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Navigation as a source of geometric knowledge: Young children's use of length, angle, distance, and direction in a reorientation task

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ABSTRACT

Geometry is one of the highest achievements of our species, but its foundations are obscure. Consistent with longstanding suggestions that geometrical knowledge is rooted in processes guiding navigation, the present study examines potential sources of geometrical knowledge in the navigation processes by which young children establish their sense of orientation. Past research reveals that children reorient both by the shape of the surface layout and the shapes of distinctive landmarks, but it fails to clarify what shape properties children use. The present study explores 2-year-old children's sensitivity to angle, length, distance and direction by testing disoriented children's search in a variety of fragmented rhombic and rectangular environments. Children reoriented themselves in accord with surface distances and directions, but they failed to use surface lengths or corner angles either for directional reorientation or as local landmarks. Thus, navigating children navigate by some but not all of the abstract properties captured by formal Euclidean geometry. While navigation systems may contribute to children's developing geometric understanding, they likely are not the sole source of abstract geometric intuitions.

1. Introduction

Human adults can conceive of lines so thin that they have no thickness, so long that they never end, and so perfectly parallel that they never meet. The nature and development of these intuitions of Euclidean geometry have long fascinated philosophers and scientists, because they are both so clear and so elusive. Geometrical intuitions are robust enough to support the development of a vast edifice of formal mathematics, and they underlie a host of cultural achievements from measurement to engineering to the visual arts. Moreover, geometrical intuitions may be universal across human cultures, despite the culturally variable ways in which geometry is used (Dehaene, Izard, Pica, & Spelke, 2006; Izard, Pica, Spelke, & Dehaene, 2011; see also Plato, ca. 380, B.C./1949). Nevertheless, the points and lines of Euclidean geometry, and the axioms and theorems that relate them, elude our direct perceptual experience. Where do these geometric intuitions come from, and how do children become sensitive to the relations that they capture?

The present research is guided by an old idea: sensitivity to the fundamental relations of Euclidean geometry arises from the systems that guide children's navigation. Although the earth is round and the local terrain is bumpy, navigation over short distances can be captured by the fundamental properties of Euclidean plane geometry including *length* (the lengths of individual surfaces and objects), *angle* (the relative orientations of two surfaces or edges with respect to one another and the size of the corner that they form when conjoined), *distance* (the displacement of a surface or object from other objects or from one's current

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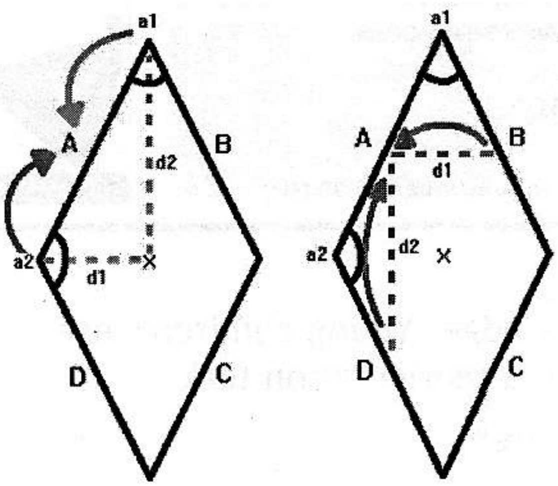


Fig. 1. Illustrations of possible ways in which distance, angle, and direction can be used for reorientation in a rhombic environment in either egocentric (left) or allocentric (right) coordinates. Given four locations (A, B, C, D) that are at the center of each wall and therefore equidistant from the center of the room (array tested in Experiment 2), the left diagram shows distance differences from the center position from which location A (and its geometric twin C), for instance, can be encoded using distance and direction (i.e., "left of the farthest point of the surrounding layout (*d2*)" or "right of the closest point of the surrounding layout (*d1*)"). The right diagram shows distance differences between the surfaces themselves from which A (or C) can be encoded (i.e., "left of the narrower space between surfaces (*d1*)" or "right of the wider space between surfaces (*d2*)") Note that distances could be measured either from the centers of walls or as the average of the distances of each point on a wall. Note that in principle, measurements of angle sizes could substitute for measurement of distances, in either reference frame (e.g., "left of the narrower angle," or "right of the wider angle").

station point) and *sense* or *direction* (the relative positions of surfaces or objects with respect to other objects or to one's own facing direction: Fig. 1).

How do geometric measurements of length, angle, distance and direction guide navigation? It has long been theorized that these measurements apply primarily to the internal encoding of proprioceptive cues to track one's movement, because movements of the eyes, head and body can be related to one another through the axioms and postulates of Euclidean geometry (Descartes, 1637/2001). According to the cognitive map hypothesis, as animals navigate from place to place, they form an allocentric internal representation of the environment that preserves the geometric relationships between the various landmarks and locations within it, based on the distances and directions that they travel (Gallistel, 1990; O'Keefe and Nadel, 1978; Tolman, 1948).

Despite the attractiveness of the view, a wealth of evidence casts doubt on the thesis that geometrical representations and computations underlie navigators' representations of the paths on which they travel. Although inertial navigation, or *path integration*, has been demonstrated in many different animals, from desert ants (Wehner & Srinivasan, 1981) to dogs (Chapuis & Varlet, 1987) to humans (Landau, Gleitman, & Spelke, 1981; Loomis, Klatzky, Golledge, & Philbeck, 1999), experiments cast doubt on the thesis that any navigating animals use path integration to build a Euclidean geometric map of the environment. Both insects (Wehner & Wehner, 1990) and humans (Foo, Warren, Duchon & Tarr, 2005) show striking limitations in their abilities to use distance, angular, and sense relations between paths navigated. Instead, a better candidate source of geometrical knowledge comes from studies of navigating animals' sensitivity to the structure of the surrounding surface layout.

"Geometry" means "the measurement of the earth." Several decades of research in spatial navigation reveal that animals, including human toddlers, use geometric properties of their environment to guide their navigation to goal locations (e.g., Cheng & Newcombe, 2005; Gallistel, 1990). Because navigating creatures can ignore all features of the terrain over short distances when they are oriented, and instead find their way by using path integration to update their position, animals' sensitivity to the geometry of the external environment is best revealed when they are disoriented. When an animal loses its own sense of direction, it must use external directional cues to reorient – to regain its heading and position with respect to locations in the environment. For example, when rats watch as food is buried in one corner of a rectangular arena and then are disoriented by covered rotation and placed back in the arena, they reorient themselves primarily according to the shape of the room, and therefore search with equal frequency at the correct corner and its diagonally opposite geometric twin (Cheng, 1986). The rats' surprising failure spontaneously to use other available cues (such as distinctive odors or wall patterns) to break the room's symmetry led to the formulation of the *geometric module* hypothesis. According to this hypothesis, disoriented animals regain their heading by establishing the congruence between the shape of the room as it is currently perceived and a representation of the room as it was previously experienced from a specific, known direction. While there is still an ongoing debate regarding whether these representations are egocentric and viewpoint-dependent (e.g., Wang & Spelke, 2002) or allocentric and viewpoint-independent (e.g., Burgess, 2006), many species of animals, from ants to chicks to human toddlers, have been shown to navigate spontaneously by the shape of the surrounding environment after they are disoriented (Brown, Spetch, & Hurd, 2007; Cheng & Newcombe, 2005; Chiandetti & Vallortigara, 2008; Sovrano, Bisazza, & Vallortigara, 2002; Wystrach & Beugnon, 2009), consistent with the hypothesis that a computation of the geometric shape of the environment underlies reorientation (Cheng, 1986; Cheng & Gallistel, 1984).

The geometric reorientation hypothesis has recently been challenged by image-matching theories of navigation that question the necessity for any abstract geometric content in the representations underlying reorientation.[1] According to image-matching theories, animals take visual

[1] A different aspect of Cheng and Gallistel's original hypothesis, that reorientation depends on an encapsulated system sensitive only to surface layout geometry, has also been challenged (e.g., Newcombe & Ratliff, 2007; Sovrano, Bisazza, & Vallortigara, 2003). We do not discuss Cheng and Gallistel's claims concerning the architecture of navigation systems in this article, but only their claims concerning the geometric content represented by these systems.

"snapshots" while exploring an environment and later match these stored retinal images to currently perceived images to guide navigation to goal locations (Cheng, 2008; Sheynikhovich, Chavarriaga, Strösslin, Arleo, & Gerstner, 2009; Stürzl, Cheung, Cheng, & Zeil, 2008). Evidence against image-matching theories comes from recent findings that both young children (3–4 years old) and young chicks spontaneously use features of the surface layout that are geometrically informative but barely visible in contrast images of the environment, while failing to use the same geometric relationships in high contrast 2D surface markings with prominent image contrast (Lee & Spelke, 2008, 2010a, 2011; Lee, Spelke, & Vallortigara, 2010). It appears, therefore, that children indeed reorient by 3D layout geometry. This research does not reveal, however, which geometric properties of the layout guide reorientation.

Disoriented children also navigate by beacon guidance: they learn the direct relationship between a local landmark object or visual feature and a nearby goal location and then relocate the goal via association with the landmark. For example, if a toy is hidden in a blue box in a geometrically uniform circular arena, disoriented 3–4-year-old children will look around the environment to locate such a box. If a second box of identical shape and color appears in the environment along with a third distinctive object, then children divide their search only between the featurally indistinguishable boxes, exhibiting both their use of an object as a beacon and their failure to reorient in that particular environment so as to distinguish between the two identical boxes (Gouteux & Spelke, 2001; Lee, Shusterman, & Spelke, 2006; Lee & Spelke, 2010a). In some experiments, the relevant location is distinguished only by a landmark's distinctive shape, and 5-year-old children have been shown to use this shape information to locate the hidden object (Nardini, Thomas, Knowland, Braddick, & Atkinson, 2009). Although younger children's use of object shape has not been tested in navigation experiments, a wealth of research using other methods (including word learning and object matching tasks) provide evidence that sensitivity to object shape grows rapidly over the first 2 years of life (Smith, 2009). Nevertheless, 3–4-year-old children fail to use the distinctive shape of an array of identical landmarks (e.g., a rectangular arrangement of columns within a circular arena; Gouteux & Spelke, 2001; Lee & Spelke, 2008) to reorient, unless they are placed at the periphery of the space and therefore are continuous with the overall, continuous surface layout (Lee & Spelke, 2010a; Lew, Gibbon, Murphy, & Bremner, 2010).

Non-human animals also use distinguishable geometric patterns to guide their search when they are disoriented, particularly after training. For example, the same rats who fail to use distinctive 2D, black and white geometric patterns located near the corners of the rectangular arena will learn, over many trials, to locate the food by using the pattern at the correct corner as a beacon to guide their search (Cheng, 1986). Many animals, even ants, have been observed to use the shape of a visual pattern as a beacon to distinguish the correct location from other potential locations (Wystrach & Beugnon, 2009; see Cheng & Newcombe, 2005 for review). As in the case of studies of reorientation, however, research does not reveal which geometric properties children or animals use to distinguish one potential landmark from another.

If reorientation and beacon guidance both reveal a purely geometric analysis of the environmental terrain, what kinds of geometric properties does each process represent? In many reorientation studies on children and animals, the shape of the tested environment is rectangular, and so the navigator's location is specified, up to a rotational ambiguity, by the lengths, distances and directions of the enclosure's walls. For example, the animal or child may encode the goal location as facing to the *left* of a *longer* wall or *closer* wall.[2] When the child or animal retrieves this information after disorientation, it will serve to reestablish the original orientation on half the trials, on average, and will yield an incorrect orientation, 180° off from the correct orientation, on the others. In other studies presenting a room with no rotational symmetries, children use the global shape of the room to distinguish the unique hiding location (Lourenco & Huttenlocher, 2006; Lee & Spelke, 2010a; Wang, Hermer, & Spelke, 1999; c.f., Lew et al., 2010). Because the enclosed room shapes used in previous studies provide distance information both in the form of wall lengths and wall distances, however, it is not clear whether the representation of environmental shape captures the lengths of surfaces or the distances of surfaces from one another or from the child or animal. Moreover, because rectangular rooms present only identical right angles, research using those rooms does not reveal whether children or animals reorient by detecting a third fundamental geometric property: the *angles* in the corners at which two surfaces meet.

Several studies have tested for reorientation in children and other animals in rooms of other shapes, with potentially informative angle information. Tommasi and Polli (2004) trained chicks to navigate to a food source in one corner of a parallelogram-shaped arena, with two long walls and two short walls meeting at two acute angles and two obtuse angles, and then tested their search when the space was transformed into a rectangular or rhombic arena. In both transformation conditions, chicks generalized their search to the two geometrically correct corners: In the rectangular arena, chicks approached the corners with the same relationships between the long and short walls as the corner that they were trained inside the parallelogram to find; in the rhombic arena, chicks approached the corners with the same angle as the corner that they were trained with. While these results demonstrate chicks' ability to use geometric properties, they do not clearly

[2] A central debate in experimental psychology and systems neuroscience concerns the frame of reference used by navigating animals: do animals represent the environments through which they navigate egocentrically (e.g., the distances and orientations of surfaces relative to the self), or allocentrically (e.g., the distances and orientations of surfaces relative to one another), or both (see Fig. 1)? This question has proved to be extremely difficult to resolve because many findings from studies of navigation can be described within either coordinate system (for reviews, see Burgess, 2006; Wang & Spelke, 2002). In the present study, therefore, we interpret our results to be applicable to either egocentric or allocentric representations. For instance, the successful use of distance in a rhombic array of objects could indicate the encoding of farther vs. closer distances from the center location of the subject (egocentric), or farther vs. closer distances between the hiding places themselves (allocentric).

provide evidence of navigation by angle, per se, even in the transformation between the parallelogram and the rhombus. In that condition, chicks performed with much higher accuracy when the goal was in the acute corner than when it was in the obtuse one. It is possible that chicks learned over trials to represent angular relationships between the sides of a parallelogram that made up an acute or obtuse angle and that acute angles are easier to learn. Alternatively, it is possible that, even without a representation of angles, chicks computed and navigated by the closer/farther distances between the walls of these environments (e.g., two walls meeting at a sharp, acute angle form a narrow stretch of space leading to that corner) and that the chicks placed alone in a new environment preferred smaller, safer spaces.

Children's disoriented search behavior has been examined in isosceles triangular rooms containing informative angle differences (Lourenco & Huttenlocher, 2006). Toddlers 18–24 months old watched a toy being hidden in one corner of the room, were picked up and disoriented, and then searched for the toy. Two different isosceles triangles were used in this study – one with two acute angles and one obtuse angle, and another with three acute angles. In both rooms, children successfully identified the correct location, regardless of whether the goal location was at the unique corner or one of the two mirror-image corners. The successful performance at the two mirror-image locations provides evidence of directional reorientation. Moreover, the lack of advantage for the unique angled corner suggests possible lack of advantage for using angle as a local landmark. Nevertheless, because length, distance, and angle were all available in these environments, successful reorientation in these environments does not reveal the types of information on which children based their disoriented search.

To isolate angles from wall lengths, a study by Hupbach and Nadel (2005) tested children 2–6 years old in a rhombus-shaped room composed of four walls of equal length that met at angles of 60° and 120°). On each trial, an object was hidden at a different corner of this rhombus, children were disoriented, and then they searched for the object. The older children searched for the object at the correct and geometrically equivalent corners, indicating a successful use of the geometrical shape. In contrast, the children aged 3 years and younger searched randomly at the four corners after disorientation, suggesting that they failed to use the angle information either as directional cues for reorientation or as local landmarks to the location of the hidden object.

This important finding raises several questions. First, what geometric property were 3- to 6-year-old children using to succeed? While the rhombus-shaped room eliminates wall length differences, there are distance differences both between the opposite corners and walls and between the children's center position and the rhombic array of goal locations. Did children navigate by the angles of the rhombus or by the distinctive lengths of its major and minor axes? Second, by what process did children locate the toy? Did they use the distinctive corners of the room as direct local landmarks, or did they reorient by this shape information?

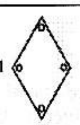
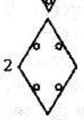
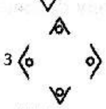
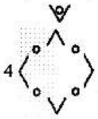
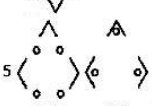
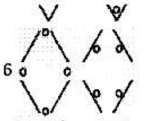
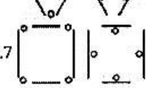
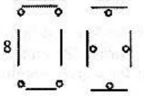
A further question concerns the failure of the 2-year-old children. Past reorientation studies have shown successful use of the shape of a rectangular space in toddlers as young as 18 months, when the same goal location is used across all trials (about 4) in a session (Hermer & Spelke, 1994; Learmonth, Newcombe, & Huttenlocher, 2001). In contrast, Hupbach and Nadel's (2005) methods involved changing the goal location to a different corner on each trial, such that children were required both to update their memory for hiding locations throughout the study and to inhibit responses to previous locations from one trial to the next. Working memory and executive control develop markedly over the preschool years, leading to developmental declines in perseverative responses (Diamond, 2006; Morton & Munakata, 2002; Munakata, 1998). It is possible, therefore, that the younger children failed to relocate the object not because they failed to use its rhomboid shape but because they lost track of the hidden object's location or made perseverative errors to a hiding place used on previous trials.

In summary, the large body of research on reorientation leaves fundamental questions unanswered concerning the shape properties that young children and animals use to reestablish their sense of place. In particular, when animals and children are tested in rectangular rooms with no distinctive angle information, do they reorient by *lengths* or *distances*? When they are tested in rooms of other shapes, do they reorient by *angle*?

Important research by Burgess and his collaborators, conducted both on rats and on adult humans using both behavioral and neuroimaging measures, suggests answers to these questions. This research provides evidence that the primary geometric relationships by which navigating rats and humans encode their own position are the distances and directions of extended surfaces, or "boundaries," relative to the navigator. The first evidence for navigation by distance and direction, but not length or angle, came from single-cell recordings of neurons in the hippocampus and surrounding cortical regions of oriented rats as they moved through a novel environment (Lever, Wills, Cacucci, Burgess, & O'Keefe, 2002; O'Keefe and Burgess, 1996; Solstad, Boccara, Kropff, Moser, & Moser, 2008). When rats explore a stable environment, many individual neurons fire in specific locations ("place cells": O'Keefe and Nadel, 1978; see Burgess, 2008, for review). When the navigable environment changes in size or shape, the response properties of these neurons are altered in ways that suggest that the representation of the rat's position is determined by distance and direction, but not angle or length, with respect to one or more of the environment's extended surfaces. For example, changes in the positions of the walls of the space that resulted in changes in the distance of a surface from the center of a hippocampal neuron's place field led to immediate and predictable changes in the neuron's firing (O'Keefe and Burgess, 1996), but changes in the corner angles of an arena that kept distances roughly constant (e.g., transformation from a square to a circle) preserved the firing patterns (Lever, Wills, Cacucci, Burgess, & O'Keefe, 2002; Solstad et al., 2008) until after many days of extensive exposure (Lever, Wills, Cacucci, Burgess, & O'Keefe, 2002). Furthermore, color or texture changes never led to remapping of the place fields, in accord with the thesis of a distinctly geometric analysis of these environments. Changes in the lengths of surfaces affected the place fields of some cells but not others, consistent with a mechanism for representing position selectively in relation to individual surfaces but not necessarily the global shape of the array (O'Keefe and Burgess, 1996).

The present research aims to clarify the nature of the geometric information guiding navigation in 2-year-old children. This age group was chosen with respect to Hupbach and Nadel's (2005) findings showing failure to reorient using the geometry of the rhombus; furthermore, we aimed to characterize the geometric navigation sense at an early stage of development, before children are exposed to geometric spatial language, spatial symbols and maps, or explicit education about geometric properties. First, we test for 2-year-old children's use of angle information both to specify a distinctive local landmark, and to specify a distinctive surface layout for reorientation. Second, we test for sensitivity to two kinds of distance information: sensitivity to surface *lengths* and sensitivity to surface *distances*. To dissociate these different properties, we test the search behavior of disoriented children in a variety of fragmented, rotationally symmetrical environments arranged in the shape of a rhombus, rectangle, or square (Table 1).

Table 1
Arrays and findings from 8 experiments testing children's use of geometry for navigation.

		Proportion of geometric searches	Success?
Exp. 1		0.73	Yes
Exp. 2		0.70	Yes
Exp. 3		0.56	No
Exp. 4		0.54	No
Exp. 5		0.45	No
Exp. 6		0.69	Yes
Exp.7		0.45	No
Exp. 8		0.69	Yes

In eight experiments, therefore, we probe disoriented children's sensitivity to *angle, length, distance*, and *direction*. The experiments focus on 2-year-old children's search performance with a single hiding location, in order to minimize demands on memory and executive function. In all the experiments, moreover, we test for use of geometric properties both for local landmark guidance and for reorientation, by comparing children's performance when an object is hidden directly at a corner or surface (such that its distinctive shape can serve as a direct cue to its location) to their performance when an object is hidden to the left or right of a corner or surface. If children use a particular geometric property to define a local landmark, then they should perform better when the hiding location is directly at the landmark. In contrast, if children use the geometric property to reestablish their own sense of orientation, then they should perform with equal success regardless of where the object is hidden.

2. General methods

Subjects: Participants were 112 children between 24 and 37 months of age (mean = 30 months, SD = 3.6) from Cambridge, Massachusetts and surrounding areas. Three children were excluded for failure to complete the session.

2.1. Experimental room

All experiments took place in a symmetrical, soundproof cylindrical room consisting of twelve curved panels and a ceiling-mounted, central camera and microphone. One of the panels had a spring hinge and functioned as the door but was indistinguishable from the other panels when closed. Centered within the room was an array of 45-cm-high surfaces (different for each experiment), with four white, opaque 2D disks that served as sticker hiding places.

2.2. Procedures

Before entering the testing room, the experimenter explained to the parents the procedures of the experiment and their role in it. Children were told that they would play a hiding and finding game, in which they would watch the hiding of a sticker, be picked up and turned, and then try to find the sticker. On each trial, the child and parent stood in the center of the room, while the experimenter pointed to the four white disks as possible sticker hiding places and oriented the child toward the actual goal location. The child watched the experimenter hide a sticker, and then was picked up by the parent, who rotated in place slowly 3–4 times while obstructing the child's vision by covering or shading the child's eyes, and while the experimenter circled the testing space and counted aloud to ten. The experimenter then stood against the wall, aligned with the predetermined facing direction for that trial, and the parent released the child and stood behind as the child was encouraged to find the sticker. If the first search was incorrect, the experimenter showed the child the correct location. Children's first searches were recorded. In past studies, as well as the present one, children tended to head directly for a particular location following disorientation. No patterns of hesitation, looking around the room, or orienting a particular way before searching were observed.

2.3. Design and analyses

Each child's session consisted of four trials, with a single hiding place and four facing directions (in clockwise order), resulting in four orders of facing directions, coupled with each of four hiding locations, resulting in 16 possible combinations. Consequently, 16 children were tested in each experiment, one child per corner-facing direction order combination. Two-tailed t-tests compared children's geometrically correct search (actual correct location, plus the geometrically identical opposite location) to a chance level of 50% in all of the arrays which tested symmetrical arrays with two geometrically correct search locations. Additionally, paired comparisons between searches at the correct location and its geometric twin confirmed disorientation and ensured that children's performance was not driven by an ability to track their own rotations.

3. Experiment 1

We begin by replicating and extending the findings of Hupbach and Nadel (2005), and revisit the question of the navigation capacity of younger children in a rhombic environment. Did younger children in Hupbach and Nadel's study search randomly because they failed to represent the geometric information in the rhombus? Or, did they fail because of the high task demands of updating the goal location on every trial? To address this question, Experiment 1 replicated Hupbach and Nadel (2005; Experiment 2, No-Landmark Condition) with 2-year-old children and an unchanging hiding location. Children were tested in a rhombic arena with potential hiding locations in its four corners (Fig. 2). In contrast to their experiments, the search location was constant across the four trials for any given child.

3.1. Method

An enclosed rhombic arena consisted of four 130-cm sides that met at two 60° corners and two 120° corners. This resulted in a rhombus with not only a 2:1 ratio in the measurement of complementary angles that cross the boundary between obtuse and acute, but also a 2:1 aspect ratio (the ratio between the long and short axes of the arena) and an environment with approximately the same size as the past studies with rectangular rooms. Participants were eight boys and eight girls, 27–37 months old (mean = 30.2, SD = 2.7).

3.2. Results

Children searched the two geometrically correct corners (correct plus geometric twin) 73.4% of the time, choosing them significantly more than the other corners (50% chance, Cohen's $d = 1.009$, $t(15) = 4.038$, $p = .001$; Table 1). Children performed equally well when the object was hidden at 60° vs. 120° corner locations (78.1% vs. 68.8%, $t < 1$). Children searched equally at the correct location and its geometric twin (34.4% vs. 39.1%, $t < 1$), providing evidence that they were successfully disoriented by the turning procedure. There were no sex differences ($t < 1$).

3.3. Discussion

The disoriented 2-year-old children used the shape of the rhomboid room to locate the hidden objects. Because children of this age have not used the geometric configuration of hiding containers to constrain their search for objects (Lee & Spelke, 2008, in review; see also Experiment 5, below), children's successful search likely depended on

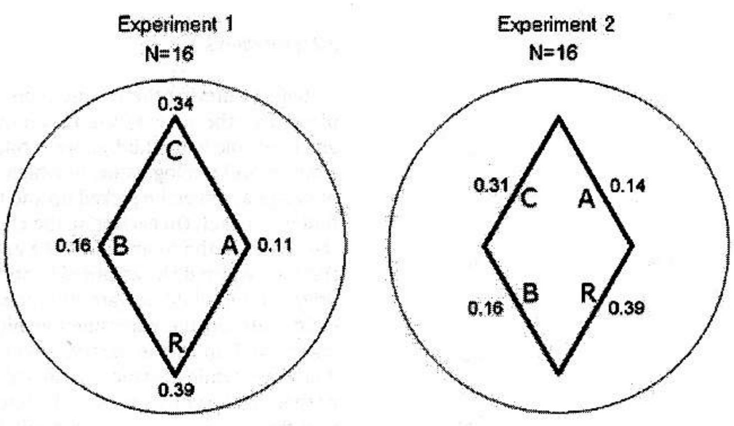


Fig. 2. Schematic diagrams of the geometric arrays in Experiments 1 (left) and 2 (right). Proportions of searches are given for the four hiding locations (represented by the letters C, R, A, and B): the correct target location (C), with the rotationally opposite geometric twin across from it (R), and the other incorrect hiding places clockwise (A) and counter-clockwise (B) from the correct location. Data are rotated so that all locations for a given condition are aligned.

some aspect of the geometrical structure of the rhombic surface array. These findings accord with Tommasi and Polli's (2004) study of chicks and contrast with Hupbach and Nadel's (2005) studies of young children, suggesting that either failures of working memory or response perseveration reduced children's performance in the latter studies.

Nevertheless, these findings do not reveal the process or the information guiding that process by which children located the hidden object. First, children may have reoriented by the overall shape of the rhombic room, as in past studies in rectangular environments. Alternatively, children may have treated the correct corner as a local landmark whose angular measure directly specified the object's location, as in past studies in which objects were hidden directly in distinctive containers in a room lacking any informative geometry (Lee et al., 2006).

To distinguish reorientation from landmark guidance, past studies in rectangular environments presented objects that were not hidden directly at a geometrically distinctive feature (e.g., at a short wall) but rather to the left or right of such a feature (e.g., left of a short wall), because children, like human adults and non-human animals, do not automatically encode the location of a hidden object relative to a spatially distant landmark (Doeller & Burgess, 2008; Lee et al., 2006; Sovrano et al., 2003; Vallortigara, Zanforlin, & Pasti, 1990). Accordingly, in the next experiment we repeated the search task with new hiding locations midway between each of the room's corners. We ask whether children's performance depends on reorientation, or on beacon guidance, by comparing performance at these spatially displaced locations (to the left or right of a potential beacon) to children's performance at the corner locations of Experiment 1 (directly at a potential beacon).

4. Experiment 2

In this experiment, we ask what process children use to locate an object in a rhomboid room. Because the object in Experiment 1 was always hidden *at* a corner, the children's success could have depended on either of the two navigational strategies: children may have reoriented themselves by the shape of their surroundings, or they may have used the differing angles at the room's corners as direct beacons marking the object's location. In Experiment 2, children were tested in the same rhombic environment, but with four hiding locations centered on the four walls of the arena (Fig. 2). Because the walls were equal in length, use of a wall as a direct landmark would not distinguish among the four hiding locations. Moreover, because the hiding places were equidistant from the center of the rhombus, a heuristic based on the distance from the child's center position would not be available. To distinguish the two geometrically correct locations from the incorrect ones, children needed to reorient using *directions* defined by the rhombus (see Fig. 1).

4.1. Method

Experiment 2 was identical to Experiment 1 except for the placement of the hiding containers, which stood at the middle of the sides of the rhombus, such that they were equidistant from the center of the rhombus and formed a rectangular array of hiding locations (Table 1). Participants were seven boys and nine girls, 26–36 months old (mean = 29.4, SD = 2.6).

4.2. Results

Children concentrated their search at geometrically correct locations (70.3% geometrically correct search, Cohen's $d = 0.892$, $t(15) = 3.569$, $p = .003$; Fig. 2). They searched equally between the correct location and its geometric twin (31.3% vs. 39.1%, $t < 1$), showing that they were successfully disoriented by the turning procedure and used the shape of the rhombic environment to reorient themselves. There were no sex differences ($t < 1$). Comparing across experiments, no difference was found in the use of geometry between the rhombus with the hiding locations in the corners (Experiment 1) vs. in the middle of the sides (Experiment 2) (Cohen's $d = 0.135$, $t < 1$).

4.3. Discussion

Children successfully used the shape of the room to guide their search for the hidden objects. Search was as consistent when the corners served as indirect landmarks, in Experiment 2, as when they served as direct landmarks, in Experiment 1. Thus, children did not single out the shape of the hiding location itself and use it as a beacon to guide their search for the object. Because children's equal search at the correct and opposite corners showed they were disoriented, these findings provide evidence that children used the shape of the environment to reorient themselves. Because children in past experiments (Lee & Spelke, 2008) and in Experiment 5 (described below) do not reorient by a rhombic arrangement of hiding containers, the relevant aspects of the environment's shape evidently were provided by the rhombic array of surfaces.

What geometric properties of the rhombic enclosure did children use to reorient themselves? A rhombic room is characterized by distinctive angles at its corners, by distinctive distance relationships between the larger and smaller corners of the array, and by distinctive distance relationships between the surfaces that meet at larger vs. smaller corners. Did children distinguish among corners by discriminating their angular size, their distance from one another or from the child's position at the room's center, or the differing separations between the walls that bounded the smaller vs. larger corners? In the next experiments, we presented children with a fragmented rhombus that allowed us to test specifically for children's sensitivity to angle in an array presenting no informative distance information.

5. Experiments 3 and 4

Experiments 3 and 4 followed the methods of Experiments 1 and 2, respectively, to test for children's use of the corner angles of the rhombus, in the absence of a distinctive aspect ratio or subject-relative array of distances.

Four corner angles of the sizes of those used in the full rhombus were placed equidistant from the room's center, and hiding locations were placed either at the corners (Experiment 3) or midway between them (Experiment 4) (see Fig. 3). If children can reorient by angle information, they should concentrate their search at geometrically correct locations in Experiments 3 and 4 as in Experiments 1 and 2. If children can use corner angles as beacons but not as cues to reorientation, then the children in Experiment 3 should confine their search to geometrically correct corner locations, but those in Experiment 4 should not. Finally, if children neither reorient nor distinguish local landmarks by angle information and instead reoriented themselves in the previous experiments by using distance information, then those in Experiments 3 and 4 should search equally at all four hiding locations.

5.1. Method

Each corner of the rhombus in Experiments 1 and 2 was detached from the walls, resulting in two 60° and two 120° corners, with sides 51 cm in length (Fig. 3). The freestanding corners were placed equidistant from one another such that their apexes formed a square array. The corners were oriented to open toward the center of the room, as in Experiments 1 and 2. In Experiment 3 (nine girls and seven boys, 26–35 months old, mean = 29.7, SD = 2.5), the hiding places were located directly inside the corners, where the corners could serve either as direct beacons marking a hidden object's location or as cues for reorientation. In Experiment 4 (seven girls and nine boys, 26–37 months old, mean = 29.4, SD = 3.4), the hiding places were located midway between the corners and served only as indirect markers of the hidden object (which was situated, e.g., to the left of an acute angled corner). Table 1 depicts these arrays, in relation to the arrays of Experiments 1 and 2.

5.2. Results

Children searched equally in the correct location and in the geometrically equivalent opposite location, both in Experiment 3 (25% and 31.3% respectively, $t < 1$) and in Experiment 4 (26.6% and 28.1%, $t < 1$), providing evidence that they were disoriented. Combining across search at these locations, children searched at geometrically correct locations no more than they searched at geometrically incorrect locations, either in Experiment 3 (56.3% geometric searches, Cohen's $d = 0.292$, $t < 1.2$, n.s.) or in Experiment 4 (54.7% geometric searches, Cohen's $d = 0.169$, $t < 1$) (Table 1). Performance in Experiments 3 and 4 did not differ, (Cohen's $d = 0.063$, $t < 1$), showing that the placement of the hiding locations did not affect children's performance. There were no sex differences ($t < 1$ for each experiment).

A further analysis compared search in Experiments 3 and 4 to search in Experiments 1 and 2. A 2 (*Search Location*: direct vs. indirect) by 2 (*Search Array*: complete vs. fragmented) by 2 (*Sex*) ANOVA on the proportion of geometrically correct searches revealed a main effect of *Search Array* ($F(1,56) = 8.038$, $p = .006$, partial $\eta^2 = 0.126$) and no other significant main effects or interactions.

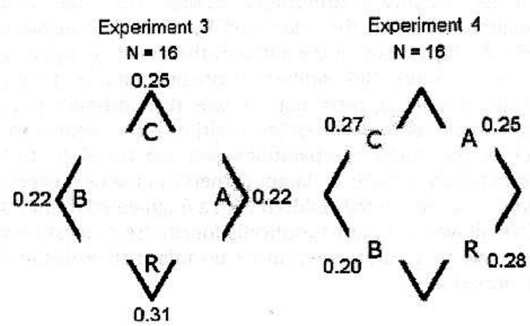


Fig. 3. Schematic diagrams of the geometric arrays in Experiments 3 (left) and 4 (right). Proportions of searches are given for the four hiding locations (represented by the letters C, R, A, and B): the correct target location (C), with the rotationally opposite geometric twin across from it (R), and the other incorrect hiding places clockwise (A) and counterclockwise (B) from the correct location. Data are rotated so that all locations for a given condition are aligned.

5.3. Discussion

When the isolated 60° corners and 120° corners were presented at equal distances from the center of the room, children searched randomly between them. Children's failure to confine their search to geometrically correct locations in Experiment 4, where those locations stood to the left or right of a distinctive corner angle, suggests that children failed to reorient by the distinctive angle information. Children's equally clear failure to confine their search to geometrically correct locations in Experiment 3, where those locations directly marked the hidden object's location, suggests that children also failed to use the distinctive corner angles as local landmarks to guide their search to an appropriate hiding location.

The findings of Experiment 3 contrast with those of earlier research in which disoriented children used the distinctive shapes and colors of landmarks as local beacons when they directly marked a goal location (e.g., Gouteux & Spelke, 2001; Lee & Spelke, 2008; Lee et al., 2006; Lew, Foster, & Bremner, 2006; Nardini et al., 2009). The findings suggest that a twofold distinction in angular size – from an acute angle of 60° to an obtuse angle of 120° – fails to define a distinctive landmark for beacon guidance, under the conditions of the present experiments. Nevertheless, many of the past findings of successful local landmark use involved remembering the features of the hiding container itself, rather than a nearby but not coincidental landmark; indeed, in the present experiment children limited their searches to the circular disks that served as hiding places.

Finally, children performed reliably less well in Experiments 3 and 4 than in Experiments 1 and 2. Because the methods and procedures of these experiments were identical and only the surface layouts differed, this contrast sheds light on the use of geometric information by navigating children. The four experiments presented exactly the same angle information, but they differed in three respects. First, the arrays of Experiments 1 and 2 were connected and continuous, whereas those of Experiments 3 and 4 were fragmented. In past research, young children have sometimes failed to reorient by fragmented arrays such as isolated corners (Gouteux & Spelke, 2001) or columns (Lee & Spelke, 2008; Lee & Spelke, 2011). Second, the arrays of Experiments 1 and 2 were characterized by a distinctive aspect ratio: the distance separating the two acute corners differed from the distance separating the two obtuse corners by a factor of two. Both in the present studies and in experiments in rectangular and isosceles triangular environments, children may reorient by aspect ratio. Third, the arrays of Experiments 1 and 2 contained extended planar walls arranged at distinctive distances and directions both from the child's position at the center of the array and from one another within the array (i.e., the walls that met at an acute angle were closer to one another than those that met at an obtuse angle), whereas those of Experiments 3 and 4 did not. Accordingly, the next experiments investigated which of these relationships are used by children to reorient themselves.

6. Experiments 5 and 6

Experiments 5 and 6 presented children with fragmented arrays with the distinctive shape and aspect ratio of the original rhombus in Experiments 1 and 2 (Fig. 4). In Experiment 5, the four freestanding corners from Experiments 3 and 4 were arranged in a rhombic array, in the same locations as they had occupied in the full rhombus conditions. In Experiment 6, four freestanding walls of equal length were arranged in the same rhombic array, separated by empty spaces where the corners had been. The total surface area of the four corners presented in Experiment 5 was equal to the total surface area of the four walls presented in Experiment 6. For half of the children in each experiment, the hiding place disks were located directly at the corners for Experiment 5 and between the sides for Experiment 6, where the array fragments directly marked the two geometrically correct object locations. For the remaining children, the hiding places were located between two corners for Experiment 5 and in the middle of the sides for Experiment 6, where the array fragments served only as indirect markers of the geometrically correct locations (Fig. 4).

Performance in these experiments should serve to determine whether children reorient only by representations of continuous arrays, by the aspect ratio but not the specific walls or corners of fragmented arrays, or by the particular distances and directions of individual walls in the array. If disoriented children navigate only by the geometric properties of continuous surface arrays, then children in both Experiments 5 and 6 should fail to locate the hidden object, and their search should resemble that of children in Experiments 3 and 4. If disoriented children successfully navigate by the distinctive aspect ratio of fragmented as well as continuous arrays, then children in both experiments should confine their search to geometrically appropriate locations. Indeed, their search should resemble that of children in Experiments 1 and 2, and it should be equally consistent across the two experiments. Finally, if disoriented children navigate by using the distance relationships among extended planar surfaces but not corner angles, then the children in Experiment 6 should search geometrically, but those in Experiment 5 should not.

6.1. Method

Nine girls and seven boys (Experiment 5: 24–37 months old, mean = 30.6, SD = 3.9; Experiment 6: 25–37 months old, mean = 30.3, SD = 3.7) participated in each experiment. Experiment 5 used the same corners as in Experiments 3 and 4 (height 45 cm, sides 51 cm each), positioned with the hiding locations as in Experiments 1 and 2. Experiment 6 presented four spatially separated freestanding surfaces (height 45 cm, length 102 cm) arranged in a rhombic array, as if the corners of the full rhombus from the first two experiments were removed (see Fig. 4).

6.2. Results

In both experiments, children searched equally at the correct corner and at its geometric twin, providing evidence that they were disoriented (Experiment 5: 21.9% vs. 23.4% searches respectively, $t < 1$; Experiment 6: 37.5% vs. 31.3%, $t < 1$; see Table 1). In Experiment 5, children searched at the geometrically correct locations only on 45.3% of the trials (Cohen's $d = 0.265$, $t < 1$). The placement of the hiding locations did not affect children's performance ($t < 1$), and there were no sex differences in accuracy ($t < 1$). Performance using the rhombic array of corners in the present experiment was significantly worse than performance in the complete rhombus (Experiments 1 and 2), Cohen's $d = 0.845$, $t(46) = 3.551$, $p = .001$. In Experiment 6, in contrast, children searched in accord with geometry 68.8% of the time (Cohen's $d = 0.750$, $t(15) = 3.000$, $p = .009$). The placement of the hiding locations did not affect children's performance ($t = 0$). There were no sex differences ($t < 1.2$, n.s.). Performance using the array of side surfaces was as high as performance using the complete rhombus in Experiments 1 and 2 (Cohen's $d = 0.131$, $t < 1$) and better than performance in Experiment 5, using the fragmented, rhombic array of corners (Cohen's $d = 0.649$, $t(30) = 2.512$, $p = .018$).

6.3. Discussion

The findings of Experiments 5 and 6 shed light on children's use of geometry for reorientation. First, they provide evidence that children reorient by fragmented as well as complete arrays of surfaces, when the surfaces are extended and planar (Experiment 6). Disoriented children successfully searched for the hidden object according to distance relationships in the array of four walls, even though the walls were separated by gaps and therefore formed no continuous array, and they performed as well with objects hidden at indirect vs. direct locations, indicating that they did not single out particular local features of the hiding location to use as beacons. These findings provide evidence that children reoriented by the surface distance relationships.

Second, the experiments reveal no effect of angle information on children's reorientation: Although Experiment 5

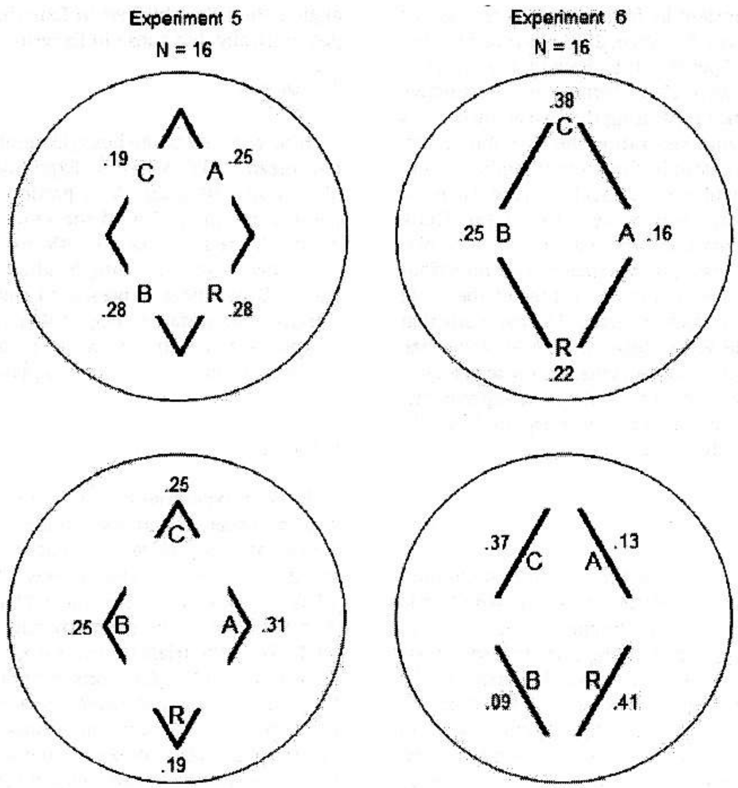


Fig. 4. Schematic diagrams of the geometric arrays in Experiment 5 (left) and Experiment 6 (right), in which the hiding places were located between the sides/corners (and required directional reorientation) (top) or directly at the sides/corners (bottom). Proportions of searches are given for the four hiding locations (represented by letters C, R, A, B): the correct target location (C), the rotationally opposite location (R), and the geometrically incorrect hiding places clockwise (A) and counter-clockwise (B) from the correct location. Data are rotated so that all locations for a given condition are aligned.

presented opposite corners both of markedly different angles and at markedly different distances from the room's center, children's random search indicated that they were not guided by either of these sources of information. Children performed as well in a rhombic array of extended surfaces without the corner angles (Experiment 6) as in a connected rhombic array that contained those angles (Experiments 1 and 2). Thus, children did not appear to navigate by angle information.

Third, children reoriented more reliably in Experiment 6, which presented wall surfaces at different distances from one another, than in Experiment 5, which presented corners at different distances from one another. This finding adds to the evidence that navigating animals and humans encode their position with respect to the distances and directions of extended surfaces that create borders in the navigable environment (Doeller & Burgess, 2008; Hartley, Trinkler, & Burgess, 2004; O'Keefe and Burgess, 1996).

These results also show that the aspect ratio of an array is not sufficient for reorientation when the aspect ratio is created by a fragmented array of corners. Children's contrasting performance in Experiments 5 and 6 is especially interesting, because the arrays in the two experiments were exactly equated in shape, surface area, and placement of the hiding containers. In particular, the four 102-cm surfaces were planar in Experiment 6 but bent at a corner angle with 51 cm on each side in Experiment 5. Children's significantly greater geometric search with the walls in Experiment 6 than with the corners in Experiment 5 provides evidence that children reoriented only by the distances and directions of the extended surfaces in the array. Experiment 6 therefore provides one of the first pieces of evidence that young children reorient solely by distance and directional relationships between identical surfaces. Because performance in the fragmented array of Experiment 6 was equal to that in the full rhombus conditions of Experiments 1 and 2, we conclude that children's performance in all these studies was based on their successful use of distance and directional relations in the layout of the side surfaces of the rhombus. When an object was hidden at the center of a wall that formed an acute angle with the wall to its left and an obtuse angle with the wall to its right, for example, children may have encoded the target location as *at the wall whose left side is further from the center* (or, equivalently, *from my position*) *than its right side*. Children's performance depends on an encoding both of distance and of direction, because in the rhombic array of Experiment 6, all four walls were equidistant from the children's position at their midpoints. For two walls, however, the more distant side was on the *left*; for the other two walls, the more distant side was on the *right*.

The finding that children reorient by distance and direction in rhombic arrays, ignoring the size of corner angles, raises the question whether they also reorient by distance and direction in rectangular arrays, ignoring the lengths of walls. The final two experiments tested further children's sensitivity to surface distance, as well as their sensitivity to the surface *length*, by presenting children with fragmented rectangular arrays. In a connected rectangle, surface length is confounded with surface distance: the two shorter walls of a rectangular room are farther from one another, and from the room's center, than are the two longer walls of the room. Furthermore, all surfaces are presented at the same orthogonal orientation from the children's view from their center position. Although myriad experiments have tested the abilities of children and animals to reorient by the shape of such a room, no study, to our knowledge, has asked whether the relevant shape property is surface distance, surface length, or both. The last two experiments address this question.

7. Experiments 7 and 8

Experiment 7 investigated whether children reorient by surface length when surface distances are held constant. To this end, we presented children with an array consisting of two short surfaces and two long surfaces arranged in a square, equating distances between surfaces while varying their lengths in a 2:1 relationship. Experiment 8 investigated whether children reorient by surface distance in a rectangular environment in which surface length is held constant. To this end, we presented children with an array consisting of four surfaces of equal length arranged in a rectangle, varying the distances between surfaces again in a 2:1 relationship, while equating the lengths of the surfaces (Fig. 5). For half of the children, the hiding places were located directly at the center of each of the surfaces to test for direct use of distance or length; for the others, the hiding places were located between the surfaces to test for reorientation by distance or length.

Children's performance in these experiments should differentiate between the potential explanations for their patterns of success in past experiments in connected rectangular arenas. If children in Experiment 8 reorient successfully in the rectangular array of equal-length surfaces, that finding will reveal that the reorientation system is sensitive to differences in the distances of surfaces, either from one another or from the child's position at the array's center. If children in Experiment 7 reorient successfully in the square array of alternating surfaces whose lengths differ in a 2:1 ratio, that finding will reveal that the reorientation system is sensitive to differences in the lengths of surfaces. Finally, if children succeed in one or both of these experiments only when the hiding places are located directly at the centers of the surfaces, that finding will reveal that children use distance or length as a direct, local cue to the hidden object's location rather than for reorientation.

7.1. Method

Participants were eight boys and eight girls (23–36 months old, mean = 32.4, SD = 3.9). Each child participated in both Experiments 7 and 8. Half participated in Experiment 7 first and the other half participated in Experiment 8 first. The test array for Experiment 7 consisted of two surfaces 68 cm in length and two surfaces 136 cm in length (total area of surfaces same as Experiments 3–6 and 8). The area of the array for Experiment 7 could not be equated to that of the rhombus experiments because of its different global shape. To ensure that the size of the square array did not affect performance, half of the children were presented with the surfaces arranged in a square array with a smaller area than the array of Experiments 1–6 (150 cm on each side), and the other half were presented with the surfaces in a square array with a larger area than in those experiments (180 cm on each side). The test array for Experiment 8 consisted of the same four surfaces used in Experiment 6 (height 45 cm, length 102 cm) in a rectangular arrangement of the same 2:1 aspect ratio tested in Experiments 1, 2, 5, and 6 (see Table 1). Finally, as in the previous experiments, half of the children in each condition were presented with the hiding locations in the empty space between the surfaces, and the other half were presented with the hiding locations at the midpoint of each of the surfaces.

7.2. Results

In both experiments, children searched equally at the correct corner and at its geometric twin, providing evidence that they were disoriented (Experiment 7: 18.8% vs. 26.7% searches respectively, $t = 1.05$, n.s.; Experiment 8: 34.4% vs. 34.4%, $t = 0$). In Experiment 7, children searched at the geometrically correct locations only on 45.3% of the trials (Cohen's $d = 0.191$, $t < 1$, see Fig. 5). There were no differences in performance dependent on the placement of the hiding locations, the size of the square array, whether the test was conducted before or after Experiment 8, or the sex of the child (all t's < 1). In Experiment 8, in contrast, children searched in accord with geometry 68.8% of the time (Cohen's $d = 0.634$, $t(15) = 2.535$, $p = 0.023$, see Fig. 5). The placement of the hiding locations did not affect children's performance ($t < 1$), there was no difference between children who participated in the length condition (Experiment 7) before or after Experiment 8 ($t < 1$), and there were no sex differences ($t < 1$). Performance using the rectangular array of side surfaces in Experiment 8 was as high as performance using the complete rhombic array of equal-length surfaces in Experiment 6 ($t = 0$) and better than performance using length differences in Experiment 7 (Cohen's $d = 0.518$, $t(15) = 2.39$, $p = 0.030$).

7.3. Discussion

Children's performance revealed successful use of *distance* relations (i.e., distances of the surfaces either to the child's position at the array's center or to each other) to guide disoriented search. In contrast, children failed to

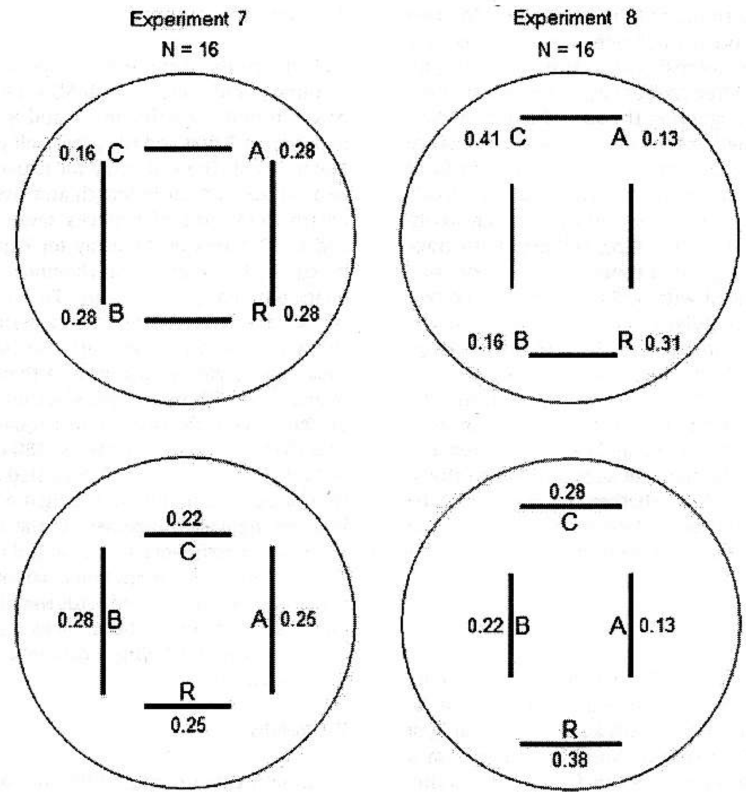


Fig. 5. Schematic diagrams of the geometric arrays in Experiment 7 (left) and Experiment 8 (right), in which the hiding places were located between the side surfaces (and required directional reorientation) (top) or directly at the surfaces (bottom). Proportions of searches are given for the four hiding locations (represented by letters C, R, A, B): the correct target location (C), the rotationally opposite location (R), and the geometrically incorrect hiding places clockwise (A) and counter-clockwise (B) from the correct location. Data are rotated so that all locations for a given condition are aligned.

use *length* relations (e.g., lengths of surfaces relative to one another and/or to the child's own body or visual field) to guide their search. Children's equal performance with hiding locations at the walls' midpoints and at the spaces between the walls reveals that children used the distance relations as guides to reorientation, not as local beacons. Furthermore, children's random search even when the hiding locations were placed directly at the center of the surfaces in Experiment 7 showed that children failed to use the distinctive lengths either to guide their reorientation or to select a beacon to guide them to the correct location.

These results illuminate the use of environmental geometry in past studies of reorientation using rectangular spaces. In a continuous rectangular arena, the child's orientation can be specified both by the relative lengths and directions of walls (e.g., the goal location is to the left of a *long* wall) and by the relative distances and directions between the wall surfaces (e.g., the goal location is to the right of a *more distant* wall). The present experiments reveal that disoriented children are able to use wall distances and directions but not wall lengths to guide their navigation.

8. General discussion

The present findings shed light on navigating children's sensitivity to geometry. They provide evidence that young children are sensitive to two abstract properties at the core of Euclidean geometry: *distance* and *direction*. In contrast, these findings provide no evidence that navigating children encode two further abstract Euclidean properties: *length* and *angle*. Neither the relative lengths of the arena's axes, nor the relative lengths of surfaces, nor the angles at which surfaces meet guided the search of the disoriented children in these studies. We begin by reviewing the evidence for these conclusions, and then we consider their implications, respectively, for research on navigation, on visual form analysis, and on mathematics education.

8.1. Summary of findings

Across eight experiments, 2-year-old children used the distance and directional relations between the surfaces of a rhombic or rectangular enclosure to guide their reorientation. In Experiments 1 and 2, children confined their search to geometrically correct locations in the rhombus, both when they searched directly at a corner of the array and when they searched to the left or right of a distinctive corner, providing evidence that they did not use the corners as local landmarks. Moreover, children searched the incorrect but geometrically appropriate location as often as the correct location, providing evidence that they did not succeed by maintaining their sense of orientation over the turning procedure. Thus, some geometric shape property of the rhombus guided children's reorientation in these initial experiments. This finding does not reveal by itself which geometric relationships children used, however, because the geometrically correct locations were distinguished by distance, length and angle.

Experiments 3–6 revealed that in a rhombic environment, children's reorientation is guided by *distance* and *direction* but not *length* or *angle*. Children failed to reorient by the distinctive corner angles of the rhombus when they were placed in an equidistant square array (Experiments 3 and 4) or in a rhombic array (Experiment 5). In Experiment 6, in contrast, children successfully reoriented by the four equal-length walls of a room lacking any corner angles, when they were arranged so that two surfaces were closer to the child on their left side and two were closer on their right side. Children's contrasting performance with corners vs. walls cannot be explained by the geometrical configuration of the hiding containers or the array as a whole, because of the contrasting findings of Experiments 5 and 6, in which children were presented with the same rhombic configuration but performed very differently. Children evidently represent their position at a particular distance and direction from one or more extended surfaces in the array.

Experiments 7 and 8 confirmed the findings of Experiment 6 and uncovered a further limitation to the geometric representations underlying children's reorientation: Not only do disoriented children fail to use angle differences when corners are presented at equal distances (as in Experiments 3–4), they also fail to use length differences when surfaces are presented at equal distances (Experiment 7). In contrast, the children in Experiment 8 reliably used the relative distances and directions of extended surfaces to guide their reorientation in a rectangular space, just as they do in a rhombic space.

The present findings reveal sharp limits to the geometric properties of landmark objects that disoriented children used for beacon-guided navigation. Children failed to use both distinctive angle sizes (Experiments 3–5) and distinctive surface lengths (Experiment 7) as direct features of landmarks specifying the location of a hidden object. Disoriented children's failure to use angle and length to distinguish between local landmarks contrasts with their early sensitivity to angle and length for visual form discrimination. It also contrasts with past navigation research, in which disoriented children have returned to hidden locations successfully when they were marked by beacon objects that were distinguishable by color and shape (Lee et al., 2006) or by shape only (Nardini et al., 2009), although these past studies tested older children (4 and 5 years of age, respectively) and properties such as size and surface area were not equated across the experiments. Nevertheless, the failure to use available local landmarks (and heavy reliance on environmental geometry) is often seen in tests of reorientation in both nonhuman animals and toddlers (Cheng, 1986; Hermer & Spelke, 1994; Wang et al., 1999), suggesting that there may be something particular about being in a state of disorientation that, in some instances, discourages the use of local landmarks. Landmark use also may have been discouraged, in the present studies, because the potential landmarks were surfaces (corner or wall fragments) rather than recognizable objects. Finally, while the present experiments show children's failure to spontaneously use angle and length for navigation across only four test trials, children may be capable of learning to use these cues to directly identify beacons or to reorient over many reinforced trials.

8.2. Navigation

The present research sheds light on the geometric information that children represent for purposes of navigation. It suggests that past claims that children, from an early age, reorient by analyzing the shape of the surrounding layout (e.g., Cheng & Newcombe, 2005; Hermer & Spelke, 1994; Lee & Spelke, 2010b) must be qualified. Children's reorientation does not depend on either of the two primary properties characterizing the shape of any form: the relative lengths of its parts and the angles at which those parts meet. Instead, reorientation depends on the distance and directional relations among the extended surfaces in the layout: either the distances and directions of surfaces with respect to one another, or their distances and directions with respect to the child's position at the center of the array. This specificity observed in children's ability to distinguish between locations provides evidence against image-matching theories that would predict successful navigation by such salient differences in the retinal images of those angles and lengths (Cheng, 2008).

The present findings bear on recent findings concerning children's reorientation in environments of various shapes. Newcombe, Ratliff, Shallcross, and Twyman (2009) showed that 3-year-old children successfully find hidden toys in an octagonal arena consisting of an array of alternating walls of two different lengths. Because this arena had no informative aspect ratio, the authors interpreted those findings as evidence that children's reorientation depends on representations of surface length and direction. While the present study was conducted with slightly younger children, Experiment 5 in the present series complements the finding that an informative aspect ratio is not necessary for reorientation, by showing that it also is not sufficient: children failed to reorient within a rhomboid arena with an informative aspect ratio but no extended surfaces. Nevertheless, the present findings suggest an alternative interpretation of the performance of children in the octagonal environment used by Newcombe et al. In an octagonal enclosure, alternating walls that differ in length also differ in distance from the child's position at the center of the display: shorter walls are more distant than longer walls. Thus, children may have reoriented by anchoring directional information to surface distance rather than length. This possibility could be tested through research with fragmented octagonal arrays.

The present experiments also bear on disoriented 1.5–3-year-old children's performance in rooms that vary in symmetry. In one experiment by Lew et al. (2010), children were disoriented in either a rectangular room or a room consisting of four walls of unequal lengths/distances, meeting at unequal angles. In further experiments, children were disoriented in a circular enclosure with 3–4 large objects at its borders providing distinctive geometric information in either a symmetrical (isosceles or rectangular) or asymmetrical arrangement. Children reoriented successfully only in the symmetric environments. This finding appears to provide the strongest evidence that children reorient by environmental shape, contrary to the present findings, because symmetry is a global shape property. Nevertheless, two features of the environments used by Lew et al. complicate the interpretation of their findings. First, the asymmetrical quadrilateral room varied more subtly in surface distance relationships than did the symmetrical rectangular room. Second, the number of distinctive distance relationships was higher in the asymmetrical environments than in the symmetrical ones. Thus, children's search failures in these studies may be explained either by the subtlety of the distance relations or by the multiplicity of different distance relations, presented in Lew et al.'s studies. Because the present experiments only test symmetrical arrays, this explanation could be tested by further research comparing children's reorientation in fragmented arenas whose surfaces are placed at distances from the center of the room that are equated, in number and magnitude, across arrays that vary in symmetry. Based on the present findings, we predict that children will reorient successfully in asymmetrical as well as symmetrical environments when the array presents surfaces at a small number of distinctive distances, and that they will fail to reorient in symmetrical as well as asymmetrical environments when surface distances are either subtle or too numerous.

The results of the present study broadly accord with research on navigation using behavioral and neurophysiological methods with animals, using behavioral and neuroimaging methods with human adults, and using the computational tools of computer science to design autonomous, mobile robots. We briefly discuss each of these broader implications.

First, these findings not only inform spatial cognition in humans but also in nonhuman animals and their neural representations of space. Behavioral search patterns of rats in a circular water maze with external landmarks show that disoriented rats rely on the distance between the hidden platform and the walls of the apparatus over the distance relationships to an array of external landmarks, even when the walls are made of transparent material designed to reduce the salience of the walls and improve the visibility of the landmarks (Maurer & Derivaz, 2000). Furthermore, the importance of distance and direction is consistent with the findings that a variety of animals represent parts of the environment with respect to its axes of elongation (Cheng & Gallistel, 2004; Kelly, Chiandetti, & Vallortigara, 2011; c.f., Pearce, Good, Jones, & McGregor, 2004; Tommasi & Polli, 2004), although our findings suggest that principal axis theories also should be qualified. Children's successful reorientation with an array of surfaces at varying distances, and their failure to use the same aspect ratio when reorienting within an array of corners, suggest that environmental axes used for navigation are computed not with respect to any environmental input but specifically from the extended surface layout. Nevertheless, the present studies do not address whether these geometric representations of the environment are local or global, and they do not reveal whether the same limits on reorientation by aspect ratio apply to non-human animals. Further research with animals using arrays that tease apart aspect ratio, axes of symmetry, and geometric properties (i.e., angle, length, distance) is needed to determine whether similar geometric representations underlie navigation across various species of animals, as some behavioral experiments (Maurer & Derivaz, 2000) and experiments using neurophysiological methods (Lever et al., 2002; O'Keefe and Burgess, 1996) suggest.

The finding that surface distances guide reorientation is in close agreement with studies of the activity of individual neurons in the hippocampus and surrounding cortex of navigating rats. As we have noted, hippocampal place cells encode an animal's position by representing location relative to extended wall-like surfaces in the environment (Lever et al., 2002; O'Keefe and Burgess, 1996; Solstad et al., 2008). A model based on place cell responses in rats proposed that place cells are attuned to barrier-like surfaces at a particular distance and direction, with sharper coding at closer distances (Hartley, Burgess, Lever, Cacucci, & O'Keefe, 2000). This boundary vector cell (BVC) model can predict the responses of place cells that remain constant over changes in the curvature of the encoded surface – from flat, to curved, to bent – consistent with a lack of sensitivity to angle. In recent research, cells that fit the BVC model appear to provide input to place cells regarding the surrounding surface layout (Lever, Burton, Jeewajee, O'Keefe & Burgess, 2009). Furthermore, although place cell firing is modulated by extended surfaces, it is not modulated by objects or discrete landmarks in the middle of the space (Cressant, Muller, & Poucet, 1997).

Our findings also accord with the performance of human adults in virtual navigation tasks. As we noted in the introduction, adults, who are given the task of returning a previously viewed object to its former position, will use information both from landmark objects and from extended surfaces to encode and retrieve that position. Adults' encoding of object positions with respect to landmarks, however, depends on attention, whereas their encoding of object positions with respect to an extended surface does not (Doeller & Burgess, 2008). Moreover, the latter encoding, but not the former, is associated with activity in the hippocampus, homologous to that shown in the neurophysiological studies just discussed (Doeller, King, & Burgess, 2008). The automaticity of the encoding of one's distance and direction from nearby extended surfaces, observed in these studies, could explain why information about surface distance and direction is available to children after disorientation. Full disorientation is a rare event, and children likely do not prepare for it in advance. Because surface distance and direction are automatically encoded, however, these relationships will be available to children, after disorientation, to allow them to reestablish their own orientation and position within the array.

Finally, our findings are consistent with those of research in computer science that aims to design navigation systems by which robots can represent both the layout through which they navigate and their own place within that layout. A large class of systems accomplishes both these tasks at once, through a process of Simultaneous Localization and Mapping (SLAM: see Thrun, 2002, for a review and discussion). This system is of special interest in the present context, because reorientation will be possible only if some process of mapping the environment has taken place. Studies in robotics therefore provide an interesting perspective on research on reorientation in humans and animals.

Effective robot navigation systems, including SLAM systems, must overcome a number of difficult problems (see Thrun, 2002), three of which are relevant to the present discussion: the problem of correspondence (navigable layouts often contain similar objects and surface features at different positions, leading to confusion of a new location with a familiar, featurally similar location), the problem of change (navigable layouts often contain moveable objects, whose displacement alters scene features in ways that can lead to failures of recognition of scenes that were previously encountered), and the problem of computational complexity (every individual scene in a natural environment contains a large amount of visual information, and every motion of the robot produces a new, similarly information-rich vista; if all this information were stored, the computational resources needed for navigation would be prohibitively large).

Investigators of robot navigation have long recognized that a focus on extended planar or near-planar surfaces can solve the third of these problems (Egerton, Callaghan, & Chernett, 2000; Thrun, 2002). Navigation by representations of extended surfaces is extremely economical, because the position and orientation of a planar surface can be represented by just three points; smoothly curved surfaces require only a few more points for their curvature to be specified (Gee, Chekhlov, Calway, & Mayol-Cuevas, 2008). In contrast, navigation by representations that include landmark objects require more information, because local landmarks often are misrecognized without highly detailed local representations (Egerton et al., 2000).

Drawing on research on animals, Gallistel (1990) has observed that a focus on extended planar or near-planar surfaces also can address the first two problems. Concerning the correspondence problem, the extended surfaces in natural layouts, and in large, artificial layouts, often have similar colors and textures, and they often stand under or behind objects that can have similar shapes, but they rarely appear in symmetrical arrangements. A navigation system that focuses on the distance relations among these surfaces therefore will make fewer correspondence errors than one that focuses on surface properties like color and texture, or on object shape properties like angle and length. Concerning the problem of change, the extended surfaces in natural layouts are less likely to move than are objects. A navigation system that focuses on extended surfaces rather than objects therefore will be less likely to fail to recognize familiar scenes as the objects within them are displaced.

In summary, the present findings accord with those of a rich array of fields from psychology to biology to applied mathematics and computer science. These findings do not accord, however, with the findings of research in what would seem to be a closely related area of developmental psychology: research on the development of form perception.

8.3. *Visual form analysis*

Although children failed to use angular or length relations in the present studies, a large literature on visual form and object perception shows that even younger children and infants are sensitive to this information in other task contexts. A long history of research using various tasks including word learning and object manipulation provides evidence that young children perceive the length and angular relations that specify the shapes of objects (DeLoache, Kolstad, & Anderson, 1991; Gentner, 1978; Landau, Smith, & Jones, 1988; Samuelson & Smith, 2005; Smith, 2009). Even infants recognize common objects by their shapes in both 2D and 3D arrays (Pierroutsakos & Deloache, 2003). When various geometric properties were presented to infants using 2D visual forms, they displayed high sensitivity to angle and length from their first months of life (Schwartz & Day, 1979; Slater, Mattock, Brown, & Bremner, 1991).

In contrast, infants and children show much lower sensitivity to the directional (or sense) information that distinguishes a form or object from its mirror image (Lourenco, Huttenlocher, & Fabian, 2005). In studies of adults, visual form analysis and object recognition are invariant both over mirror image reflection and over changes in scale (Biederman & Cooper, 1992; Malach et al., 1995): they therefore generalize over relationships of distance and direction. Both scale- and directional invariance characterize visual form perception throughout development (Izard & Spelke, 2009). Children's evident lack of sensitivity to these properties in the present studies therefore does not reflect either a general failure to perceive length or angular relationships or a general success in perception of distance and directional relationships, in visual arrays. Instead, navigation and object recognition evidently depend on different geometric properties and relations (Landau & Jackendoff, 1993).

Why do children fail to represent the shapes of the arrays in the present studies, when they succeed at representing the shapes of forms and objects? First, children may readily extract the shapes of small forms, but less readily extract the shapes of larger (room-sized) ones. Second, children may readily perceive the shapes of forms that are convex (like an object) but not the shapes of forms that are concave (like a hole in a surface or the space enclosed within a room). Third, children may extract shape with equal readiness from arrays large and small, and from both convex objects and the holes or spaces they enclose, but they may fail to use this shape information to guide their navigation. Future research focusing on tasks of shape recognition is needed to test these possibilities (for a start, see Shutts, Ornkloo, von Hofsten, Keen, & Spelke, 2009).

Regardless of the reasons for children's failure to reorient by length or angle information, the contrast between the findings of present experiments and those of the large literature on children's visual form perception suggests that distinct systems of geometrical analysis serve to guide children's analysis of the shapes of objects and visual forms, on one hand, and the shapes of 3D navigable layouts, on the other (Spelke, Lee, & Izard, 2010). Much remains to be learned concerning the conditions under which each of these systems is activated. The convergence of the present findings with those of studies of oriented navigation (Doeller & Burgess, 2008; Hartley et al., 2004) suggests that the system for analyzing distance and directional relationships to extended surfaces is activated not only after disorientation but during oriented navigation as well. It is not clear, however, whether the differing systems are activated by the differing tasks that children are given (finding vs. recognizing objects) or the differing displays they encounter (large 3D enclosing surfaces vs. small objects or 2D forms).

8.4. Implications for development and education

The present research gives a new perspective to our starting hypothesis, that human knowledge of abstract geometry is grounded in mechanisms of navigation. Consistent with this hypothesis, these experiments provide evidence that disoriented children, from a very young age, reorient with respect to two abstract properties at the core of Euclidean geometry: *distance* and *direction*. Nevertheless, disoriented children are not sensitive to all properties on which Euclidean geometry builds. These findings both allow for, and limit, the possible roles that the child's navigation systems can play in the development of universal geometric intuitions (Izard et al., 2011; Newcombe & Uttal, 2006), of symbolic, map-based navigation (Dehaene et al., 2006; Shusterman, Lee, & Spelke, 2008; Uttal, 2000), and of abstract geometrical reasoning (Izard et al., 2011).

There is, nevertheless, a second potential foundational system of core geometry that includes geometrical content missing from the navigational system – object form analysis. Processes of form analysis also capture some, but not all, of the fundamental properties of Euclidean geometry: they are sensitive to length and angle but tend not to preserve either the directional information that distinguishes a form from its mirror image or the distance information that distinguishes forms with similar shapes but different sizes. These observations suggest a hypothesis: the construction of an abstract geometry that includes *length* and *angle*, as well as *distance* and *direction*, may arise from the combination of the systems of navigation and form analysis, each of which is specific and limited (Spelke et al., 2010). If our most abstract geometrical intuitions are rooted, in part, in systems of representation that children activate when they navigate, then future research specifying the geometrical content of young children's spatial representations and identifying the causes of changes in those representations with age and experience may help to enhance the development of our most abstract, mature geometrical concepts.

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