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Brief non-symbolic, approximate number practice enhances subsequent exact symbolic arithmetic in children

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ABSTRACT

Recent research reveals a link between individual differences in mathematics achievement and performance on tasks that activate the approximate number system (ANS): a primitive cognitive system shared by diverse animal species and by humans of all ages. Here we used a brief experimental paradigm to test one causal hypothesis suggested by this relationship: activation of the ANS may enhance children's performance of symbolic arithmetic. Over 2 experiments, children who briefly practiced tasks that engaged primitive approximate numerical quantities performed better on subsequent exact, symbolic arithmetic problems than did children given other tasks involving comparison and manipulation of non-numerical magnitudes (brightness and length). The practice effect appeared specific to mathematics, as no differences between groups were observed on a comparable sentence completion task. These results move beyond correlational research and provide evidence that the exercise of non-symbolic numerical processes can enhance children's performance of symbolic mathematics.

1. Introduction

Recent evidence suggests that symbolic mathematics arises, in part, from the reuse of a phylogenetically ancient and ontogenetically primitive cognitive system for making quantitative judgments and decisions: the approximate number system (ANS) (e.g. Dehaene, 2005; Hubbard et al., 2008). To date, however, most of the evidence suggesting a role for the ANS in symbolic mathematics is indirect, and the mechanism(s) driving this relationship are not well understood (e.g. Bugden & Ansari, 2011; Gilmore, McCarthy, & Spelke, 2010; Halberda, Mazzocco, & Feigenson, 2008; Libertus, Feigenson, & Halberda, 2011; Libertus, Odic, & Halberda, 2012; Halberda, Ly, Wilmer, Naiman, & Germine, 2012; Holloway & Ansari, 2009; Lourenco, Bonny, Fernandez, & Rao, 2012; but see Lyons & Beilock, 2011; Price, Palmer, Battista, & Ansari, 2012, and Sasanguie, Defever, Maertens, & Reynvoet, in press). We used an experimental procedure to test the extent to which engaging the ANS causally enhances subsequent symbolic arithmetic performance in children learning symbolic mathematics in school. By systematically manipulating the content of experimental tasks and analyzing the resulting effects, we also begin to clarify the specificity of the relationship between the ANS and symbolic mathematics.

1.1. Primitive number representations

A wealth of research reveals that even infants can discriminate between arrays of visual elements on the basis of number (e.g. Brannon, 2002; Xu, 2003; Xu & Spelke, 2000; Xu, Spelke, & Goddard, 2005). This ability is present from birth, persists over the lifespan, and is common to a wide variety of non-human animals (Feigenson, Dehaene,

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& Spelke, 2004; Izard, Sann, Spelke, & Steri, 2009). Studies in infants, preschool children, and non-human primates reveal that the ANS supports computations as diverse as numerical discrimination, ordinal comparison, addition, and subtraction (Brannon & Terrace, 1998; Cantlon & Brannon, 2006a, 2006b, 2007; Gilmore et al., 2010; McCrink & Wynn, 2004). Nevertheless, the ANS represents number imprecisely: Precision in the mental representations of number decreases as number increases, and comparison of two numbers is possible only when they differ by a sufficient ratio (Halberda et al., 2008). The signature ratio-dependent imprecision of the ANS stands in stark contrast to the exact meaning and precision associated with the symbolic number system that is acquired in early childhood and is used to learn and perform higher symbolic mathematical computations (for reviews see Carey, 2009; Le Corre & Carey, 2007; Le Corre, Van de Walle, Brannon, & Carey, 2006).

1.2. Links between the ANS and symbolic mathematics

Despite the differences between the approximate number system and later acquired symbolic numbers and mathematics, three lines of evidence suggest a functional link between them. First, tasks involving purely symbolic numbers and exact arithmetic reveal signatures of non-symbolic, approximate number representations (see Piazza, 2010 for a review). For example, when adults or older children are asked to determine which of two symbolic numbers is larger, their performance depends on the numerical distance between the numbers to be compared (e.g. Deheane & Akhavein, 1995; Deheane, Deheane-Lambertz, & Cohen, 1998; Moyer & Landauer, 1967; Temple & Posner, 1998). Similarly, speed of processing a symbolic number depends on its numerical distance from a covertly presented, antecedent numerical prime (e.g. Van Opstal, Gevers, De Moor, & Verguts, 2008). Finally, in adults and older children, overlapping parietal brain regions are activated during processing of number in both symbolic and non-symbolic number formats, and these regions show similar release from adaptation to numerical changes independent of the format of presentation (symbolic or non-symbolic) (see Piazza, 2010; Piazza, Pinel, Bihan, & Deheane, 2007 or Dehaene, Piazza, Pinel, & Cohen, 2003 for reviews).

Second, individual differences in ANS acuity correlate with mathematics achievement scores (e.g. Bugden & Ansari, 2011; DeWind & Brannon, 2012; Halberda et al., 2008; Libertus et al., 2011, 2012; Bugden & Ansari, 2011; Gilmore et al., 2010; Halberda et al., 2012; Lourenco et al., 2012; but see Lyons & Beilock, 2011). Several studies show concurrent or retrospective correlations between ANS acuity and mathematics achievement scores (e.g. Halberda et al., 2008; Libertus et al., 2011, 2012; Lourenco et al., 2012). For example, individual differences in the acuity of approximate, non-symbolic number comparisons, tested at 14 years, were significantly associated with past mathematics achievement scores as far back as kindergarten (Halberda et al., 2008). In these correlational studies, it is unclear whether individual differences in ANS acuity play a causal role in creating individual differences in mathematics development, whether symbolic mathematics development causes changes in ANS acuity (e.g. Piazza, Pica, Izard, Spelke, & Dehaene, 2013), or whether a third, mediating factor, such as differences in the facility of operations on number symbols (e.g. Lyons & Beilock, 2011) or differences in aspects of executive function (e.g., Fuhs & McNeil, 2013; Gilmore et al., 2013) explain the relationship. Other studies show that individual differences in ANS acuity predict future mathematics achievement even after controlling for variables like general intelligence, verbal abilities, age (e.g. Gilmore et al., 2010; Libertus, Feigenson, & Halberda, 2013; Mazzocco, Feigenson, & Halberda, 2011), and even when non-symbolic numerical processing is measured in infancy (Starr, Libertus, & Brannon, in press). These studies, however, do not show that individual differences in ANS acuity cause the later changes in mathematics performance, because both the earlier differences in ANS acuity and the later differences in school mathematics learning could depend on one or more additional common factors.

Third, recent work suggests that practice with or training of the ANS, either alone or together with training of symbolic numbers, leads to gains in symbolic mathematics performance (Park & Brannon, 2013; Räsänen, Salminen, Wilson, Aunio, & Dehaene, 2009; Wilson, Dehaene, Dubois, & Fayol, 2009; Wilson, Dehaene, et al., 2006; Wilson, Revkin, Cohen, Cohen, & Dehaene, 2006). One line of work showed that children who practiced a variety of symbolic number skills related to the ANS, including games involving approximate numerical comparisons, verbal counting, and mapping numbers to space, showed improvement on symbolic number tasks (Räsänen et al., 2009; Wilson, Dehaene, et al., 2006; Wilson, Revkin, et al., 2006; Wilson et al., 2009). From this work, however, it is unclear which aspects of the training – targeted practice with the ANS, explicit practice mapping the ANS to symbols, symbolic number practice alone, or something else – contributed to the observed gains. More recently, Park and Brannon (2013) showed that several days of training on a non-symbolic approximate numerical addition task led to improvements in ANS acuity and symbolic mathematics performance in adults. Individual differences in ANS acuity change, although modest, correlated with individual differences in change on the symbolic arithmetic measures. Similar improvements were not seen in control groups with no training task, in a non-numerical, factual knowledge-training task, or in adults who practiced a symbolic number ordering task. These results provide the strongest evidence to date of a causal and specialized relationship between the ANS and symbolic mathematics. However, it is unclear whether such training depends on a mature mapping between the symbolic number system and the ANS or whether such training would also improve symbolic mathematics in children who are still acquiring mathematics skill and ANS precision. It is also unclear whether engagement of the ANS, the cognitive operations involved (including comparison and addition), magnitude representations in general, or something else contributed to the improvements in symbolic arithmetic.

1.3. Theories of the relationship between the ANS and mathematics

Several theories have been proposed to explain the link between the ANS and symbolic mathematics. One view is that symbolic mathematics depends specifically on the ANS (e.g. Barth, Beckmann, & Spelke, 2008; Barth, La Mont, Lipton, & Spelke, 2005; Barth et al., 2006; Dehaene, 1997; Gilmore et al., 2010; Nieder & Dehaene, 2009; Park & Brannon, 2013). In addition to the correlational studies and training studies cited above, further research consistent with this position comes from neuropsychological and trans-cranial magnetic stimulation research showing that damage or impairment of parietal brain regions thought to underlie the ANS alters the ability to performance symbolic numerical computations (e.g. Cappelletti, Barth, Fregni, Spelke, & Pascual-Leone, 2007; see Dehaene et al., 2003 for a review). Similarly, individuals with dyscalculia, a mathematics-specific learning disability, also show poor ANS acuity (e.g. Butterworth, 2010; Piazza et al., 2010; Price, Holloway, Vesterinen, Rasanen, & Ansari, 2007).

Alternatively, the relationship between performance on tasks involving the ANS and on tests of symbolic mathematics may reflect a broader underlying relationship between symbolic mathematics and magnitude representations (see Lourenco et al., 2012). On this view, a generalized magnitude system underlies the representation of all magnitudes regardless of dimension (physical size, number, duration, etc.) (for reviews see Walsh, 2003 or Lourenco & Longo, 2011). The hypothesis of a generalized magnitude system is supported by evidence showing overlap at the behavioral, cortical, and neuronal level between magnitude domains (e.g. Fias, Lammertyn, Reynvoet, Dupont, & Orban, 2003; Henik & Tzelgov, 1982; Lourenco & Longo, 2010, 2011; Tudusciuc & Nieder, 2007). Thus, individual differences in the generalized magnitude system (which includes number), rather than the ANS specifically, may be linked with individual differences in symbolic mathematics. Some evidence for this position comes from research with children showing that spatial magnitudes promote earlier understanding of higher numerical concepts (e.g. Mix, Levine, & Huttenlocher, 1999; Gunderson, Ramirez, Beilock, & Levine, 2012). Other evidence with adults shows individual differences in both discrimination of spatial extent and discrimination of number correlate with higher mathematics performance (Lourenco et al., 2012). However, further analysis of these results revealed that differences in spatial discrimination were uniquely associated with performance in the domain of geometry, whereas differences in numerical discrimination were uniquely associated with performance of symbolic arithmetic, suggesting a more specific role for the ANS in mathematical reasoning (Lourenco et al., 2012).

On a third family of views, the relationship between the symbolic and non-symbolic number is mediated by other general cognitive operations or abilities common to both tasks (see Fuhs & McNeil, 2013; Gilmore et al., 2013; Holloway & Ansari, 2008; Lyons & Beilock, 2009, 2011). Several recent studies, for example, provide evidence that the relationship between number comparison and mathematics achievement could be explained by variation in general inhibitory ability, rather than ANS acuity (Fuhs & McNeil, 2013; Gilmore et al., 2013). Other studies have found that domain-general cognitive operations, like the ability to compare one quantity to another, account for a significant portion of individual variation on non-symbolic number tasks (Holloway & Ansari, 2008). In one study, for example, the relationship between performance on a symbolic and a non-symbolic numerical task was mediated by symbol-ordering operations (Lyons & Beilock, 2009). These studies suggest that the relationship between the ANS and mathematics may be mediated by more general-purpose cognitive operations, such as ordering, comparison, or addition, common to both symbolic and non-symbolic tasks, or more domain general cognitive abilities such as inhibitory or executive control.

In sum, previous work shows clear correlations between performance on tasks that involve the ANS and symbolic mathematics performance (e.g. Gilmore et al., 2010; Halberda et al., 2008; Libertus et al., 2011; Lourenco et al., 2012) and some evidence of a causal relationship between ANS training and symbolic mathematics performance in adults (Park & Brannon, 2013). However, the mechanisms responsible for this relationship remain unclear and are highly debated. Furthermore, it is unclear from previous research if symbolic mathematics is dependent on the ANS in children, without years of associations between the symbolic and non-symbolic systems. We addressed these questions by assigning children to participate in one of several training conditions, each aimed at engaging a particular mechanism hypothesized to explain the relationship between the ANS and mathematics, and then subsequently tested the groups on exact, symbolic arithmetic performance. If the ANS contributes to the cognitive mechanisms responsible for symbolic arithmetic in children, then engaging the ANS may enhance children's subsequent symbolic arithmetic performance.

2. Experiment 1

2.1. Experiment overview

Children participated in one of four training conditions: a non-symbolic numerical addition task, a line-length (or area) addition task, a non-symbolic number comparison task, or a brightness comparison task (see Fig. 1). Each condition targeted the engagement of a particular non-symbolic magnitude skill hypothesized to play a role in symbolic mathematics. In all these conditions, children practiced adding or comparing approximate, non-symbolic magnitudes. During and immediately after the training task, children were asked to complete a symbolic addition test worksheet to assess the effects of the training task on the speed and accuracy of symbolic mathematics. Finally, at the end of the experiment, children's approximate numerical acuity was measured (Halberda et al., 2008).

2.1.1. Non-symbolic numerical addition

One condition involved numerical addition of non-symbolic dot arrays (see Barth et al., 2005, 2006; Gilmore et al., 2010). In this condition, children were asked to estimate

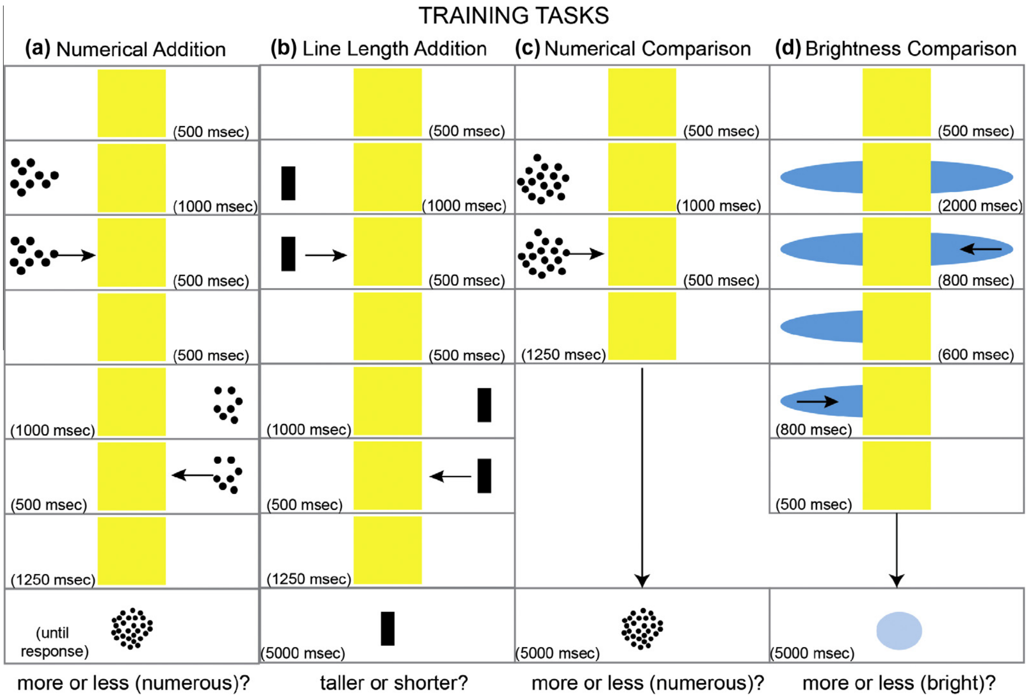

Fig. 1. Schematic depiction of training tasks. Stimulus events are organized horizontally from start (top) to finish (bottom). The numbers indicate the duration of presentation. The horizontal arrows indicate stimulus movement. The vertical arrows indicate the following event.

the numerical sum of two sequentially presented arrays of dots (addends) and judge whether an outcome array was more or less numerous than the actual sum (see Fig. 1a). Previous research has shown that performance on this task correlates with mathematics achievement scores in young elementary school children (see Gilmore et al., 2010). Furthermore, a recent training experiment with adults showed that practice with this task improved symbolic arithmetic (Park & Brannon, 2013). In addition to requiring the engagement of the ANS, this task may require transformational operations at the core of symbolic arithmetic concepts, making it an ideal task to engage cognitive mechanisms in common with those used for symbolic mathematics (Barth et al., 2005; Gilmore et al., 2010). If the ANS and/or the arithmetic operations involved in non-symbolic addition overlap with those used in symbolic arithmetic, then we might observe enhanced performance on symbolic addition in children who first practice non-symbolic addition compared to children who practice tasks involving other quantities or other operations.

2.1.2. Line length addition

The second condition involved addition of line lengths (i.e. spatial extent). This condition was equal to the non-symbolic numerical condition in terms of timing, difficulty, and cognitive demands, but involved the addition of spatial magnitudes rather than numerical magnitudes (see Fig. 1b). This condition was motivated by the generalized magnitude system hypothesis (Lourenco & Longo, 2011; Walsh, 2003), as well as by recent findings of a relationship between spatial magnitude representation and achievement in mathematics (Lourenco et al., 2012). If generalized magnitude representations drive the link between symbolic mathematics and performance on tasks involving the ANS, then practice adding lines (non-symbolic addition of lengths) may enhance subsequent symbolic arithmetic as much as practice adding arrays of dots (non-symbolic addition of numbers).

2.1.3. Non-symbolic numerical comparison

A third condition involved approximate, non-symbolic numerical comparison. In this condition, subjects saw two sequentially presented arrays of dots and had to judge whether the second array was more or less numerous than the first (see Fig. 1c). As reviewed above, emerging work suggests that the ability to compare arrays of objects on the basis of number correlates with mathematics achievement scores in a variety of contexts (e.g. Bugden & Ansari, 2011; Halberda et al., 2008; Lourenco et al., 2012). If the ANS alone plays a functional role in symbolic arithmetic, rather than co-activation of the ANS and cognitive arithmetic computations as in the non-symbolic numerical addition condition, then performance on symbolic arithmetic problems may be enhanced in children who previously engaged the ANS through comparison or addition, relative to children who receive other the non-numerical training conditions.

2.1.4. Brightness comparison

A fourth condition involved comparing the brightness magnitude of two objects (see Fig. 1d). Cognitive and neural overlap between representations of numerical magnitudes and brightness magnitudes has been highly debated (see Lourenco & Longo, 2011; Walsh, 2003). Some evidence suggests brightness to be included with space and number in the generalized magnitude system (e.g. Cohen Kadosh & Henik, 2006a, 2006b, 2006c), whereas other evidence suggests it may be distinct (e.g. Pinel, Piazza, LeBihan, & Dehaene, 2004). If previously observed associations between the ANS and symbolic mathematics development are due to commonalities in processing and comparing magnitudes in general, then no differences should be observed in symbolic arithmetic performance between the children in any of the training conditions. On the other hand, if approximate number or length representations play a functional role in symbolic arithmetic, then better performance may be seen in conditions where the ANS or length is engaged than in cases where brightness is engaged.

2.2. Material and methods

2.2.1. Participants

Participants were 96 first grade children from the greater Boston area (44 females, *M* age = 6 years 327 days, *SD* = 79 days, range: 6 years 150 days–7 years 237 days). Twenty-four children were assigned to each condition. An additional 21 children were eliminated from the study for failure to complete all the training sets and at least one test set of the experiment (16), not following directions regarding the sequence of tasks (1), or because of an experimenter error in the procedure (4). All children and their parents gave written consent before participation in the study and were offered $5 for travel reimbursement and a small appreciation gift (toy or t-shirt).

2.2.2. Displays and tasks

Training games were computer-animated, non-symbolic addition or comparison problems (Barth et al., 2005, 2006; McCrink & Wynn, 2004). All problems started with a rectangular occluder in the middle of the screen (see Fig. 1). For the non-symbolic numerical addition condition, one dot array appeared to the left of the occluder (addend 1) and moved quickly behind it, a second dot array (addend 2) appeared to the right of the occluder and moved similarly behind it, and then the occluder disappeared to reveal a collection of dots (foil) that was numerically larger or smaller than the actual sum (see Fig. 1a). Children indicated by button-press whether the test array (foil) was more or less numerous than the total number of items that had moved behind the occluder (sum). Numerical arrays were controlled for intensive and extensive parameters (see S2 for details). In addition, we implemented a number of design features shown by other researchers to discourage symbolic number strategies (Ballinger & Barth, 2007; Barth et al., 2008; Gilmore, McCarthy, & Spelke, 2007). Specifically, we used relatively large addends (7-43 dots, average 17 dots) that were unable to be enumerated exactly under the time constraints of the experiment and whose sums were unlikely to be previously memorized by the child participants (average sum/outcome = 34 dots; range for sum/outcomes = 16–56 dots) (Details regarding timing of events can be found in Fig. 1 and Supplementary materials S2).

For the line addition condition, one line segment appeared to the left of the occluder (addend 1) and moved quickly behind it, another line segment (addend 2) appeared to the right of the occluder and moved similarly behind it, and then the occluder disappeared to reveal the third line segment (test) that was taller or shorter than the sum of the first two segments (see Fig. 1b). Children indicated by button-press whether the test segment was more or less tall than the sum of the first two segments that had moved behind the occluder. The ratio of sum to foil and the timing of events were held constant with the numerical addition condition.

For the number comparison condition, one dot array appeared either to the left or right of the occluder and moved behind it. After a delay, the occluder was removed to reveal a test array (see Fig. 1c and S1 for details). Children were asked to judge whether the test array was more or less numerous than the initial array (see Fig. 1c). The numerical values for the first and second array matched those used for the sum and foil in the numerical addition condition. Timing of dot array presence, movement, and occlusion was equal to that of the numerical addition training condition.

For the brightness comparison condition, an oval-shaped form appeared behind and to the sides of the occluder, shrank to fit behind the occluder first from the left and then from the right, and then the occluder disappeared to reveal the form in the shape of a circle at a different level of brightness (see Fig. 1d). Children indicated by button-press whether the circle had increased or decreased in brightness. Trial timing and total trial duration was similar to the numerical addition and line length addition conditions.

Training tasks were conducted on a laptop computer and programmed using E-prime software (Psychological Software Tools, Pittsburgh, PA), which recorded reaction time and accuracy. The animated problems were presented in the context of game to maintain children's attention (see S2 for more details on the game context). The experimenter was not blind to condition, as she had to instruct children on the introductory trials and continually monitor progress. After each problem a "bing" sound indicated a correct answer and "bong" sound indicated an incorrect answer, the meaning of which was described in the initial practice problems.

Symbolic arithmetic test problems were presented on 4 sheets of paper and completed with a pencil. The time to complete each page of symbolic addition problems was recorded by the experimenter with a stopwatch, and accuracy was calculated after the testing session by assigning 1 point for each correct answer. Reliability of the experimenter's timing measurements were confirmed in a random sample of 16 subjects by an independent coder using offline video recordings of the sessions ($r = .99$, $p < .001$).

Finally, after training and test problems were completed, the ANS acuity of each participant was assessed by means of an approximate number comparison task using the Panamath computer game (see Halberda et al., 2008 for details). In this task, children saw a collection of yellow dots, associated with a yellow cartoon character, and a collection of blue dots, associated with a blue cartoon character, simultaneously presented on a computer screen for 2 s. Children were asked to choose the more numerous array (blue or yellow) by pressing a corresponding colored button (blue or yellow). Arrays ranged from 4–15 dots and included numerical comparisons at 6:5, 4:3, 3:2, and 2:1 ratios. Based on accuracy at each ratio, the Panamath software generated a psychophysical model of performance and an estimate of numerical acuity (a Weber fraction). Details regarding the freely available software, the task, or the calculation of a Weber fraction can be found at www.panamath.org.

2.2.3. Design and procedure

Participants were assigned quasi-randomly to experimental conditions, equating for gender and age, and, as best as possible for time of testing relative to the school year (see S2 for details). Over the experiment, children completed 2 sets of training trials (60 problems total), 4 sets of symbolic addition test problems (40 problems total), and an approximate, non-symbolic numerical acuity task (actual training and test problems appear in Supplementary materials S1).

Symbolic arithmetic problems were interleaved with the experimental task in an attempt to ensure participants gave equal care to all problems. After completing 8 practice trials, all participants were given the first 50 trials of their assigned non-symbolic training task, followed by 20 symbolic arithmetic test problems presented on two sheets of paper: 10 very easy problems on the first sheet and 10 moderately easy problems on the second sheet. After a brief break (if desired), participants received 10 more training trials, followed by 20 more exact symbolic addition problems: 10 somewhat more difficult problems on the third sheet and 10 moderately difficult problems on the last sheet (see S1 for all problems used). Finally, children completed (6 practice) and 60 trials of the test of approximate numerical acuity.

2.2.4. Analysis

ANOVAs were used to compare the different training groups on age and approximate numerical acuity. Training task performance was analyzed by separate mixed-factor ANOVAs on average reaction time and accuracy with the within-subjects factors of Ratio (2 levels), Time (first half vs. second half), and the between-subjects factor of Training Condition (4 levels: numerical addition, line addition, number comparison, brightness comparison). Test performance (speed and accuracy) was computed by averaging responses across completed test sets. A majority of children, 71, completed all four test problem sets, 16 children completed 3 out of 4 test problem sets, 7 children completed 2 out of 4 problem sets, and 2 children completed only 1 problem set. Missing problem sets appeared to be distributed equally among experimental conditions (see S2 for details). Test performance was analyzed using ANOVAs on average time to complete test sets (speed) and accuracy, with the between-subjects factor of Training Condition (4 levels). Main effects or interactions with Training Condition were followed up with post-hoc pairwise comparisons using *t*-tests.

2.3. Results

2.3.1. Participant factors

The children in the different conditions did not differ in average age ($F(3,95) = 1.697$, $p = .173$: numerical addition, $M = 6$ years, 311 days, $SD = 73$ days; line addition $M = 6$ years 311 days, $SD = 77$ days; numerical comparison $M = 6$ years 355 days, $SD = 67$ days; brightness comparison $M = 6$ years, 332 days, $SD = 94$ days) or approximate numerical acuity ($F(3,95) = 0.766$, $p = .516$: numerical addition $M = .17$, $SD = .11$; line addition $M = .21$, $SD = .12$; numerical comparison $M = .18$, $SD = .08$; brightness comparison $M = .17$, $SD = .08$).

2.3.2. Training task performance

The analysis of the average reaction time during each training task revealed main effects of Ratio ($F(1,92) = 4.197$, $p < .05$, $\eta_p^2 = .044$), Time ($F(1,92) = 19.385$, $p < .001$, $\eta_p^2 = .174$), and Experimental Condition ($F(3,92) = 7.222$, $p < .001$, $\eta_p^2 = .191$), and an interaction between Ratio and Time ($F(1,92) = 5.078$, $p < .05$, $\eta_p^2 = .052$). Regardless of condition, subjects were faster on the second half compared to the first half of the training trials ($F(1,95) = 19.297$, $p < .001$) (see Fig. 2). Further analysis of the interaction between Ratio and Time revealed ratio differences averaged across all conditions emerged only on the second half of training problems ($t(95) = -3.054$, $p < .005$), with longer average response times to problems involving close ratios compared to problems involving far ratios (see Fig. 3). Further post hoc analysis of the main effect of Training Condition on speed revealed significantly faster performance on the brightness comparison task compared to all other tasks (brightness vs. numerical addition: $t(46) = -4.750$, $p < .001$; brightness vs. line addition: $t(46) = -2.919$, $p < .01$; brightness vs. number comparison: $t(46) = -3.312$, $p < .005$) (numerical addition: $M = 1951$ ms, $SD = 284$ ms, Range = 1416–2542 ms; line addition: $M = 1826$ ms, SD 346 ms, 1140–2764 m; number comparison: $M = 1835$ ms, $SD = 294$ ms, Range = 1111–2313 ms; brightness comparison: $M = 1555$ ms, $SD = 292$, Range = 895–2040 ms) (see Fig. 2). No other significant differences were seen in speed of the different tasks (all other $ps > .17$).

On the measure of training task accuracy, the analysis revealed main effects of Ratio ($F(1,92) = 57.859$, $p < .001$, $\eta_p^2 = .386$) and Training Condition ($F(3,92) = 14.764$, $p < .001$, $\eta_p^2 = .325$) (Fig. 2). No main effects of Time or interactions between factors were observed (see Fig. 2). Participants were less accurate on problems involving closer ratios, regardless of the experimental task. In addition, post hoc pairwise comparisons of accuracy revealed that subjects in the brightness condition were more accurate than all other groups (brightness vs. numerical addition: $t(46) = 4.546$, $p < .001$; brightness vs. line addition: $t(46) = 7.530$, $p < .001$; brightness vs. number comparison: $t(46) = 5.723$, $p < .001$), and the numerical comparison group was more accurate than the line addition group (line addition vs. number comparison: $t(46) = -2.436$,

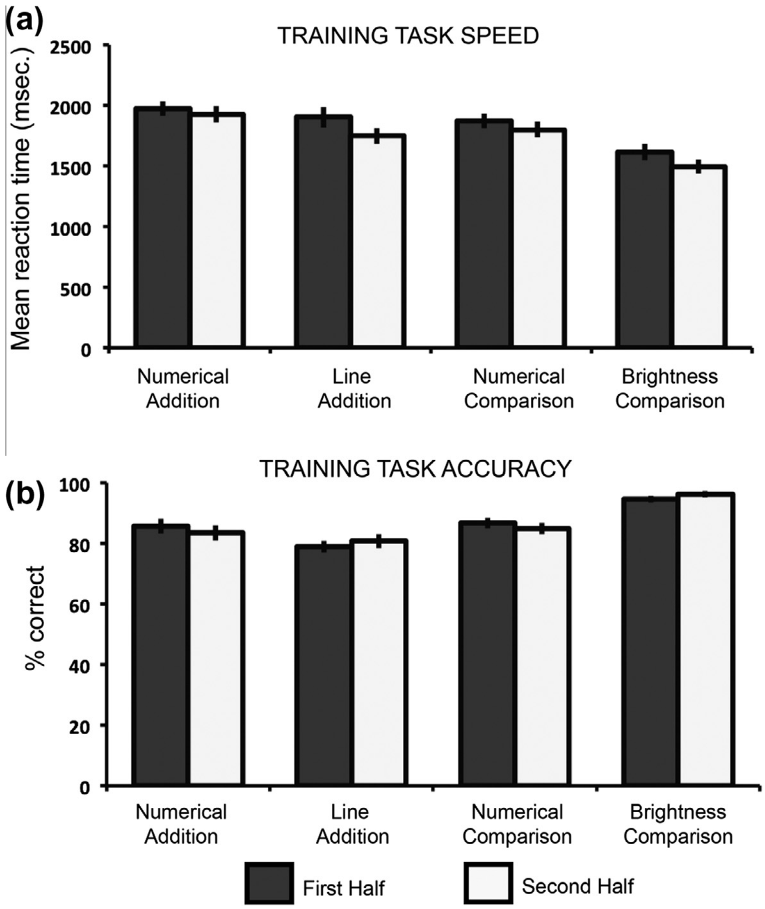

Fig. 2. Average training task performance over time in Experiment 1. (a) Average reaction time (in milliseconds) for each condition. (b) Average task accuracy (expressed as percent correct) for each condition.

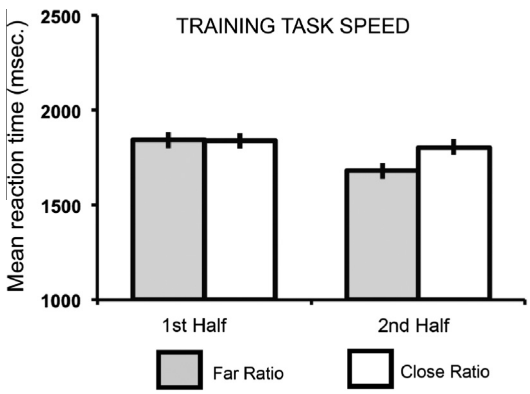

Fig. 3. Effects of ratio on average training performance over time in Experiment 1.

$p < .05$) (see Fig. 2). However, neither the line addition group nor the numerical comparison group differed significantly from the numerical addition group in accuracy (numerical addition vs. line addition: $t(46) = 1.596$, $p = .117$; numerical addition vs. numerical comparison: $t(46) = -.440$, $p = .662$).

In sum, the analysis of performance on the four tasks of numerical addition, line length addition, numerical comparison, and brightness comparison suggests that subjects improved in speed in a ratio-dependent manner over the course of each task, independent of the actual training condition. Furthermore, those completing the brightness comparison task performed better than those in the other groups: they were both faster and more accurate. On the other hand, no differences on any of the performance measures were observed between the numerical addition group and the numerical comparison group or between the numerical addition group and the line-length addition group.

2.3.3. Exact symbolic arithmetic test performance

The analysis of the average time taken by children to complete each page of the written arithmetic test problems revealed a main effect of Training Condition ($F(1,95) = 3.366$, $p < .05$) (see Fig. 4). Pairwise post hoc analysis revealed that children in the numerical addition and numerical comparison conditions completed symbolic arithmetic problems faster than children in the non-numerical conditions (numerical addition vs. brightness comparison: $t(46) = -2.176$, $p < .05$; numerical addition vs. line addition: $t(46) = -2.030$, $p < .05$; brightness vs.

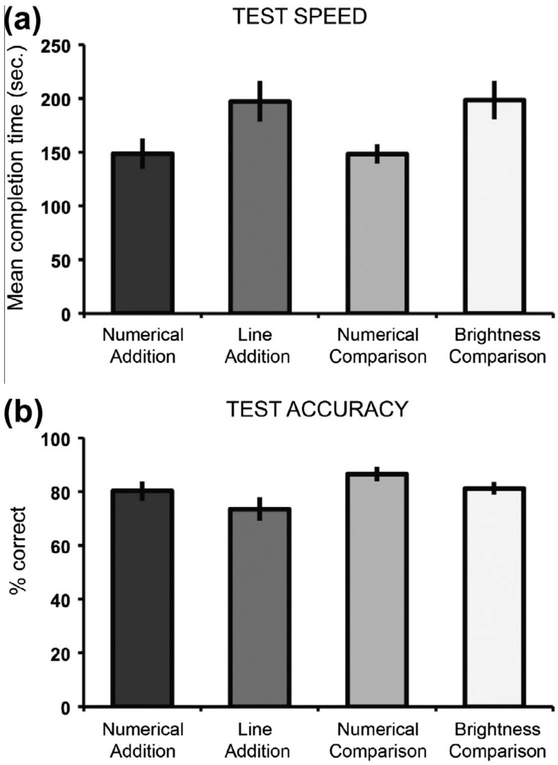

Fig. 4. Average symbolic arithmetic test performance in Experiment 1. (a) Average speed of test completion (in seconds) for each condition. (b) Average test accuracy (expressed as percent correct) for each condition.

number comparison: $t(46) = 2.527$, $p < .05$; line addition vs. number comparison: $t(46) = 2.327$, $p < .05$) (see Fig. 4). No differences in speed on symbolic arithmetic tests were observed between non-numerical conditions (brightness comparison vs. line addition: $t(46) = .049$, $p = .961$) or between numerical conditions (numerical addition vs. number comparison: $t(46) = .032$, $p = .975$) (Fig. 4).

The analysis of performance accuracy on the symbolic arithmetic test revealed a marginally significant main effect of Training Condition ($F(1,95) = 2.598$, $p = .057$). However, post hoc pairwise comparisons revealed that the only pair of groups showing a difference in accuracy was the line-length addition group and the numerical comparison group pair, with the numerical comparison group subsequently performing more accurately on the symbolic arithmetic problems ($t(46) = -2.576$, $p < .05$) (see Fig. 4).

2.3.4. Further analyses

An alternative account of the differing effects of the different training conditions on arithmetic tests appeals not to their differences in content but the extent to which they presented problems that were challenging or engaging. Two aspects of the findings presented above cast doubt on such an account. First, differences in training task performance did not consistently predict the effects of the different training conditions on subsequent test problems. For example, reaction time and accuracy on training problems were not different from each other in the numerical addition and line-length addition conditions, yet those in the numerical addition condition performed significantly faster on subsequent test problems compared to those in the line-length addition condition. Second, no differences were observed in the extent of learning on the different training tasks (i.e., the change in performance from the first half to the second half of the session), suggesting that participants where equally engaged or attentive in their given task. Nevertheless, additional analyses were undertaken to address this alternative account further. We tested for the practice effect after controlling for effects of training task reaction time and accuracy. The critical main effect of Training Condition on speed remained significant even after effects of training task reaction time ($F(3,91) = 8.680$, $p < .001$) and accuracy ($F(3,91) = 4.285$, $p < .01$) were accounted for as covariates. Thus, Training Condition had an effect on the time it took participants to complete exact symbolic addition problems that cannot be explained by differences in performance, attention to, or engagement with the different training tasks.

2.4. Discussion

The findings of Experiment 1 provide evidence that the ANS plays a functional role in symbolic arithmetic. Children who practiced either a non-symbolic approximate numerical comparison or numerical addition task were faster to complete subsequent exact, symbolic addition test problems than were children who performed comparable tasks involving non-numerical magnitudes (length, brightness). While one of these training tasks was easier than the others (brightness comparison), our results do not appear to be due to differences in the general difficulty of the training tasks in which the different groups of children engaged, because differential test performance was seen between numerical and non-numerical tasks of equal difficulty (e.g. line-length addition and numerical addition), and because entering performance on the four training tasks as a covariate over all tasks did not eliminate the critical main effect of training condition. Our results also do not appear to depend on differential levels of learning during the training phase, as we observed that participants improved in speed over time on the initial experimental task regardless of condition.

We also observed two established signatures of the ANS in performance on the two training tasks involving numerical magnitudes. First, reaction time was a function of the ratio between the two numbers to be compared (sum vs. foil or first array vs. second array) (Barth et al., 2005, 2008; Izard & Dehaene, 2008; Pica, Lemer, Izard, & Dehaene, 2004). Second, no significant differences were observed in performance between the numerical comparison and the numerical addition tasks (Barth et al., 2006; Gilmore et al., 2007). These results provide strong evidence that subjects used the ANS to solve experimental tasks involving non-symbolic numerical magnitude.

Our experimental design and analyses provide evidence against several alternative hypotheses related to the relationship between the ANS and symbolic arithmetic. First, is does not appear from our data that a generalized magnitude system (Walsh, 2003), rather than a number-specific system (Dehaene, 1997), explains the relationship between the ANS and symbolic arithmetic (Lourenco et al., 2012), as the experimental conditions that involved non-numerical magnitudes did not lead to better subsequent performance compared to the experimental conditions involving non-symbolic numerical magnitudes. Second, it does not appear that common cognitive operations inherent in symbolic and non-symbolic tasks (Holloway & Ansari, 2008; Lyons & Beilock, 2009), rather than the ANS in particular, are responsible for correlations between the ANS and symbolic mathematics, as participants showed enhanced performance on symbolic arithmetic after practicing comparison or addition of numerical magnitudes but not after practicing tasks involving the same cognitive operations (ordering, comparison, and/or addition) over non-numerical magnitudes. In a similar vein, a deflationary account that our results can be explained as an easier arithmetic exercise "warming-up" or priming more difficult symbolic arithmetic (e.g. Fuchs et al., 2013) does not hold, as practicing non-symbolic numerical comparison worked equally as well as practicing addition to improve subsequent symbolic arithmetic.

Our findings also provide some evidence against the claim that the inhibitory demands of tasks involving the ANS drive correlations with symbolic mathematics (Fuhs & McNeil, 2013; Gilmore et al., 2013). It is possible, as some have argued, that non-symbolic numerical tasks engage executive function (EF) to a greater extent than do non-symbolic spatial or brightness tasks, because they require children to inhibit responses to continuous variables that are anti-correlated with number on some trials in order to respond correctly. Under this view, greater commonalities in EF engagement between the numerical training tasks and the symbolic arithmetic test, rather than specific overlap in the ANS and symbolic mathematics, might explain better subsequent symbolic arithmetic performance in the numerical training groups compared to the non-numerical training groups. For several reasons, this is not likely the case in our dataset. First, unlike previous studies reporting that the relationship between the ANS and symbolic mathematics is mediated by inhibitory control (Fuhs & McNeil, 2013; Gilmore et al., 2013), we used stimulus controls where continuous properties could not be reliably used to solve the tasks because they were not systematically related to the answer. The non-numerical continuous properties of each numerical array within each trial and between trials in our study were randomly chosen, in contrast to previous work where non-numerical properties of each numerical arrays within a problem were reliability and systematically related to the answer on a given trial (either all positively or all negatively correlated with number, although the direction of the relationship was manipulated across problems). Second, if the numerical tasks required substantially more inhibitory processes than other non-numerical tasks, this would likely be reflected in behavioral performance. However, the approximate numerical addition task was no harder than the line addition task, suggesting no substantial differences in the inhibitory control required, yet significant differences were observed in subsequent symbolic addition test performance. Third, exercising executive function appears to deplete rather than enhance performance on subsequent tasks also involving EF (Baumeister, Bratslavsky, Muraven, & Tice, 1998; Hagger, Wood, Stiff, & Chatzisarantis, 2010; Hofmann, Schmeichel, & Baddeley, 2012; Powell & Carey, in preparation; Schmeichel, 2007). Given the temporal structure of our experiment, with ANS training and symbolic mathematics testing occurring in immediate succession, a common role for EF in both tasks would be predicted to lead to impairment rather than to enhancement of symbolic arithmetic performance.

Some may also argue that visuo-spatial working memory is differentially engaged between numerical and non-numerical training tasks and could mediate the observed relationship between approximate numerical training tasks and symbolic math performance. Most of the arguments provided against the idea of inhibitory control mediating the effect, apply equally well against a differential working memory account. Specifically, substantial differences in working memory between training conditions should have been evident in training task performance, but equal performance was observed between the numerical conditions and the non-numerical line length addition condition, for example. Also, contrary to the obtained test results, it is likely that a training task that taxed the working memory system would lead to worse rather than better performance on a subsequent task. Finally, the numerical addition task clearly should tax working memory more than the numerical comparison task, yet these two tasks had equal effects on children's subsequent symbolic arithmetic performance. Nevertheless, further research should investigate the role of EF and working memory more directly in children's ANS practice and symbolic arithmetic performance.

Finally, our results run contrary to the suggestion that non-symbolic numerical addition is a better task for improving symbolic mathematics than numerical comparison (Gilmore et al., 2010; Park & Brannon, 2013), at least under conditions of brief exercise and immediate testing. In our experiment, practice of numerical comparison and numerical addition produced similar effects.

The scope of the observed practice effect, however, remains unclear. One possibility is that the practice effect is specific to the domain of number or mathematics. Alternatively, engaging the ANS may have more general effects on motivation, reasoning, or cognition that would translate to an entirely different cognitive task outside the domain of number or magnitude. In a second experiment, we tested this hypothesis by extending the rationale and method of Experiment 1 to include a cognitive test in the domain of reading.

3. Experiment 2

To investigate whether engagement of the ANS enhances subsequent cognitive performance more generally, we compared the effects of one numerical and one non-numerical training task from Experiment 1 on children's performance within and outside the domain of mathematics. Specifically, we developed a new test of exact symbolic addition and a reading test involving sentence completion. Like the addition test, the reading test was presented on paper and required that children evaluate a statement and write in a missing item to complete the statement. In contrast to the addition test, the statement consisted not of a mathematical equation but of a sentence, and the item to be supplied was not a number (to be written in Arabic notation) but a word. Crucially, the sentence completion task did not contain magnitude judgments or operations thought common to symbolic arithmetic. If the number practice effect observed in the above studies is specific to mathematics, then exercise of the non-symbolic approximate addition task should enhance performance on the symbolic arithmetic problems but not on the sentence completion problems. On the other hand, if more general motivational or cognitive factors explain the effect observed in Experiment 1, then improved performance might be observed on both the symbolic mathematics problems and the sentence completion problems in the group of children who practiced numerical addition, relative to children who practiced brightness comparison.

A further motivation for Experiment 2 was to investigate whether practicing non-symbolic numerical addition could yield benefits in accuracy as well as speed. In Experiment 1, we presented children with relatively easy symbolic addition problems that generated little variability in accuracy. In Experiment 2, in contrast, we presented children with more difficult arithmetic problems in an attempt to generate more variability in accuracy. We reasoned that if practicing non-symbolic addition can enhance accuracy as well as speed, then these changes in method might lead to an effect on accuracy in addition to, or instead of, the effect on speed.

3.1. Materials and methods

3.1.1. Participants

Forty-eight first grade children (24 females, *M* age = 7 - years 200 days, *SD* = 93 days) were included in the final dataset. An additional 12 children participated in the study but were excluded from analysis for not completing the training session (7), reported developmental/language delays (2), not being a native English speaker (1), taking an extremely long time to complete the study (1), and because age did not allow appropriate counterbalancing between groups (1).

3.1.2. Procedure and design

The interleaved experimental-test procedure was modified from that of Experiment 1 to obtain a more consistent amount of test data across all subjects. Over the course of the entire experiment, children completed 60 training problems, 20 symbolic arithmetic problems, and 20 sentence completion problems in an interleaved fashion. Specifically, each participant performed 24 trials of their assigned training task (either non-symbolic approximate addition or brightness comparison) and was then given 10 sentence completion problems or 10 exact, symbolic addition problems. Children then received 12 more trials of their assigned training task followed by 10 more symbolic addition or 10 more sentence completion problems (see S1 for actual problems). The same procedure (12 training problems followed by 10 test problems) was repeated twice more except those who completed mathematics problems during the first half were given sentence completion problems during this second block of testing and visa versa. The order of symbolic arithmetic and sentence test problems was counterbalanced across children and gender. With these changes, all children who completed the training trials (requisite for inclusion of data in Experiment 1) also completed all the test trials. Thus, there were no missing test data in this experiment.

3.1.3. Displays and tasks

Non-symbolic training problems were a randomly chosen subset of those used in Experiment 1 for the non-symbolic numerical addition and brightness comparison conditions, and included equal numbers of close and far ratios (see Fig. 1 and S1). The exact, symbolic addition test items consisted of new and old problems from the previous experiments (see S1). Critically, we only reused test problems from previous test sets that were challenging to children, as evidenced by their error rates: only problems from Experiment 1 that had been incorrectly answered by at least one child were included. New problems were created to be equal or more difficult than the old problems. Sentence completion problems (see S1) were developed from basic vocabulary word lists for 1st–4th grade. Each sentence included a blank with the first letter of a word that would serve to form a meaningful, complete sentence. Children's task was to use the context of the sentence so as to supply a word, beginning with the given first letter, which created a meaningful sentence. Correctly answered blanks filled with a vocabulary word that created a meaningful sentence were scored as 1 (see S2 for accuracy scoring details). Scoring procedures were the same as those in Experiment 1 for non-symbolic training and symbolic test problems. Reliability between the original experimenter's speed measurements on the test sets and an independent coder, calculated for 8 randomly chosen subjects, was high ($r = .999$, $p < .001$). Inferential analyses were the same as in Experiment 1 except that the factor of Experimental Condition only included two levels (non-symbolic numerical addition/brightness comparison) and the analysis of symbolic test performance included an additional within-subjects factor of Test Type (symbolic addition vs. sentence completion).

3.2. Results

3.2.1. Participant characteristics

Children in the numerical addition and brightness comparison conditions did not differ in mean age (non-symbolic addition: *M* age = 7 years 204 days, *SD* = 88 days; brightness: *M* age = 7 years 196 days, *SD* = 100 days) ($F(1,47) = .089$, $p = .766$) or in approximate number acuity (non-symbolic addition: $M = .18$, $SD = .07$; brightness comparison task: $M = .17$, $SD = .\ 07$) ($F(1,46) = .092$, $p = .763$).

3.2.2. Training task performance

Main effects of Ratio ($F(1,46) = 27.395$, $p < .001$, $\eta_p^2 = .373$), Time ($F(1,46) = 39.263$, $p < .001$, $\eta_p^2 = .460$), and Training Condition ($F(1,46) = 42.916$, $p < .001$, $\eta_p^2 = .483$), and an interaction between Time and Training Condition ($F(1,46) = 16.892$, $p < .001$, $\eta_p^2 = .269$) were observed on average reaction time (numerical addition: $M = 1883$ ms, $SD = 332$ ms, Range = 1278–2516 ms; brightness comparison: $M = 1318$ ms, $SD = 261$ ms, Range = 861–1913 ms). Participants were slower at answering problems involving close ratios. Participants in the brightness training condition responded faster than those in the numerical addition condition. Post hoc analysis revealed that the interaction between Training Condition and Time resulted from a larger difference between the two training conditions during the first half of the training trials ($F(1,46) = 72.811$, $p < .001$) than during the second half of the training trials ($F(1,46) = 13.452$, $p < .005$) (see Fig. 5).

Main effects of Ratio ($F(1,46) = 58.255$, $p < .001$, $\eta_p^2 = .559$), Time ($F(1,46) = 9.559$, $p < .005$, $\eta_p^2 = .172$), Training Condition ($F(1,46) = 40.443$, $p < .001$, $\eta_p^2 = .468$), and an interaction between Ratio and Training Condition ($F(1,46) = 6.915$, $p < .05$, $\eta_p^2 = .131$) on accuracy were observed (see Fig. 5). Subjects were more accurate on the second half of problems compared to the first half of problems regardless of task, suggesting a general effect of practice in both conditions (Fig. 5). Post hoc analysis revealed that participants in the brightness condition were more accurate than those in the non-symbolic numerical addition condition, participants were less accurate on closer ratio problems regardless of condition, and the interaction resulted from a larger difference between training conditions on the harder ratio problems ($F(1,46) = 37.318$, $p < .001$) compared to the easier ratio problems ($F(1,46) = 19.332$, $p < .001$).

3.2.3. Symbolic addition and sentence co0mpletion test performance

The analysis of test performance speed revealed a main effect of Test Type ($F(1,46) = 4.269$, $p < .05$, $\eta_p^2 = .085$) (see Fig. 6). Sentence completion problem sets were completed faster than symbolic addition problem sets. There was no main effect of Training Condition on the speed of children's performance. The analysis of test performance accuracy revealed main effects of Training Condition ($F(1,46) = 4.840$, $p < .05$, $\eta_p^2 = .095$), and an interaction between Test Type and Training Condition ($F(1,46) = 5.234$, $p < .05$, $\eta_p^2 = .102$) (see Fig. 6). Post hoc independent samples *t*-tests revealed that the children who received the non-symbolic, numerical addition task were more accurate on symbolic mathematics problems than those who received the brightness comparison task ($t(46) = -2.814$, $p < .01$), whereas there was no difference between the two groups on the test of sentence completion test problems ($t(46) = -.480$, $p = .633$) (see Fig. 6). Additional analyses revealed test order had no main effect or interaction with Test Type or Training Condition on accuracy (all $ps > .39$) (see S2 for analysis details).

3.2.4. Further analyses

An alternative account of the finding that children performed more accurately on the symbolic mathematics test after practicing non-symbolic numerical addition (Experiments 1 and 2) is that participants were engaging symbolic number representations jointly with ANS representations in the numerical addition training task. Thus, the symbolic number representations may have primed symbolic arithmetic, and the role of the ANS representations may simply have been to activate number symbols. If this account were correct, then one would expect that direct presentation of symbolic numbers also would enhance subsequent symbolic addition performance. This prediction can be tested by comparing children's performance on the second set of symbolic addition test problems, relative to the first set of problems, in the children who were given the non-numerical training task of brightness comparison. Accordingly, we compared the performance of children on the two sets of numerical addition problems in an analysis with Test Set (1st or 2nd) and Training Condition (Numerical Addition vs. Brightness Comparison). This analysis revealed a main effect of Test Set on speed and accuracy but no interactions with Training Condition (speed: Test Set = $F(1,46) = 7.427$, $p < .01$; Test Set × Training Condition = $F(1,46) = .016$, $p > .90$) (accuracy: Test Set = $F(1,46) = 21.454$, $p < .001$; Test Set × Training

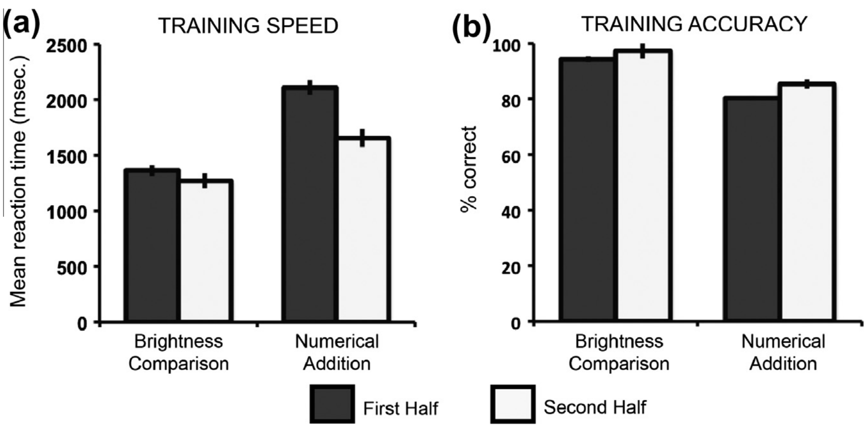

Fig. 5. Average training task performance over time in Experiment 2. (a) Average reaction time (in milliseconds) for each condition. (b) Average accuracy (expressed as percent correct) for each condition.

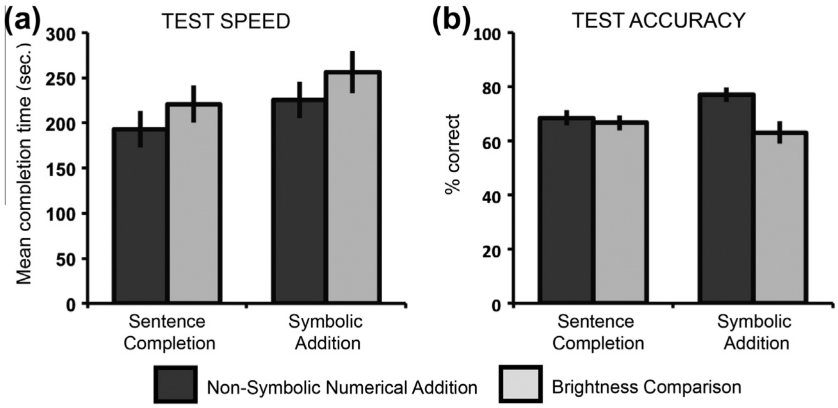

Fig. 6. Average arithmetic and sentence completion test performance in Experiment 2. (a) Average speed (in seconds) on each test type for each condition. (b) Average accuracy (expressed as percent correct) on each test type or each condition.

Condition = $F(1,46) = .265$, $p > .60$). Examination of the means for each test set for each training condition revealed that performance declined rather than improved during the second set of test problems (see Fig. 7), suggesting that engaging symbolic arithmetic did not improve subsequent symbolic arithmetic under the brief parameters of our experiment. These findings also suggest that the engagement of symbolic number representations does not likely explain advantages in the training conditions involving numerical magnitudes, relative to those involving non-numerical magnitudes. If practicing the ANS task had enhanced children's symbolic arithmetic performance because it led to activation of numerical symbols, then practicing the symbolic addition task also should have produced such an enhancement.

3.3. Discussion

Children who first practiced a non-symbolic approximate addition task subsequently performed more accurately on exact, symbolic addition problems than did children who practiced a control task involving brightness magnitude comparison. Their greater accuracy was achieved with no loss in speed. The benefits of ANS engagement were limited to performance on problems in the domain of mathematics, as children trained on non-symbolic addition performed more accurately only on the test of exact, symbolic addition, not the sentence completion test. Thus, the observed effects are likely explained by a specialized relationship between the ANS and symbolic mathematics, rather than by mediating factors such as effects of practice on children's general motivation or cognitive engagement, as such mechanisms would likely generalize to enhanced performance on cognitive tasks more broadly (including the sentence completion task). Finally, it appears that simply activating symbolic number representations in our brief paradigm is not sufficient to prime better performance on subsequent symbolic addition, as the presentation of symbolic numbers on the first symbolic addition test led to no enhancement of performance on the second symbolic addition test. These findings suggest that the present effects of the ANS on symbolic arithmetic do not simply depend on co-activation of symbolic number representations.

4. General discussion

Two experiments provide evidence that brief practice on a non-symbolic approximate numerical task enhances the performance of 6–7 year old children on a subsequent

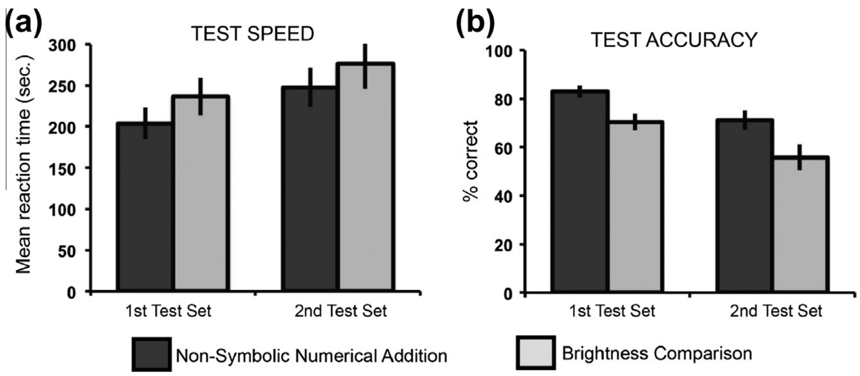

Fig. 7. Average symbolic arithmetic test performance over time in Experiment 2. (a) Average speed (expressed in seconds) on each test set for each condition. (b) Average accuracy (expressed as percent correct) on each test set for each condition.

test of exact, symbolic arithmetic. The pattern of data obtained across the different conditions indicates that these results are not due to engagement of a generalized magnitude system, engagement of common cognitive operations (such as comparison or addition), or difficulty differences between the training tasks. Rather, our data provide evidence that symbolic arithmetic draws on at least some overlapping cognitive and/or neural structures used to represent approximate number. The pattern of data obtained across two different test conditions in Experiment 2, indicates that the enhancing effects of approximate number representations are limited to the domain of symbolic mathematics or number, as comparable enhancements were not observed in children's performance of the sentence completion task. This dissociation also provides evidence that participants who practiced approximate number tasks were not simply more motivated, focused, or engaged than those assigned to a training task involving other quantities or variables, and that numerical comparisons did not prime general cognitive abilities to a greater extent than did other tasks.

Our data also argue against symbolic number representations underlying the observed effect. First, based on observed performance, it appears that children used the ANS to solve the non-symbolic addition and comparison training tasks. This claim is supported by evidence of two well-established signatures of the ANS in our data: the ratio effect and the equality of comparison and addition performance (Barth et al., 2005, 2008; Gilmore et al., 2007; Izard & Dehaene, 2008; Pica et al., 2004). Children were slower and less accurate on problems where the actual answer and outcome were closer in ratio compared to problems where the ratio between answer and outcome were more distant. Children also showed equal performance on numerical comparison and addition. In contrast, if exact symbolic comparison and addition strategies had been used, numerical comparison should have been easier than numerical addition, as the comparison involves only two numbers, not combining two numbers to compare to a third. Moreover, no children were noted to have used verbal counting or called out verbal numbers during the task; if such strategies were being used, they were being done covertly. Second, the design of the task employed established procedures to discourage the use of symbolic numbers to answer the questions (see Ballinger & Barth, 2007; Barth et al., 2006, 2008). We presented the numerical arrays too quickly to be enumerated exactly (1 second) and we used large numbers (average sum/outcome = 34; range for sum/outcomes = 16–56; average addend = 17; range for addends 7–40) to discourage rapid identification, serial enumeration, or memorized answers to addition problems. Third, previous work suggests that this type of task can be performed without symbolic arithmetic knowledge (preschool children: Gilmore et al., 2007; monkeys: Cantlon & Brannon, 2007) and the use of a symbolic number strategy does not facilitate performance (e.g. Ballinger & Barth, 2007; Barth et al., 2008; Gilmore et al., 2007). Fourth, Park and Brannon (2013) showed that training on a task involving ordering symbolic number does not lead to as significant gains in symbolic arithmetic as a training task engaging the ANS. Consistent with their findings, the participants in both conditions of Experiment 2 engaged symbolic numbers during the first block of symbolic addition test problems, but this engagement did not yield improvements on the second set of test problems. In fact, subjects in Experiment 2 performed worse on the second set of symbolic addition problems, regardless of training condition. These findings cast doubt on the possibility that symbolic number engagement over non-symbolic numerical arrays, rather than the ANS itself, drives the observed enhancements seen in the numerical training conditions of our experiments. While we cannot entirely rule out the possibility that symbolic number representations were co-activated with ANS representations, our results, our design, and previous research all suggest that the ANS rather than symbolic number representations was used to solve the tasks and likely drives the observed effect. Future research using the method of Experiment 2 with different symbolic tests as outcome measures may add further insight into this issue.

In sum, the present findings move beyond the findings of correlational studies (Gilmore et al., 2010; Halberda et al., 2008; Libertus et al., 2011; Mazzocco et al., 2011; Mundy & Gilmore, 2009) and build on recent training experiments (Park & Brannon, 2013) to provide experimental evidence that exercising the primitive system of approximate number representation can enhance both the speed and the accuracy of children's performance of symbolic mathematics. However, the results also raise a number of questions regarding the nature of this effect.

First, the developmental origins of the relationship between the ANS and symbolic number remain unclear. ANS acuity is associated with facility at symbolic mathematics across the lifespan, from infants (Starr et al., in press) to preschool children (Halberda et al., 2008) to octogenerians (Halberda et al., 2012). Experimental studies in children (current study) and adults (Park & Brannon, 2013) seem to suggest that practice or training with the ANS enhances symbolic mathematics. Our results show that the functional and causal link between ANS activation and symbolic arithmetic performance does not require a lengthy history of education in symbolic mathematics, as it occurs in children who are only in their second year of formal schooling and participants in most previous studies have had at least some working knowledge of symbolic number and formed initial mappings between symbolic number representations and the ANS. It is unclear if earlier interventions (such as those in infants or toddlers) centered on engaging and exercising the ANS, would lead to better mathematics outcomes later in life. It also is unclear if later interventions on participants whose manipulations of number systems are fully automatic (e.g. Bugden & Ansari, 2011; Girelli, Lucangeli, & Butterworth, 2000) would show the same immediate effects found in the present experiments. On one view, both initial learning and mature performance of symbolic mathematical computations such as arithmetic depend on the ANS (Dehaene & Cohen, 1997; Isaacs, Edmonds, Lucas, & Gadian, 2001; Lee, 2000; Levy, Reis, & Grafman, 1999; Molko et al., 2003; Takayama, Sugishita, Akiguchi, & Kimura, 1994), which plays an obligatory role in exact symbolic numerical representations and arithmetic operations. On a different view, the ANS and symbolic number representations become linked because they are repeatedly associated with one another over the course of children's learning of number symbols; thus, the ANS plays a habitual rather than obligatory role in symbolic mathematics performance (e.g. Lyons & Beilock, 2011; Sasanguie, De Smedt, Defever, & Reynvoet, 2011). On a third view, symbolic mathematics performance may depend on the ANS at early points in learning, but its influence may decline or become more habitual once symbolic arithmetic skills are fully automatic. Future research using the same training methods at different ages may adjudicate between these views.

A second open question concerns the symmetry or asymmetry of the causal relationship between the symbolic and non-symbolic number systems. Although the present experiments tested only for a relationship in one direction, and showed that exercising the ANS can enhance symbolic number processing, it is possible that causal effects operate in the reverse direction as well. Consistent with the latter possibility, the Munduruku of the Brazilian Amazon provide suggestive evidence of an effect of symbolic number training on the acuity of the ANS (Piazza et al., 2013). The Munduruku language has a limited numerical vocabulary and no formal symbolic number system. However, some Munduruku have learned the Portuguese numerical language and some have studied symbolic arithmetic in school. Individual differences among the Munduruku in ANS acuity are associated with both of these factors (Piazza et al., 2013).

Finally, the depth and temporal extent of the effects of ANS activation on symbolic number processing are not known. Recent work shows that extended, intense practice with the ANS through an approximate addition task can change both ANS acuity and symbolic mathematics ability and extent of ANS acuity change in individual participants correlates with individual increases symbolic arithmetic (Park & Brannon, 2013). No significant differences in ANS acuity were observed between children in the different training conditions of our study, casting doubt on the possibility that the mechanism of symbolic mathematics enhancement in our study was an ANS acuity change. Instead, it appears that simply preceding symbolic arithmetic with focused engagement of the ANS was sufficient to produce the effects on symbolic arithmetic. We speculate our effects arose through engagement of common cognitive mechanisms in the two tasks. Because the present research involved very brief practice and immediate testing, we do not know whether the effects on symbolic arithmetic reported here are momentary or enduring. Future work should contrast the extent and duration of symbolic mathematics outcomes after tasks involving engagement of, compared to change in, the ANS.

Regardless of the answers to these questions, our studies provide evidence for a causal relationship between non-symbolic approximate number and exact, symbolic arithmetic by children, and they move beyond previous work to delineate the specificity of this relationship. The fact that a single session of practice on an approximate number task can improve both the speed with which children solve easier symbolic mathematics problems, and the accuracy with which they solve harder mathematics problems, raises important possibilities for future educational research. In particular, it is possible that exercises engaging the ANS will provide a way not only to speed up mathematics performance in an immediately following test but also to boost performance of school mathematics in a more enduring way. In light of the importance of mathematics both in the elementary school curriculum and in diverse disciplines and professions, this possibility deserves to be tested.

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Appendix A. Supplementary material

Supplementary data associated with this article can be found, in the online version, at http://dx.doi.org/10.1016/j.cognition.2013.12.007.

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