 y and with little training or feedback, young children can use simple maps that symbolize abstract distance and angle relations by depicting an overhead view of an array of objects or surfaces (21–24). Not until the age of 6 to 10 y, however, do children begin to integrate distance and angle information when they reason about the properties of triangles and the behavior of dimensionless points and perfectly straight lines of infinite extent (19). Although it is unclear how this ability emerges, examining children's use of geometry in spatial symbols such as overhead maps may shed light on the development of the powerful geometric concepts achieved by adulthood. Uniquely human spatial symbols may serve as a medium in which children engage abstract interpretations of distance and angle. If this early understanding of the abstract geometry in spatial symbols arises from the core geometric foundations that humans share with other animals, young children's map-based navigation should be related to their performance on two distinct tasks eliciting core knowledge of geometry to navigate the environment and to recognize objects. Unlike older children, however, 4-y-old children might fail to integrate the distance and angle information represented in such maps.

Previous research by Huang and Spelke (25) showed that children's use of distance in a nonsymbolic navigation task correlated with their use of a map to locate targets at the surface midpoints of a continuous triangular environment. Moreover, children's use of angle and relative length in a nonsymbolic shape recognition task correlated with their use of a map to locate targets at the corners of the same triangular environment. Because these map tasks used continuous triangular arrays, however, both tasks could be solved by representing either distance and direction or relative length and angle. In the present research, we investigate whether children's early-emerging and shared geometric sensitivities to distance, direction, length, and angle make specific contributions to their use of the abstract geometry presented in spatial symbols.

We address this question by presenting children with one set of purely geometric maps that serve to represent two differently fragmented 3D environments. In one map task, children had to navigate a triangular array in which the corners were removed, leaving three sides of equal length placed at distinct distances and directions from the array's center. In the other map task, the sides of the triangular array were interrupted at their centers, leaving three corners of distinct angles. Previous research found that children navigated by distance and directional information to find both side and corner locations in fragmented rectangular or rhomboidal arrays displaying equal-length surfaces at distinct distances. In contrast, children failed to use distance, direction, length, or angle to locate targets in a fragmented square array displaying equidistant sides of different lengths or a fragmented rhomboidal array displaying corners at distinct distances with distinct angles (16). Children's failure in this last condition does not stem from a general lack of sensitivity to angle information, however, because even infants, like adults, use similar arrangements of fragmented angles to perceive the shapes of visual forms (26, 27). Thus, because absolute length information was

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held constant across our two map tasks, and because corners were removed in one map task and sides were interrupted in the other map task, the environments presented arrays in which only distance and directional relations or relative length and angle relations, respectively, were available to guide map use (16). We tested the specificity of children's core geometry to interpret the spatial symbols representing these fragmented arrays by controlling for the effects of age, verbal intelligence, and other spatial abilities.

Forty-five 4-y-old children (23 female, mean age = 4 y 6 mo, age range = 4 y 0 mo–4 y 11 mo were tested during two laboratory visits. In one visit, they completed two nonsymbolic tasks used to elicit core geometric representations in young children and animals. In the navigation task, children were disoriented within three rectangular environments with different aspect ratios and then were allowed to reorient by the distance and directional relations in each environment to locate a hidden object (10, 11, 16, 28) (Fig. 1*A*). In the visual form analysis task, children were presented with a succession of visual arrays displaying five similar shapes and one deviant shape that differed in one of a variety of properties, including proportional length, angle size, global shape, relations of parallelism and alignment, and symmetry, as well as the sense relations that distinguish a form from its mirror image (13) (Fig. 1*B*). In another visit, children completed two symbolic tasks in which they used the same geometric maps (Fig. 2*A*) to locate targets in a triangular array formed either by walls at distinct distances (Fig. 2*B*) or by corners of distinct angles (Fig. 2*C*). Following the map tasks, children completed a test of verbal intelligence.

Performance on the tests of reorientation and form analysis was consistent with past research using these tasks with infants and animals. On the reorientation task, children searched most often and equally at the correct and opposite corner locations, indicating that they were disoriented (Fig. 2*C* and Table S1) and that they used the distance and directional relations in the enclosure to reorient themselves. Children's performance exceeded chance in the two more elongated rectangular enclosures (6:9 rectangle [$t(44) = 6.64$, $P < 0.001$] and 6:8 rectangle [$t(44) = 2.85$, $P = 0.007$]) but not in the least elongated enclosure (6:7 rectangle [$t(44) = 0.88$, $P = 0.382$]). Finally, children used distance relations with greater difficulty as the relative distances of the extended surfaces became harder to distinguish [$F(1, 44) = 21.42$, $P < 0.001$; Fig. 1*C*]. Reorientation scores were calculated as an average of the two above-chance conditions.

Children successfully located the deviant shape on 11 of the 16 form analysis trials (Fig. 1*B*). They performed at chance in two trials assessing their sensitivity to the sense relations that distinguish a form from its mirror image, two trials targeting their sensitivity to symmetry, and one trial presenting forms characterized by both relative length and angle but varying considerably in size. An analysis of error patterns from this task revealed that most children relied on absolute size rather than shape in this last case, choosing the smallest figure significantly more often than any other figure in the array (Fig. 1*B*). In all the above-chance trials, the deviant form differed from the others by one or more geometric properties, including proportional length, angle size, global shape, or relations of parallelism and alignment; forms that shared these properties varied in absolute size, orientation, or

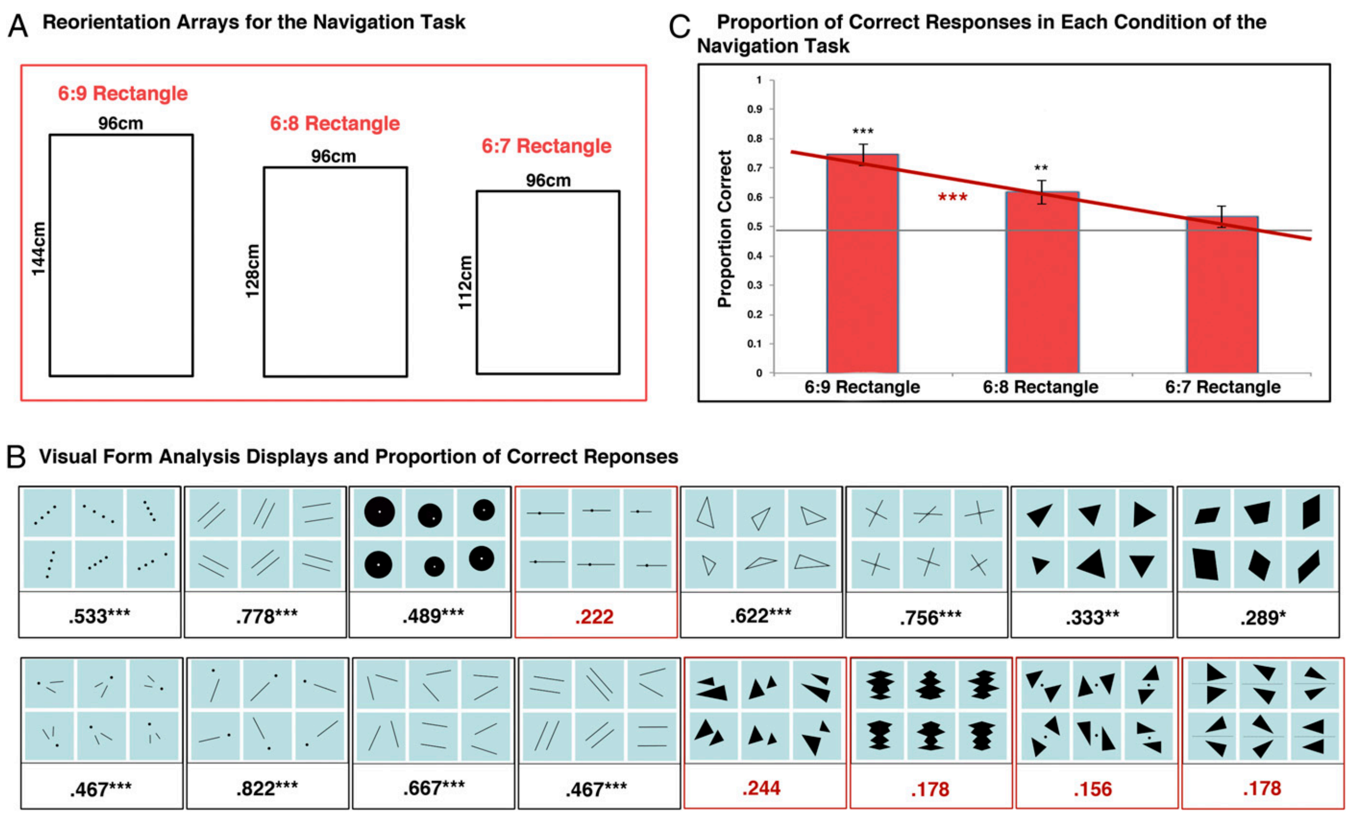

Fig. 1. Two nonsymbolic geometry tasks. (*A*) Schematics of the three rectangular enclosures that were used in the navigation task. (*B*) All 16 displays used in the visual form analysis task, which required children to locate the geometric deviant in a group of shapes. Children performed above chance in 11 of the 16 trials and at chance in the five trials outlined in red (binomial test: $***P < 0.001$; $**P < 0.01$; $*P < 0.05$). (*C*) Proportion of correct responses in each condition of the navigation task. Children performed above chance in both the 6:9 and 6:8 conditions. They used the enclosures' relative wall distances with greater difficulty as their aspect ratio approached 1 ($***P < 0.001$; $**P < 0.01$).

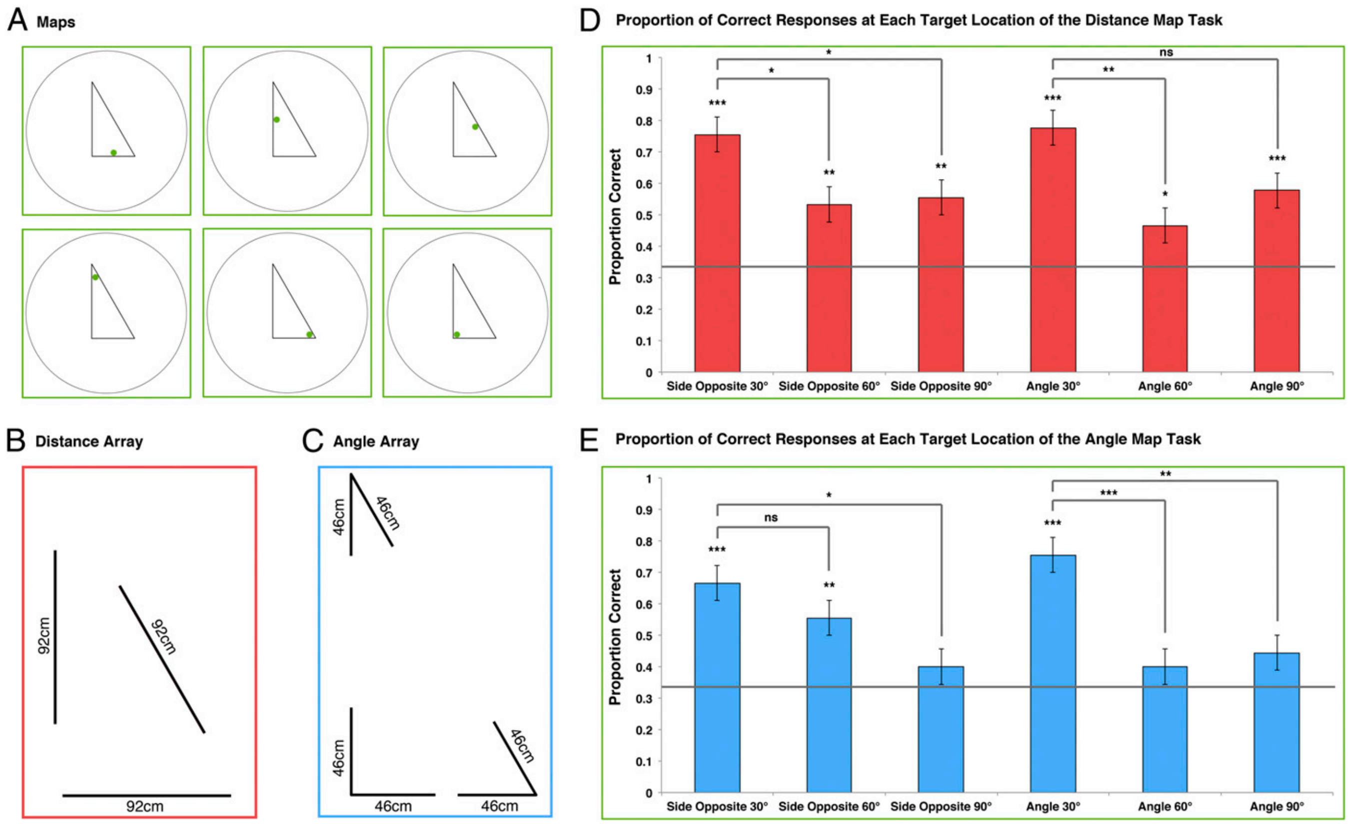

Fig. 2. Maps and schematics of the 3D environments in the two map tasks. (*A*) Six maps used in both map tasks, which depicted intact triangles at a 0.13:1 scale. Each map was presented at a constant orientation relative to the child, who faced a different direction relative to the array on each trial (0°, 60°, 120°, 180°, 240°, or 300°). (*B*) Overhead view of the triangular array for the distance map task. Three boards of white foam core (25 cm × 92 cm) were arranged as the sides of a 30-60-90 triangle (102 cm × 176.67 cm × 204 cm). (*C*) Overhead view of the triangular array for the angle map task. Three corners of white foam core (25 cm high) were arranged as the corners of a 30-60-90 triangle with two 46-cm segments defining each corner. Proportion of correct responses at each target location in the distance map task (*D*) and the angle map task (*E*). The gray horizontal line indicates chance-level (0.33) performance (binomial test: ***$P < 0.001$; **$P < 0.01$; *$P < 0.05$). In both tasks, more correct responses occurred at the most geometrically distinct locations, indicating that children were using this geometric information when searching for targets (McNemar test: ***$P < 0.001$; **$P < 0.01$; *$P < 0.05$).

both. Children's form analysis scores were calculated as an average of the 11 above-chance trials.

On the map tasks, children were successful overall when the map designated target locations in an array with walls at distinct distances [$t(44) = 8.61$, $P < 0.001$] and with corners of distinct angles [$t(44) = 5.34$, $P < 0.001$]. Performance did not differ significantly between these tasks [$t(44)$ 1.70, $P = 0.096$] or between trials in which targets appeared directly at a side or corner location and trials in which targets appeared at the gap between two sides or two corners [$t(44) = 0.063$, $P = 0.950$]. As was the case with reorientation, children's performance scaled with the geometric distinctiveness of the target locations; children successfully located targets on all six of the distance map trials and on three of the six angle map trials (Fig. 2 *D* and *E*). Children's distance and angle map scores were calculated as an average of their performance on the above-chance trials.

We first tested for relationships between children's use of core geometry for navigation and visual form analysis. Strikingly, a bivariate correlation revealed no significant association between performance on the reorientation task and performance on the form analysis task [$r(43) = 0.026$, $P = 0.867$; Fig. S1]. Thus, children's use of geometry for navigation showed no evidence of being related to their use of geometry for analyzing visual forms.

Do children nevertheless engage these different core geometric representations when interpreting the same spatial symbol? We conducted hierarchical regression analyses to address this question. The first analysis tested whether children recruited representations of distance as used for navigation when finding targets in the distance map task. Children's reorientation scores predicted their ability to use the map to locate targets within an array of surfaces at distinct distances, over and above the effects of age and verbal intelligence [β(Reorientation) = 0.334, P = 0.027; Fig. 3*A*]. Still, it is possible that children used multiple strategies for locating targets in the distance map task. To test for the specificity of children's reorientation ability as a predictor of their score on the distance map task, we further controlled for children's performance on both the visual form analysis task and the angle map task. Children's performance on the visual form analysis task did not significantly predict their performance on the distance map task [β(Form Analysis) = 0.255, $P = 0.104$], and their reorientation scores still predicted a significant amount of variance after controlling for individual differences in visual form analysis and in performance on the angle map task [β(Reorientation) = 0.320, $P = 0.032$].

The second analysis tested whether children recruited representations of relative length and angle as used for object recognition when finding targets in the angle map task. Children's scores on the visual form analysis task predicted their ability to use the same maps to locate targets within an array of corners of distinct angles, over and above the effects of age and verbal intelligence [β(Form Analysis) = 0.325, $P = 0.023$; Fig. 3*B*]. To test for the specificity of children's visual form analysis as a predictor of their score on the angle map task, we further controlled for

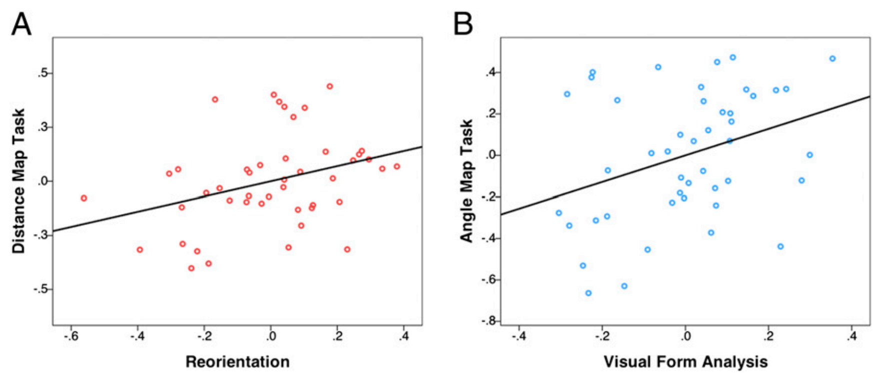

Fig. 3. Partial regression plots controlling for the effects of age and verbal intelligence and showing that reorientation performance predicted performance on the distance map task (*A*) and visual form analysis performance predicted performance on the angle map task (*B*).

children's performance on both the reorientation task and the distance map task. Children's performance on the reorientation task did not significantly predict their performance on the angle map task [β(Reorientation) = 0.034, $P = 0.825$], and their ability to analyze visual forms still predicted a significant amount of variance after controlling for individual differences both in reorientation and in performance on the distance map task [β(Form Analysis) = 0.322, $P = 0.035$].

These analyses reveal a striking pattern of relationships between children's reliance on distance for both the reorientation and distance map tasks and their reliance on object shape information for both the visual form analysis and angle map tasks. To investigate whether the two map tests elicited any common processes, we tested for a relationship between children's performance on the two map tasks. A bivariate correlation revealed no significant association between performance on the distance and angle map tasks [$r(43) = 0.182$, $P = 0.230$; Fig. S2]. Although the two map tasks used identical instructions and map displays to test children's interpretation of symbolic geometry, the children recruited different representations in applying the map to two different 3D environments.* Consistent with past findings that young children fail to integrate relations of distance and angle in tests probing more abstract geometric intuitions (19), children in the present studies showed no evidence of integrating core geometric representations used for navigation and form analysis when interpreting simple symbolic geometric maps.

In summary, performance on tasks engaging children's early-arising, nonsymbolic knowledge of geometry specifically predicted performance on two tasks evaluating their use of spatial symbols. Children's sensitivity to distance and directional relations in a navigation task predicted their use of a map to find targets in a 3D array with surfaces at distinct distances; their sensitivity to properties of object shapes in a form analysis task predicted their use of the same map to find targets in a 3D array with corners of distinct angles. This pattern of findings provides evidence that in their untutored interpretations of symbolic maps, children flexibly recruit the core geometric representations that emerge in infancy (10, 11, 30, 31), are shared by other animals (2, 7, 32, 33), and are used by children and adults throughout their lives.

Children's performance on both nonsymbolic tests of navigation and form analysis and symbolic tests of map understanding show no evidence of integrated representations of distance and angle. Such integration would have been indicated by convergent use of geometry in both of the symbolic spatial tasks and would have enhanced children's performance on all the tasks. For adults, who have achieved more abstract Euclidean intuitions, a triangle can be described by the distances between its corners, by the angles at its corners, or by a triplet of distance and angle combinations. Adults are sensitive to these geometric relations (19) and likely would apply the same shape description to all the arrays used in our symbolic map tasks (Fig. 2 *C* and *D*). Nevertheless, 4-y-old children show no evidence of having constructed the abstract geometric concepts that relate distances to angles and that specify shape descriptions applying to both surfaces and corners, even when reading spatial symbols, a skill achieved early in development (34, 35). Research using the present methods with older children applied to tasks engaging abstract geometry that develops over the lifespan may offer clues to the processes by which these integrated and uniquely human geometric intuitions emerge.

Materials and Methods

Participants. Children participated in the two testing sessions within a 2-wk time window. Three additional children participated in at least one task but were excluded due to a misunderstanding of task directions (two) or failure to return for the second appointment (one). Twenty-three children completed the set of nonsymbolic tasks on their first visit (followed by the set of symbolic tasks on their second visit), and 22 children completed these sets of tasks in the opposite order. There were no performance differences between these two groups (Table S2) or between male and female children on any of the tasks or conditions (Table S3). Informed consent was obtained from all participants. The use of human subjects was approved by the Committee on the Use of Human Subjects at Harvard University.

Statistical Methods. Reliability of the measures subject to regression and correlational analyses was maximized by the following: randomizing task order; selecting tasks, items, and difficulty levels based on research investigating human navigation, form analysis, and map reading (13, 16, 22, 23, 28, 36); excluding measures yielding chance performance; and confirming that mean performance levels and observed variance were similar across tasks and conditions (37) (Table S4). As confirmation that parametric hierarchical regressions were appropriate for these data, approximate normality of regression residuals was confirmed on the basis of comparison with the standard bell curve and examination of Q–Q plots.

*When targets were hidden at a side in the distance map task, our analyses and past research (16, 29) suggest that children located targets by relying on their distance to the sides of the array. When targets were hidden at a corner in the angle map task, children located targets by relying on the angular sizes of the array corners (16, 29) (Fig. 2*D*). It is less clear what geometric information children used when finding targets located at the gaps in the arrays. In the distance map task, children may have located a corner target either by finding a location to the left or to the right of a side at a particular distance or by finding a location between two sides at particular distances. In the angle map task, searches to the side locations likely did not depend on the distance relations between the corner locations. Children's performance on the side locations in the angle map task correlated with their performance on the corner locations of the angle map task (Spearman's $\rho = 0.300$, $P = 0.045$) and not with their performance on the side locations in the distance map task (Spearman's $\rho = 0.040$, $P = 0.793$). Nevertheless, children may have located the side between two corners either by inferring the relative length (rather than the absolute distance) of the side implied by two corners or by evaluating the relative sizes of the two corner angles themselves. Further research is needed to distinguish between these possibilities.

Reorientation. Both the experimenter and the child stood inside one of three 50-cm-high rectangular enclosures made of white foam core and differing only in aspect ratio (Fig. 1*A*). Each enclosure was placed in the center of a round room with white paneled walls, symmetrical lighting, and a concealed spring-loaded door (providing no distinguishing landmark or geometric information). For each of the four trials of each condition, the experimenter hid a sticker under a disk at one corner location of the rectangle while the child watched. Then, the experimenter blindfolded the child, turned him or her around in place for three to four full rotations until the child was disoriented, and stopped the child facing the center of one wall (a different wall on each trial). Finally, the participant removed the mask and searched for the sticker. The hiding locations were constant across all trials and conditions for any given child but were counterbalanced across children; the order of the three conditions also was counterbalanced across children. Children's search locations (measured as the first lifted disk) were judged offline from an overhead video feed by observers who were unaware of the children's performance in any of the other tasks. A single summary variable of successful use of geometry (proportion of searches to the two geometrically correct corners) was computed across the two conditions yielding above-chance performance.

Visual Form Analysis. Children were presented with 16 trials on a computer screen, each trial depicting an array of six 2D shapes. They were asked to examine all the shapes and to locate the shape that did not belong with the rest. In each array, five of the shapes were similar with respect to proportional length, angle size, global shape, parallelism and alignment, or left/right symmetry, whereas one shape differed on that property (Fig. 1*B*). Trials were randomly intermixed during the testing session. After a child indicated his or her choice on the screen, an experimenter, who was unaware of the child's performance on the other tasks, pressed the associated key on the keyboard and the response was recorded by the presentation software. Although children were not given instructive feedback during the testing session, they did complete and receive feedback on two practice trials, using figures that did not differ on the target properties, to ensure that the task was understood. A summary variable of children's scores on the visual form analysis task was calculated based on an average of the 11 above-chance trials.

Map-Based Navigation. For each map task, the child stood in the center of a triangular array composed of sides (distance task) or corners (angle task) while the experimenter showed him or her a continuous triangle depicting the shape of the array from an overhead view (Fig. 2*A*). Because the same complete shape was depicted for both side and corner arrays, children viewed the same maps in both tasks. All maps were presented at a constant orientation with the 30° angle at the top, but for each trial, the child faced a different direction relative to the array (0°, 60°, 120°, 180°, 240°, or 300°). A child indicated his or her choice by putting a small stuffed animal on one of six green caps located either at the corners of the triangle formed by the array or at the centers of the sides of the array. Because the arrays were fragmented, targets were located at an environmental feature, a physically present side or corner, on only three trials in each task. On the other three trials, targets were located at the gap between two environmental features (i.e., at the corner formed by the continuation of two flanking sides or at the side formed by the continuation of two flanking corners). Corner and side target locations were tested in blocks, with block order, map task order, and map orientation counterbalanced across children. Before each map task, two practice trials were presented, using color rather than geometry to specify a target location: The child had to find either a purple or pink cap in the center of the room after the experimenter pointed to either a purple or pink dot in the center of a laminated sheet of paper that depicted nothing else. Performance was assessed from an overhead video feed by observers who were unaware of the child's performance on other tasks. The proportion of correct responses was calculated separately for each of the two tasks, and a summary variable was calculated based on targets where children showed above-chance performance.

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