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Reading Angles in Maps

Véronique Izard
Université Paris Descartes, Sorbonne Paris Cité, CNRS, and Harvard University

Evan O'Donnell and Elizabeth S. Spelke
Harvard University

Preschool children can navigate by simple geometric maps of the environment, but the nature of the geometric relations they use in map reading remains unclear. Here, children were tested specifically on their sensitivity to angle. Forty-eight children (age 47:15–53:30 months) were presented with *fragments* of geometric maps, in which angle sections appeared without any relevant length or distance information. Children were able to read these map fragments and compare two-dimensional to three-dimensional angles. However, this ability appeared both variable and fragile among the youngest children of the sample. These findings suggest that 4-year-old children begin to form an abstract concept of angle that applies both to two-dimsional and three-dimensional displays and that serves to interpret novel spatial symbols.

Geometry defines abstract concepts that apply to various types of spatial entities. For example, we may find angles in two-dimensional line drawings, in the contours of three-dimensional objects, in the arrangement of the walls surrounding us, or even in the projections of star patterns in the sky. Similarly, in geometry the length of an object and the distance between two landmarks or walls are captured by the same abstract concept of metric distance. Comparing shapes of different entities is challenging, because one must abstract away from the particular medium in which these shapes appear. In young children, this ability might take time to develop: For example, 1½-year-old toddlers still fail to recognize the matching shapes of blocks and holes or flat patterns (Shutts, Örnkloo, von Hofsten, Keen, & Spelke, 2009), and even in adults, the presence of convex or concave segments induce different descriptions of interlocking shapes (Cohen & Singh, 2007). In an effort to understand children's developing sensitivity to abstract geometry, here we focus on angle, a central concept in Euclidean geometry. We ask whether preschool children possess an abstract representation of angle that applies both to large three-dimensional surface arrays and to small two-dimensional figures.

Angle is a particularly interesting case in the study of abstract geometric concepts, because children's performance with angle appears greatly affected by stimulus format. On one hand, young preschool children can detect angle variations in small, two-dimensional figures, even when these figures also vary in size (Izard & Spelke, 2009). Preschoolers' sensitivity to angles and metric properties of two-dimensional figures is so pervasive that, when they start learning the names of geometric shapes (e.g., "triangle" and "square"), they do not apply these names to nonprototypical figures (e.g., an irregular triangle; Clements, Swaminathan, Hannibal, & Sarama, 1999), yet they are willing to generalize the categories to disrupted figures (e.g., a triangle with a corner cut off, or with an interrupted side), provided that these disrupted figures retain the metric properties of the prototype (Satlow & Newcombe, 1998). Sensitivity to angles in two-dimensional figures may already be present in infants (Lourenco & Huttenlocher, 2008; Schwartz & Day, 1979; Younger & Gotlieb, 1988), and even newborns (Slater, Mattock, Brown, & Bremner, 1991); although in all these studies, it is not clear whether infants represented angles per se, or reacted to length and distance variations (see next). Besides two-dimensional figures, children in their 2nd year of life can compute heading angles and find short-

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Correspondence concerning this article should be addressed to Véronique Izard, UMR 8158, 45 rue des Saints-Pères, 75006 Paris, France. Electronic mail may be sent to veronique.izard@polytechnique.org.

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cuts between locations, even without visual input (Landau, Gleitman, & Spelke, 1981; Landau, Spelke, & Gleitman, 1984); an ability that subsequently improves with age (Morrongiello, Timney, Humphrey, Anderson, & Skory, 1995). When tested with configurations of three-dimensional surfaces, however, young navigating children appear to ignore the angles formed between surfaces (Hupbach & Nadel, 2005; Lee, Sovrano, & Spelke, 2012), even though they can process other geometric properties of a configuration of walls, such as the distances that separate them (Hermer & Spelke, 1994; Lee et al., 2012).

Comparing two-dimensional figures to the three-dimensional environment is a central component of map reading (Shusterman, Lee, & Spelke, 2008; Uttal, 1996, 2000; Vasilyeva & Bowers, 2006). In everyday life, most of the maps we use contain a mixture of geometric and nongeometric information. Moreover, different maps may convey spatial information at different levels: Road maps usually conserve metric information, whereas subway maps sometimes carry only topological information about the connections between stations, distorting distances and angular relations. Prototypical maps (Uttal, 2000) are locally Euclidean: They conserve metric information from an overhead projection of the navigable layout. In terms of geometric information, such maps provide three types of geometric cues to the reader: angles (between axes and within shapes), relative distances (information pertaining both to the lengths of objects and other structures, and to the distances separating objects and other structures), and sense (left, right directions between and within objects and other structures).

Because map reading requires matching the geometric properties of two- and three-dimensional spatial arrays, researchers have used maps to probe the development of children's competence in Euclidean geometry. Preschoolers are able to read simple geometric maps (Huttenlocher, Newcombe, & Vasilyeva, 1999; Huttenlocher, Vasilyeva, Newcombe, & Duffy, 2008; Landau et al., 1984; Shusterman et al., 2008; Vasilyeva & Bowers, 2006; Vasilyeva & Huttenlocher, 2004), with no apparent cost for reading a flat two-dimensional map compared to a three-dimensional scale model (Huttenlocher et al., 2008). By varying the configurations used, these studies have shed some light on the type of geometric relations that children are able to process. In one type of study, the configuration was a rectangular form varying only in one dimension or a trio of objects arranged in a line, and the cued position was specified solely in terms of length (along the rectangle) or distance (between the aligned objects; Huttenlocher et al., 1999; Shusterman et al., 2008). Children as young as 3 or 4 years of age were found to detect and use length and distance relations from these linear maps (Huttenlocher et al., 1999) and they did so spontaneously, with no instruction to use geometry or feedback concerning their performance (Shusterman et al., 2008).

Other tests used an isosceles triangular configuration: either three objects placed to mark the corners of an isosceles triangle, or boards attached to form a continuous triangle. With these configurations, a combination of length, distance or angle with sense is needed to distinguish among all three corners of the triangle. More specifically, length (in the case of the continuous triangle), distance (in both cases), and angle are each sufficient to define the unique corner with respect to the two nonunique corners, while sense relations are necessary to distinguish among the two nonunique corners. Tested with isosceles displays made of three separate objects, members of a remote Amazonian community responded reliably to all three positions on the map (Dehaene, Izard, Pica, & Spelke, 2006), thus manifesting an ability to use either distance or angle, and also sense. In the same test, 4-year-old children from the United States successfully located the unique corner of the triangle from the map as well, but in contrast to the older participants from the Amazon or from the United States, they failed to distinguish between the other two similar corners (Shusterman et al., 2008). In fact, even 6-year-old U.S. children do not reliably discriminate between two similar corners of an isosceles triangle (Vasilyeva & Bowers, 2006). This failure indicates that children do not use sense relations on a map to guide their navigation in a three-dimensional array. On the other hand, because the apex of an isosceles triangle can be defined in terms of either angle, length, or distance (or all three), these experiments do not reveal whether children used angle at all to make this distinction.

The failure to distinguish between children's use of angle and their use of length–distance reflects a fundamental limitation of any task presenting purely geometric, complete planar figures. In Euclidean geometry, variations in angle are inevitably accompanied by variations in length or distance. As the angular size of one corner of a triangle increases, for example, so does the length of the opposite side (Euclid's Proposition 25; Heath, 1956). Similarly, if a rectangle is changed into a parallelogram by modifying the angles between the sides, the distances between the sides change as well. Because all two-dimensional geometric forms can be constructed purely from an array of triangles, the covariation in length–distance and angle found in triangles applies to all forms. Thus, it is in principle not possible to dissociate angle from length–distance in any complete two-dimensional map of an array.

Spelke, Gilmore, and McCarthy (2011) attempted to probe children's use of each type of geometric cue separately, by introducing a new type of display and map task to separate angle, length, and sense from each other. Here, instead of a triangle, at each trial they installed two wall configurations shaped as L's, which differed from each other in terms of the angle between the two branches, the length of the branches, or in the sense relation across branches. For example, in angle trials, the two L's had branches of similar length, pointing in the same direction, and forming different angles. In length trials, the branches of the two structures were oriented in the same way (same angle, same sense), but their lengths differed. In sense trials, the two structures were identical except that one pointed to the left and the other to the right. Instead of presenting a complete map of this array, moreover, the experimenter presented children with a depiction of just one of the L-shapes and recorded whether the children were able to recognize which of the two arrays was represented on the map. The authors found that 5- and 6-year-old children successfully recognized an L-shape structure from two-dimensional drawings in both "angle" and "length" trials. However, success at the angle trials does not prove that children represent angle relations, because the arrays with different angles differed also with respect to the distances between branches: With the length of the branches held constant, any increase in angle is accompanied by an increase in the distance between the branches' endpoints. Ascribing the children's responses in Spelke et al.'s study to distance seems particularly plausible in light of two findings. First, 4-year-old children can read distances from a map of objects arranged in a line, where distance is the only available cue (Shusterman et al., 2008). Second, after being disoriented by repeated turning without vision, 2-year-old toddlers reorient in a chamber using exclusively the distances between the walls, fully ignoring wall angles and lengths (Lee et al., 2012).

Here, we introduce a new experimental paradigm that isolates angle from both length and distance relations in a map reading task. The children were tested on a placement task where they needed to choose between two locations, marked by buckets situated in two of the corners of a large wooden triangular structure. The correct bucket was indicated to them on a two-dimensional map, representing the blueprint of the wooden triangle. In some critical trials, the map was taken apart before the experimenter indicated the placement location, such that the angle information was presented in the absence of any informative length or distance relations. If children are sensitive to angle in maps, then they should succeed on these fragmented map trials as well as on the trials with complete maps.

Experiment 1

Children were tested on a map task using isosceles triangle configurations. Experiment 1 included two types of configuration, allowing the extraction of precut fragments from the maps which contained either only angle information (in experimental trials) or only length information (in control trials). We chose a placement task rather than a search task, because the former has been found to be easier for young children (Huttenlocher et al., 2008) and because placement tasks provide no corrective feedback over the course of the experiment.

Two target buckets were placed either in two corners or along two of the sides of an isosceles triangle. Depending on the configuration, fragments were cut out of a map of the array either around the corners (angle fragments trials) or along the sides of the triangle (length fragments trials; Figure 1). The angle fragments were circular and centered on the corners of the triangle, thus presenting angle information in the absence of any informative length or distance information. These trials tested whether preschool children are able to generalize angle across two-dimensional and three-dimensional displays. The length fragment trials were rectangular, of similar width but different elongation, and presented a portion of two of the triangle sides without showing its corners, thus presenting length information in the absence of any informative angle information. Given previous reports that 4-year-old children can successfully read length from maps, we expected success at the length trials and included them as a control to check that the children were able to understand the fragmentation manipulation.

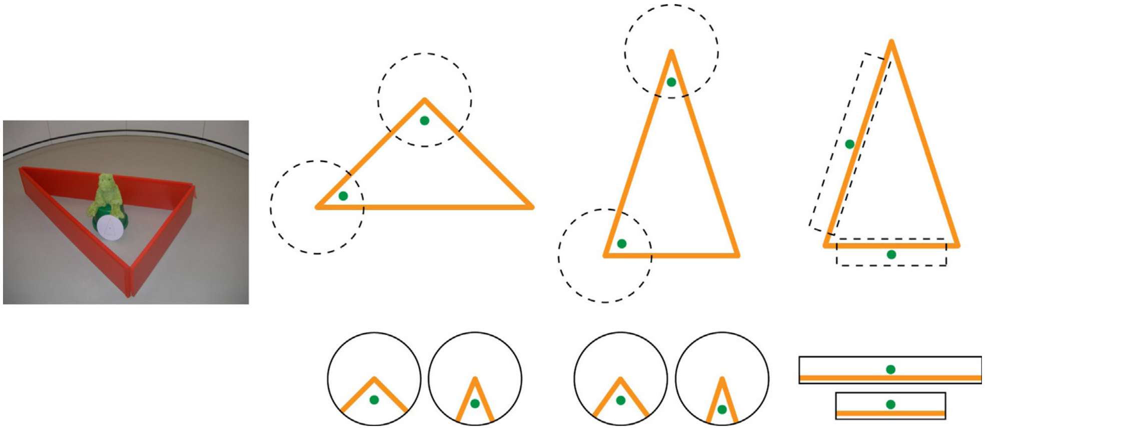

Figure 1. Displays for Experiment 1. The photograph shows the material used, presented here in a familiarization configuration: wooden triangular structure, inverted bucket, circular board serving as map, and toy frog. The drawings presented on the right show the configurations used in the test, as they appeared on the map: Two configurations for angle (right triangle, acute isosceles triangle) and one for length. The dashed lines were not visible on the maps but are added here to show the fragments precut to be extracted from the map in each condition. The bottom line shows the fragments aligned as they were presented to the children.

Method

Participants. Thirty-two children (M_{age} = 50.7 months, range = 47:15–53:30; 14 female children) participated in the study. Children came mostly from Caucasian middle-class families. Birth records were obtained from local city halls in the greater Boston area, and families were invited to come to the lab and participate by phone or by mail. The parents were reimbursed $5 for their travel expenses, and the children were thanked of their participation with a small toy. An additional three children were tested but excluded from the final sample for experimenter error (1) or excessive distraction (2).

Displays. Children were tested in a perfectly round room in the laboratory (diameter 12.5 ft) containing a large orange wooden triangular structure placed at its center (longest side 60 in., height 12 in.). The structure was made of boards rather than isolated objects to create configurations allowing map fragmentation. Two green inverted buckets were placed either at two interior corners of the triangle, or along two of its sides, and served as targets in the placement task. A circular board made of foam core, with an orange triangle identical in shape to the wooden triangle and two green dots indicating the buckets, served as the map of the room (Figure 1). Two fragments could be extracted from the pictures so as to show only parts of the setting in isolation (walls or corners), exhibiting just one geometric feature (length or angle). A toy frog served as the object used in the placement trials.

Two isosceles triangles were presented. The first triangle had an acute angle of 36° at its unique corner, and two angles of 72° at nonunique corners (length of sides: 60 × 60 × 37 in.). The second triangle had a right angle at its unique corner (90°) and two 45° angles at nonunique corners (length of sides: 60 × 42.5 × 42.5 in.). In the angle configurations, one bucket was placed in the unique corner and one was placed in a nonunique corner (Figure 1). The acute isosceles triangle served also in a length configuration, where the two buckets were placed, respectively, along the short side and along one of the long sides of the triangle. In the angle configurations, the two precut map fragments were circular and centered on the two corners with buckets. In the length configuration, the fragments were rectangular and ran alongside the sides with buckets, with the endpoints of the sides excluded so as to remove all angle information.

Design. Children were first given three familiarization trials to introduce the task without focusing on any metric relations of length or angle (see the Procedure section) and then tested with three blocks of trials corresponding to the three test configurations described above and in Figure 1. Each block contained 2 trials with the full maps (1 for each bucket), followed by 2 trials with the map fragments (1 for each bucket), for a total of 12 experimental trials. Across children, the trials were presented in four fixed orders so as to counterbalance the order of presentation of the configurations, with the constraint that the two configurations with the acute triangles followed each other, thus minimizing changes in setting. The four orders counterbalanced also the position of the correct bucket in the first trial for each configuration, both in the full map condition and in the map fragments condition.

Procedure. The familiarization procedure used either the acute or right triangle depending on the condition that would appear first in testing, and it focused on the topological relation of containment. In the first familiarization trial, there was only one bucket at the center of the triangle. The corresponding map showed one dot at the center of a triangle, which was geometrically identical to the wooden triangle. When first introducing the map, the experimenter stressed the correspondence between the picture and the room display by pointing successively to the triangle (the "house") and the dot (the "chair") on the picture and then in the room. Then, the experimenter pointed to the dot on the map, and asked the child to help the frog go sit on her favorite chair. In the second familiarization trial, a second bucket was added outside the triangle and a new corresponding map was produced. The experimenter pointed to the map to request that the child place the frog on the outside bucket. Last, the third familiarization trial served to introduce the map fragmentation manipulation. The map was precut so that two circular sections, centered on each bucket, could be extracted. One section showed a dot in the center of a triangle, the other showed an isolated dot. The fragments were shuffled in front of the child, and then the fragment with the dot inside the triangle was indicated as the target for the placement task. Throughout familiarization, children were provided with feedback if they placed the toy in the wrong location (the need for feedback was rare).

Following familiarization, the children were tested on the three different configurations. At the beginning of each block, the experimenter first introduced the configuration by pointing to the triangle ("house") and the dots ("chairs") on the map, then in the room, stressing the correspondence between the map and the room. For each trial, the experimenter pointed to one of the two possible locations on the map. The child had to place a sticker on the map (to check that they had encoded the location of the correct bucket on the map), and then go place the toy on the corresponding bucket. The experimenter looked either to the floor or straight toward the child, to avoid cuing to one of the response locations. The children were allowed to come back and look at the map if they wanted. In the fragments trials, before the experimenter indicated the position of the target bucket, the two precut fragments were taken apart from the map, shuffled, and then placed in front of the child, with the rest of the map removed from view. The two fragments were presented aligned with each other such that their position did not reproduce the position of these elements in the whole configuration (Figure 1). The relative orientation of the map and the room was randomized by walking the child around the display and stopping at various locations across trials, while the map was presented in a constant orientation with respect to the child. Children were given neutral feedback throughout the task.

Data coding and analyses. The sessions were videotaped from a camera built in the ceiling for later coding. The videos showed the child transporting the toy in the large triangle, but the resolution was not sufficient to perceive the details on the map. Each session was replayed offline without the sound and the experimenter, who was blind to the location of the correct response, coded the location where the child left the toy in each trial.

For the purpose of the analyses, the trials were grouped according to the type of relations that were available (full maps: angle, distance and length relations; angle fragments: angle relations only; length fragments: length relations only). A preliminary analysis indicated that gender had no effect on performance, so this factor was dropped from further analyses. Accuracy data were first analyzed in an analysis of covariance (ANCOVA) with one within-subject factor for condition and one covariate for age. Next, performance in each condition was compared to 50% chance level in a t-test analysis. Finally, to compare performance across conditions, planned comparisons were conducted across all three conditions using t tests (Bonferroni corrected).

Results

The ANCOVA revealed a significant main effect of condition, $F(2, 60) = 4.2$, $p = .020$, $\eta_p^2 = .12$ (Figure 2). Children performed above chance on the full map trials: 72.9%, $t(31) = 6.3$, $p < .001$, as well as on the fragmented trials testing angle: 63.3%, $t(31) = 2.5$, $p = .019$. However, they were at chance on the fragmented trials testing length: 50.0%, $t(31) = 0.0$, $p = 1.0$, thus yielding a significant difference between the full map and the length fragments trials: $t(31) = 2.8$, $p = .026$, Bonferroni corrected. The performance on the angle fragments trials did not differ significantly either from the full map performance or the length fragments performance (p values > .30, Bonferroni corrected). Performance also

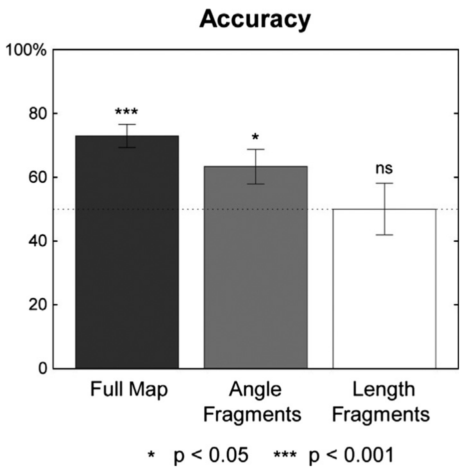

Figure 2. Results of Experiment 1. For each condition (full map, angle fragments, length fragments) the performance was tested against chance (50%) by means of a *t* test. Error bars represent the standard error of the mean.

improved with age, $F(1, 30) = 4.2$, $p = .049$, $\eta_p^2 = .12$, but the factors of age and condition did not interact, $F(2, 60) < 1$.

Because the analyses of Experiment 1 were based on categorical data whose conformity to parametric statistical assumptions is difficult to verify, we performed the same analysis using a mixed model logistic regression. This technique is specifically designed for the analysis of categorical data and is more robust over variations in the distribution of data across conditions and subjects (Jaeger, 2008). The findings of this analysis, reported in the online Supporting Information (see Appendix S1), fully accorded with the findings reported here.

To explore the effect of age in further detail, we separated the children into two subgroups of 16 participants, based on age (Group 1: M_{age} = 49.2 months, range = 47:15–50:30; Group 2: M_{age} = 52.2 months, range = 51:04–53:30) and tested performance against chance in each condition for each subgroup. The older children succeeded on the full map trials: 79.2%, $t(15) = 5.7$, $p < .001$, and on the angle fragments trials: 71.9%, $t(15) = 3.2$, $p = .0058$, but their performance did not reach significance on the length fragments trials: 62.5%, $t(15) = 1.1$, $p = .30$. The younger children performed above chance when presented with the full map: 68.8%, $t(15) = 3.5$, $p = .0035$, but failed to solve either the length fragments trials: 37.5%, $t(15) = -1.2$, $p = .26$, or the angle fragments trials: 54.7%, $t(15) = 0.59$, $p = .57$. The difference in performance between the two subgroups did not reach significance in any of the conditions, however, $ts(30) < 1.9$, $ps > .08$. As an illustration, scatter plots of the performance by age are presented in the online Supporting Information (Figure S1).

Discussion

Experiment 1 confirmed that 4-year-old children are able to read a complete geometric map. Children performed above chance when they were presented with the full maps, which redundantly combined cues of angle, length and distance. However, performance was more mixed when children were tested with a partial map. As a group, participants were able to match two-dimensional angle map fragments with the corresponding corners of a three-dimensional triangle, thus providing evidence that by 4½ years of age, children can compute abstract angles over two-dimensional and three-dimensional arrays. However, this ability appeared fragile and variable across individuals, especially among the youngest children tested (see online Supporting Information Figure S1C). Last, the children failed to solve the task when the map fragments presented two different lengths.

The results of the length fragments condition were unexpected, given a previous report that even 3-year-old children use length relations in complete maps (Huttenlocher et al., 1999). Three different explanations can be considered to account for this finding. First, perhaps some children failed to understand our fragmentation manipulation altogether. Although almost all children understood the perforation in the third familiarization trial (27/32 children succeeded at the first attempt), this configuration may have been intrinsically easier, because no portion of the figure was cut in the fragmentation. In contrast, to understand how the angle or length fragments related to the triangle structure, children needed to compare a whole pattern with its isolated parts, a task that may be difficult for 4-year-olds. This explanation may account for the performance of those children who failed at both our fragments conditions, but a different explanation is required for the children who succeeded with angle fragments.

Second, perhaps the children were misled by the stimuli used as length fragments, and failed to understand that the length of the pieces was related to the length of the sides of triangle. Indeed, in the fragmentation manipulation, the endpoints of the segments were cut off on purpose to avoid including angle cues, but their absence could have unfortunately led the children to think that the length of the pieces was not necessarily related to the length of the sides of the triangle. After all, it is entirely possible to cut off a longer segment from a shorter side, and a shorter segment from a longer side.

Third, the children's failure at the length fragments trials may reveal essential limitations to their representation of length. Indeed, although children's ability to read length from maps has been clearly established for one-dimensional displays, reports have been mixed for configurations extending in two dimensions (Uttal, 1996; Vasilyeva & Huttenlocher, 2004). With two-dimensional displays, it is possible that children do not represent and compare length or distance ratios, but simply use distance to parse the configuration into subgroups of close items, a strategy that was not available in our task.

In the context of our experiment, the children's failure in the length fragments condition raises a question: Did some children fail to compare two-dimensional to three-dimensional angles because they lacked an abstract representation of angles or because they failed to understand the fragmentation manipulation altogether? To address this question, we first analyzed whether children's performance at the angle fragment trials correlated with their performance with the different full map configurations. In the full map trials, length–distance cues conveyed less distinctive information in the right triangle configuration, compared to the acute triangle configuration, because the length and distance ratios were larger in the latter condition (length ratio between sides: right triangle 1.4, acute triangle 1.6; ratio of the distances between sides, measured at the middle point of each sides: right triangle 1.1, acute triangle 1.8). Therefore, children's ability to read the full right triangle map should correlate with their ability to read angles, whereas they may succeed at the full acute triangle even if they cannot read angle cues. Consistent with this prediction, a multiple regression analysis revealed a correlation between performance on the angle fragment trials and on the right-angle full map condition, $t(31)= 2.3$, $p = .030$, whereas performance on the other full map conditions did not contribute to the fit (ts < 0.29, ps $> .78$). Therefore, the variations in children's ability to solve the angle fragment condition appear to reflect their individual competencies at reading angle from maps in general, rather than their understanding of map fragmentation.

To explore further the source of the variability in young 4-year-old children's generalization of angles across two- and three-dimensional stimuli, we designed a new experiment, this time using color as a control condition rather than length.

Experiment 2

In this experiment, the length configuration was replaced by two configurations where the location of the correct bucket was specified by color. In both the angle and the color fragment conditions, we cut circular fragments of the map centered on each bucket: These two fragments differed from each other either in terms of angle (in the angle condition) or in terms of color (in the color condition). If the failure of some children to read angle map fragments stems from a general difficulty with fragmentation, these children should fail with color map fragments as well. Accordingly, in this experiment we narrowed the age range to span only the younger end of the range used in Experiment 1, in an effort to test more children who failed the fragmented angle map task.

Method

Participants. Sixteen children ($M_{age} = 49.15$ months, range = 47:24–50:18; 8 female children) participated in Experiment 2. They were recruited from the same population as in Experiment 1. Three additional children were tested but excluded from the sample for video equipment failure (2) or refusal to participate (1).

Stimuli, design, and procedure. The procedure was identical to Experiment 1, except that the children were tested on four different configurations (4 trials each), for a total of 16 experimental trials. The order of presentation of the four blocks was controlled across children in a Latin square design. There were two configurations testing the use of color, and two configurations testing the use of angle (Figure 3). The two angle configurations were identical to those used in Experiment 1: an acute isosceles triangle and a right isosceles triangle. The first color configuration corresponded to an equilateral triangle (length of sides: 48 × 48 × 48 in.), in which one of the corners was painted in blue. Correspondingly, on the map, one of the corners of the equilateral triangle appeared in blue. Two buckets were placed, respectively, in the blue corner and in one of the orange corners. In the second color configuration, the same three boards were placed as three sides of a square, making two corners, one of which was painted in blue. To isolate the color information in the map fragments, circular sections were precut in the map around each corner with bucket: The two corners had a similar angle but one appeared entirely orange and the other entirely blue. In the equilateral condition, the corners measured both

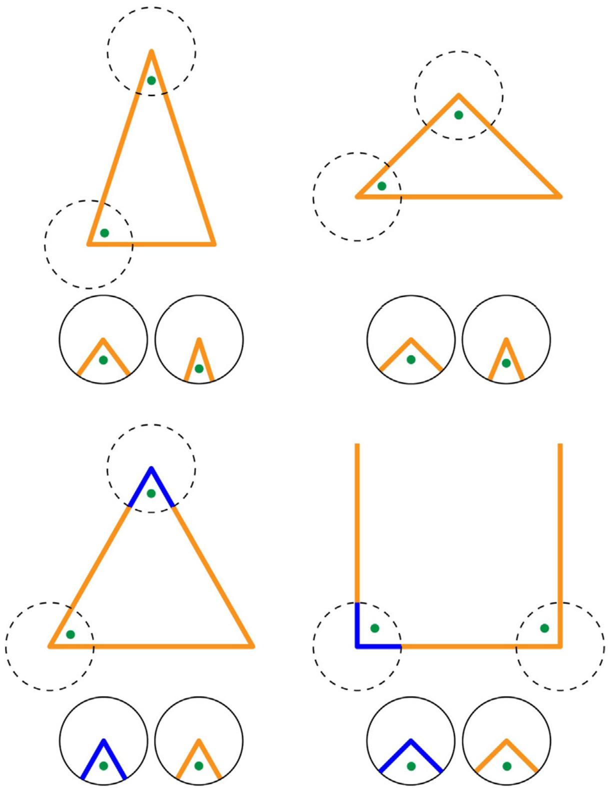

Figure 3. Configurations used in Experiment 2. The two upper configurations test the use of angle relations, and are similar to the angle configurations of Experiment 1. The two lower configurations test the use of color: The corners appearing with a darker shade on the grayscale figure were colored in blue in the display (the reader is referred to the online version of the article to see real colors). As in Figure 1, the dashed lines were not visible on the maps but are added here to indicate the fragments presented in the fragmented condition. Fragments are shown with the same orientation as when they were presented to the children.

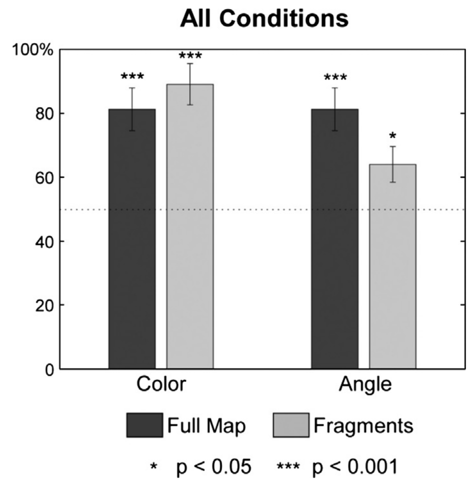

Figure 4. Results of Experiment 2. The performance was tested against chance (50%) in each condition. Error bars represent the standard error of the mean.

60°, and in the square condition, the corners measured 90°.

Data analysis. In this experiment, the full map conditions were not comparable as they were in Experiment 1: Whereas the children needed to attend to geometric cues to interpret the full map in the angle configurations, they could only interpret the colored full maps by attending to color. Accordingly, in Experiment 2, we defined two within-subject factors for configuration (color/angle) and fragmentation (full map/map fragments). Preliminary analyses showed that children's performance was not affected by either gender or age. Therefore, the data were analyzed using an analysis of variance (ANOVA) with two within-subject factors for configuration and fragmentation, as described above. Performance in each condition was tested against chance (50%) by means of a *t* test.

To test whether failures with angle fragments could be explained in terms of a general difficulty of some children to understand map fragments, we used a linear regression analysis to compare the performance across the two fragments conditions (angle and color) across subjects. Last, we also extracted a subgroup of children who clearly failed at the angle fragments condition (performance 50% or less), and tested whether this subgroup succeeded at the color fragment condition by means of a *t* test.

Results

We observed a significant effect of condition, $F(1, 15) = 8.6$, $p = .0083$, $\eta_p^2 = .38$, which was qualified by a interaction between condition and fragmentation, $F(1, 15) = 4.8$, $p = .045$, $\eta_p^2 = .24$ (Figure 4). The performance was identical in the full map conditions across the angle and the color configurations: 81.3% versus 81.3%, $t(15) = 0.0$, $p = 1$, but children had more difficulties solving the angle fragments than the color fragments: 64.1% versus 89.1%, $t(15) = 3.2$, $p = .0064$. Nevertheless, children performed above chance in all conditions (all p values $< .05$). As for Experiment 1, analyses using a mixed model logistic regression yielded the same results (see the online Supporting Information).

Did the children who failed to solve the angle fragment trials nonetheless succeed on the color fragment conditions? Across subjects, there was no correlation between the performance at the angle and color fragments trials, $F(1, 14) = .28$, $R^2 = .02$, $p = .60$. There was nonetheless some variability in the performance for the angle fragments trials, as 9 of the 16 children tested performed at or below 50% in this condition. When the analyses were restricted to this subgroup of children, performance at the color fragments condition was still above chance: 83.3% correct, $t(8) = 3.0$, $p = .016$—only 1 child scored below 75%, and did not differ from the color full map performance: 72.2% versus 83.3%, $t(8) = 0.84$, $p = .43$.

Discussion

In Experiment 2, children succeeded at reading both full maps and fragmented maps, both when the maps were purely geometric maps and when they contained a nongeometric color cue. In the case of the fragmented maps in particular, young children succeeded at mapping angle information from the partial two-dimensional array to the full three-dimensional layout, and at using this mapping to navigate by purely geometric angle information. These findings confirm that young 4-year-old children can understand our fragmentation manipulation, and that they can use abstract angle in maps.

Even though 4-year-old children were able to read full geometric maps, their ability to read angles from fragments again was variable. The findings of Experiment 2 shed light on the source of children's difficulty, revealing that it did not stem from a general misunderstanding of fragmentation in maps. First, there was no correlation between the children's performance at reading map fragments showing different angles, and their performance with another type of fragment, which provided nongeometric information (color). Moreover, in Experiment 2 the children who failed to solve the angle fragment trials were nonetheless able to find the correct bucket based on color map fragments, thus providing evidence that they understood the fragmentation process and understood that the fragments still referred to the three-dimensional layout. Third, and more generally, for the whole sample included in Experiment 2, children were more affected by the fragmentation in the angle condition than in the color condition: Generalizing angles across the map fragments and the three-dimensional wooden structure poses a bigger challenge to the children than generalizing color.

Contrary to the younger subgroup of children in Experiment 1, in Experiment 2 the participants performed above chance on the angle fragment trials. More generally, when considering all the trials at the angle configurations together (the angle configurations were identical in both experiments), young children tended to perform better in Experiment 2 than in Experiment 1: 72.7% versus 58.6%; effect of Experiment in an ANOVA with two factors for experiment and fragmentation. $F(1, 30) = 3.9$, $p = .058$, $\eta_p^2 = .12$—a trend that was present both with the map fragments (62.5% vs. 54.7%) and the full map (81.3% vs. 64.1%). Could the design of this second experiment have contributed to the better success of the children? Given that the improvement on angle configurations generalized to the full map trials, it seems unlikely that the improvement resulted simply from a better understanding of fragmentation. Perhaps the easy color trials contributed to maintain a high motivation throughout the task. Perhaps also, the inclusion of configurations where the corners of the triangle were painted in different colors helped the children by directing their attention to the corners of the triangle, therefore leading them to discover how they sometimes differed in angle. However, given that the performance difference across experiments was only marginally significant, these hypotheses should be regarded with caution. Further research is needed to understand what type of experience can boost children's abilities to read angles in maps.

General Analysis of the Full-Map and Fragments Angle Conditions

The results of Experiments 1 and 2 indicate that children's ability with angles is quite variable at the onset of their 5th year of age. First, in Experiment 1 the subgroup of the 16 oldest children succeeded reliably at reading map fragments, whereas the 16 youngest children did not. Second, the younger children's performance fluctuated across samples, as performance was above chance in Experiment 2 but not Experiment 1.

To explore further the hypothesis that children's ability to read maps with angles improves during the first half of their 5th year, we pooled the data from the full map and fragmented angle conditions of the two experiments, as these conditions used the same geometric configurations (Figures 1 and 3). Data from the 48 participants were sorted according to the participants' ages and then smoothed by taking a running average over subgroups of 16 children. Using these processed data, we tested (a) whether performance with angles improved with age, for each of the full-map and fragments condition and (b) at which age performance exceeded chance reliably—whether children were above chance across the whole age range or only in the older end of the age range.

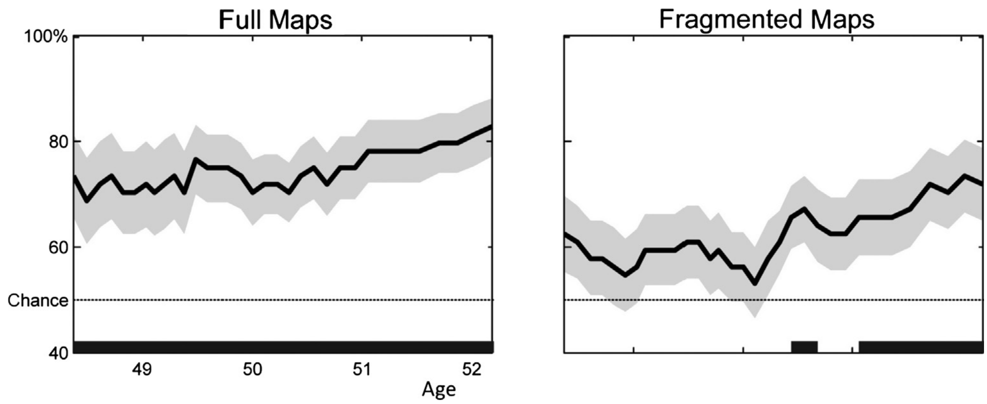

Figure 5. Running average of performance; data pooled across the angle conditions of Experiments 1 and 2. The shaded area indicates the standard error of the mean. The bar on the bottom indicates the ages at which the performance of the subgroups exceeds chance level (*t* test, $p = .05$; $N = 16$ in each subgroup).

The data are presented in Figure 5. First, we tested for the presence of an age effect on these smoothed data using a linear regression between the mean age and mean performance of the subgroups. For both the full map and the fragmented map angle conditions, average performance increased with the average age of the subgroups: R^2s = .64, respectively, .63, Fs(1, 31) = 55.3, respectively, 51.7; ps < .001. Next, we tested whether the performance of each running subgroup exceeded chance by means of a *t* test ($p < .05$). All subgroups performed reliably above chance in the full map condition. In the fragmented condition, in contrast, performance exceeded chance only in the subgroups spanning the higher end of the age range (M_{age} above 50 months).

This analysis confirms that children's abilities with angles are fragile around their fourth birthday, despite their success at reading full maps. In addition, children's performance increased with age on both the full map and the fragments conditions. Both effects may be related to the children's increasing ability to read angles, as indicated by a correlation between the mean performance at the full map and fragment conditions: $R^2 = .66$, $F(1, 31) = 61.5$, $p < .001$. By using angle information along with distance and length information, older children may enhance their map reading performance.

General Discussion

Our results add to current knowledge of young children's competence at reading maps in two ways. First, preschoolers are able to read maps not only when they depict a whole figure, but also when they represent only parts of an array. Second, children in their 5th year of life can compare angles across two- and three-dimensional displays so as to use this information in their map reading. These two points shed light on preschooler's representations of angle, which must be abstract enough to encompass angle in spaces of different dimensions, presenting different geometric configurations (figures or isolated sectors), and made of different materials (two-dimensional drawings and three-dimensional boards).

Could the children have succeeded in our task without representing angles—by using exclusively representations of distance and length? Our experimental procedure was designed so as to exclude this possibility. In the angle fragments condition, the children were presented with two circular fragments of the map, cut around two of the corners of the triangle. First, the length properties of these two fragments were equivalent, because the two circles cut from the map were of the same size (note that as the size of the two fragments was identical and the fragments were shuffled before the experimenter indicated the target, children could also not solve the task by remembering where in the full map each fragment came from). Moreover, the length of the two branches of each sector, which were identical, did not correspond to the length relations between the sides of the full triangle. On the other hand, because the two fragments cut were of identical size, for the larger angle, the endpoints of the two branches were further apart. This difference could not have served as a cue in our task, however, as the endpoints of the sectors were not marked on the three-dimensional triangle, where the sides of the triangles were continuous. To use these cues, the children needed to normalize the distance between sides by the distance from the corner—in short, they needed to compute angles.

Our experiments also reveal that individual 4-year-old children vary in their ability to read angles from maps. In both Experiments 1 and 2, even though the group performance was above chance in the angle fragments condition, a considerable proportion of children performed at chance level (50% accuracy or less, 14/32 children in Experiment 1, 9/16 children in Experiment 2). Experiment 2 was undertaken to better understand the reason for these failures. It revealed that all the children who failed with angle fragments were able to read map fragments with color information: Thus, children's difficulty did not stem from the fragmentation process in itself. Moreover, Experiment 2 revealed a significant difference between young 4-year-old children's competence with color and with angle: Even though our participants succeeded at using angle on the maps, they performed better with color. Both findings suggest that angle poses a particular challenge for young children, and that children's representation of abstract angle undergoes a developmental change around their fourth birthday.

This suggestion accords with numerous findings concerning young children's performance in other tasks. Although children and infants are sensitive to angles in two-dimensional figures at ages younger than those tested here (Izard & Spelke, 2009; Slater et al., 1991; Younger & Gotlieb, 1988), young children have far greater difficulty encoding the angles formed by three-dimensional arrays of surfaces (Hupbach & Nadel, 2005; Lee et al., 2012), be they large surfaces such as walls or smaller surfaces arranged on tabletop arrays. Two different tasks suggest that the ability to navigate by the angular relations between surfaces emerges during the 5th year of age: A reorientation task in a large enclosed arena whose walls were arranged in the shape of a rhombus, and a search task with a rhombic tabletop display that rotated between trials (Hupbach & Nadel, 2005). These studies suggest that before their fourth birthday, children have a partial concept of angle, which encompasses only two-dimensional figures and not angles formed by surfaces (Spelke, Lee, & Izard, 2010). As indicated by our findings, most children overcome this limitation and develop a more abstract concept of angle by the age of 4½ years.

Our suggestion of a developmental change raises the question of how children gain wider, more abstract representations of angles. On one hand, the children could learn to encode three-dimensional surfaces in the same format as they encode two-dimensional drawings, by encoding a projection of these surfaces (akin to taking a footprint of a structure). By doing so, they would then be able to generalize the kind of analysis they apply to planar figures to a greater variety of arrays. Alternatively, children might develop an ability to encode angles from three-dimensional displays directly, using the original format of representation they use to encode other geometric properties of surfaces, and restructuring this representation to include angles.

A related question concerns whether the ability to perceive angles in two-dimensional figures fosters the development of the perception of angles in three-dimensional displays. In particular, exposure to maps may boost the development of an integrated concept of space and of angles (Landau & Lakusta, 2009; Uttal, 2000). By analyzing two-dimensional angular relations in maps, children may either develop an abstract concept of angle on the way to understanding angles in three-dimensional stimuli, or they may acquire this concept as a second step after learning to perceive three-dimensional angles.

Children's competence with angles eventually extends beyond that of comparing and matching forms and objects, when children start being able to reason about the abstract geometric properties of shapes. For example, Izard, Pica, Spelke, and Dehaene (2011) recently presented children with the task of constructing the missing angle from an incomplete triangle, and observed that 5- and 6-year-old children from the United States are deeply misled by a faulty heuristic. Instead of taking into account the base angles of the triangle, which fully determine the size of the third angle on a plane, young children based their estimations exclusively on the length of the triangle base, which is irrelevant to the third angle's size. Interestingly, this limitation was overcome in older children (aged 7–13 years) living either in an industrialized society (France) or in an indigene group from the Amazon (the Mundurucu).

In conclusion, at the age of 4 years, children are already able to generalize angles both from small planar maps to larger surface layouts, and from fragments to complete figures. The fragility of the youngest children's performance, and the limitations on older children's grasp of abstract geometry, suggest that the preschool years are a pivotal time for the development of abstract geometric concepts. Further studies of children should be undertaken to probe the nature of the changes in sensitivity to geometry in tasks of map-guided navigation and abstract form analysis, to address fundamental questions concerning the nature and development of this system of knowledge.

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Supporting Information

Additional supporting information may be found in the online version of this article at the publisher's website:

Appendix S1. Mixed Model Logistic Regression Analyses of Experiments 1 and 2.

Figure S1. Scatterplot of Children's Individual Performance Across the Conditions of Experiment 1 Indexed by Age.