CHAPTER

18

Natural Number and Natural Geometry

Elizabeth S. Spelke

Harvard University, Cambridge, USA

Summary

How does the human brain support abstract concepts such as *seven* or *square*? Studies of non-human animals, of human infants, and of children and adults in diverse cultures suggest these concepts arise from a set of cognitive systems that are phylogenetically ancient, innate, and universal across humans: systems of *core knowledge*. Two of these systems—for tracking small numbers of objects and for assessing, comparing and combining the approximate cardinal values of sets—capture the primary information in the system of positive integers. Two other systems—for representing the shapes of small-scale forms and the distances and directions of surfaces in the large-scale navigable layout—capture the primary information in the system of Euclidean plane geometry. As children learn language and other symbol systems, they begin to combine their core numerical and geometrical representations productively, in uniquely human ways. These combinations may give rise to the first truly abstract concepts at the foundations of mathematics.

For millenia, philosophers and scientists have pondered the existence, nature and origins of abstract numerical and geometrical concepts, because these concepts have striking features. First, the integers, and the figures of the Euclidean plane, are so intuitive to human adults that the systems underlying them are called "natural number" and, by some, "natural geometry" [1]. Second, these two systems are extremely useful: it is hard to find any important human cultural achievement—from money to measurement, to the arts, sciences, and mathematics—that does not depend on them. Third, these conceptual systems are simple: five postulates, together with some general axioms of logic, suffice to define all the objects of Euclidean geometry, and an even smaller set of postulates defines the positive integers. It is perhaps not surprising, therefore, that explorations of the origins of abstract concepts, from Plato [2] and Kant [3] to Piaget [4] and Carey [5], often use number or geometry as case studies.

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Although natural number and natural geometry are important, intuitive, and formally simple, they are also highly puzzling from the standpoint of psychology and neuroscience. One puzzle concerns the sources of these concepts. Most concepts, like *crow* and *cell phone*, appear to be shaped by experience, but it is far from obvious how experience could produce these abstract concepts. All our perceptions have finite resolution, yet the objects of Euclidean geometry are points so small they have no extent, and lines so thin they have no thickness. All our actions, moreover, have finite extent and duration, yet we conceive of lines as infinitely long and of integers as arrayed in an endless sequence. A second puzzle concerns the application of these concepts. Most concepts apply to some things but not others: *cow* applies to cows and *brown* to a property of brown things. In contrast, concepts of number and geometry apply to everything: anything that we can conceive of at all can be characterized with numerical or spatial terms (*seven samurai, seas or sins; a distant village, era, or cousin*).

Where do these concepts come from? Despite the efforts of the world's greatest thinkers and experimenters, from Socrates to Helmholtz, this question has not been answered [6]. I believe, however, that it can be addressed by research that takes four comparative approaches. First, comparisons across species tracing continuity and change in the evolution of spatial and numerical abilities. Second, comparisons over human development can trace both the origins of these abilities and their changes with growth and education. Third, comparisons across cultures, or across individual humans with different degrees of access to the products of their culture, can elucidate universal and variable aspects of these abilities. Fourth, comparisons across levels of analysis, from cognition and action to brain systems, neurons and genes, can probe the systems on which our spatial and numerical concepts depend.

These strands of research provide converging evidence for at least four cognitive systems that are sources of our numerical and geometrical intuitions: a system for comparing and combining sets based on their cardinal values, a system for selecting and tracking small numbers of numerically distinct individuals, a system for representing the shape of the large-scale layout so as to determine one's own position, and a system for distinguishing the shapes of small-scale objects and visual forms so as to identify objects of particular kinds. Each of these systems has a long evolutionary history; none is unique to humans. Each system emerges early in development, largely independently of any specific experiences with the entities to which it applies. Above all, each of the systems has genuine numerical or spatial content on which we draw when we learn and perform symbolic mathematics. Because they are innate systems with mathematical content, I will call them *core systems of number and geometry*.

This chapter begins with a brief review of the evidence for the existence and properties of two of these core knowledge systems. Drawing on this evidence, I suggest that research has effectively addressed two of the more difficult problems in cognitive science: the problem of determining the content of non-linguistic representations, and the problem of determining the role of innate capacities in the development of those representations. Solutions to both problems hinge critically on the discovery that these systems are shared by humans and nonhuman animals. Thus, studies of educated humans shed light on animal minds, and studies of animals shed light on the development of human concepts.

This review will also elucidate two properties of these systems that distinguish them from truly abstract number and geometry: each system captures some but not all of the properties of natural number and Euclidean geometry, and each system applies to some but not all of the objects to which our abstract mathematical concepts apply. I conclude that no core system, by itself, accounts for our capacity to learn and use these fully abstract concepts. Finally, I offer

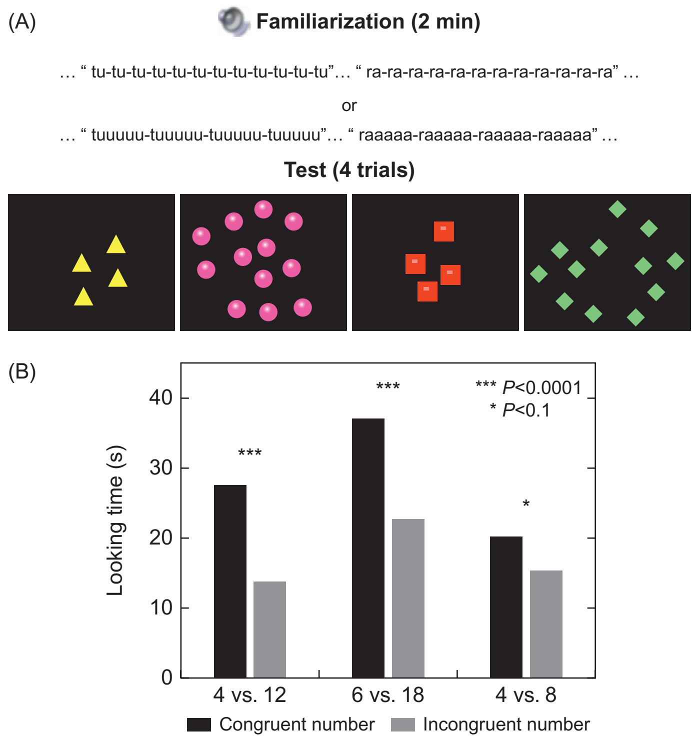


FIGURE 18.1 (A) Auditory and visual displays for an experiment on number representations by newborn infants, and (B) infants' looking times to the visual arrays that corresponded or differed in number from the accompanying auditory sequences ([7], reprinted with permission from PNAS).

two suggestions concerning the latter capacity: children and adults build systems of abstract number and geometry by productively combining the outputs of these four systems, and this process depends, in some way, on the acquisition and use of natural language.

A CORE SYSTEM OF NUMBER

An experiment by Izard *et al.* [7] serves to introduce the first core system. Newborn infants in a hospital nursery were presented with a series of syllable trains varying in pitch and duration. Across the different trains, the particular repeating syllable changed but the number of syllables per train was constant: four for half the infants and 12 for the others. After 2 min, a visual array of either four or 12 objects appeared on a large video screen (Fig. 18.1A). To ensure that no non-numerical variables connected these sounds to either array, the trains of four and 12 syllables were equated in extensive variables (train duration, total amount of sound) and differed in intensive variables (syllable frequency, duration of individual syllables), whereas the arrays of visible objects were equated in intensive variables (item size and density) and differed in extensive variables (array size, total amount of filled area on the screen). Across trials, the number of objects alternated between four and 12 while the syllable trains continued, and infants' looking time at each array was recorded, guided by findings that infants tend to look longer at visual arrays that correspond to an ongoing sound sequence (e.g., [8]). The newborn infants looked longer at the numerically corresponding visual arrays, detecting the numerical relationship between the auditory temporal sequences and visual spatial arrays (Fig. 18.1B, left).

Many experiments, conducted over the last decade, support the conclusion that prelinguistic infants are sensitive to number in spatial arrays (e.g., [9–11]) and temporal sequences (e.g., [12,13]). Evidence for these abilities comes from experiments using a variety of measures including preferential looking (e.g., Brannon, Chapter 14 of this volume), habituation of looking time (e.g., [9]), anticipatory head turning (e.g., [12]), exploratory reaching (e.g., [14]), and the neuroimaging measures of electroencephalography (e.g., [15,16]) and near infrared spectroscopy (e.g., [17]). Most important, this research reveals five important, non-obvious *signatures* of infants' numerical representations.

First, infants' ability to discriminate one numerical value from another depends on the ratio of the two values. In Izard *et al.*'s studies, newborn infants reliably discriminated between numbers that differed by a ratio of 3 (12 *vs* 4, 18 *vs* 6) but performed markedly less well when numbers differed by a ratio of 2 (8 *vs* 4; Fig. 18.1B). Over the first year, the critical ratio drops to 2 at six months and to 1.5 at 9 to 10 months (e.g., [12,18]). Second, at any given age, infants show the same ratio limit for different types of arrays: an infant who can just discriminate arrays of 8 *vs* 16 dots also will just discriminate sequences of 8 *vs* 16 sounds or actions [12,13]. Third, infants do not only discriminate numbers but can order them [9] and add two successively presented numbers and compare their sum to a third number [10]. Comparison and addition accuracy appear to be subject to the same critical ratio limit as discrimination. Fourth, numerical discrimination is impaired or abolished when arrays are presented under conditions that favor the attentive selection and tracking of individual objects. Although six-month-old infants can discriminate arrays of 4 *vs* 8 dots or sounds, even older infants tend to fail this discrimination when presented with objects that appear individually and move sequentially out of view ([19], Feigenson, Chapter 2 of this volume). Moreover, when infants are presented with arrays containing just a few simultaneously presented objects, each of which can be tracked in parallel, they tend to focus attention on the objects and ignore the cardinal value of the set [20–22].[1] Finally, infants spontaneously relate changes in number to changes in a different quantitative variable, line length. For example, infants who are habituated to arrays of dots that progressively increase (or decrease) in number will generalize habituation to arrays consisting of a line that progressively increases (or decreases) in length [27].

These five signatures provide clues to the nature and limits of infants' numerical representations. In particular, the ratio signature suggests that infants represent number

[1] These findings have sometimes been taken to suggest that infants cannot represent the approximate cardinal values of small numerical magnitudes, but more recent evidence refutes that suggestion (e.g., [23,24], see Brannon, Chapter 14 of this volume). Approximate numerical representations extend to the smallest numbers, but they are inhibited by processes of attentive object tracking, both in infants and in adults ([16,25], Burr, Chapter 3 and Cavanagh, Chapter 12 of this volume). When stimulus variations block object-directed attention or present objects that are usually perceived in large collections, representations of small cardinal values emerge [14,26].

imprecisely, and the evidence for a common ratio limit across diverse types of arrays and operations suggests that the source of the limit is to be found in the numerical system itself, which functions both to compare numerosities and to combine them in accord with the operations of arithmetic. For these reasons, this system has been called the *Approximate Number System*, or ANS: a term I will use hereafter. Infants' failures to enumerate the objects to which they are attending suggest that their ANS does not make explicit the identity or properties of the individual entities that it enumerates. Indeed, representations of individual entities may block the operation of this system: infants can see the forest and the trees, but they may not readily see both at once. Finally, the linkage between representations of number and length suggest that this system of numerical representation is one part of a more general sensitivity to magnitude (Lourenco, Chapter 15 of this volume).

The five signatures also provide a means to track this system of representation over the time-scales of evolution and human development, across different cultures and tasks, and into the human and animal brain. Experiments reveal all five signatures in nonhuman primates, providing evidence that the ANS is not unique to humans (see Brannon, Chapter 14 of this volume). Studies of children and adults in North America and Europe reveal the same five signatures, provided that the participants are tested under conditions that prevent verbal counting or other symbolic forms of enumeration. Thus, the system persists over human development and education, although its precision increases with growth and learning (see [28]). Studies of adults in remote cultures, lacking formal education, again reveal these signatures, indicating that the system is maintained over the lifespan without support from instruction in mathematics. Finally, studies of human and animal brains at the levels of cortical regions and single neurons reveal these systems as well (see Chapters 8 and 17 of this volume), opening the door to detailed studies of the neural coding of abstract number.

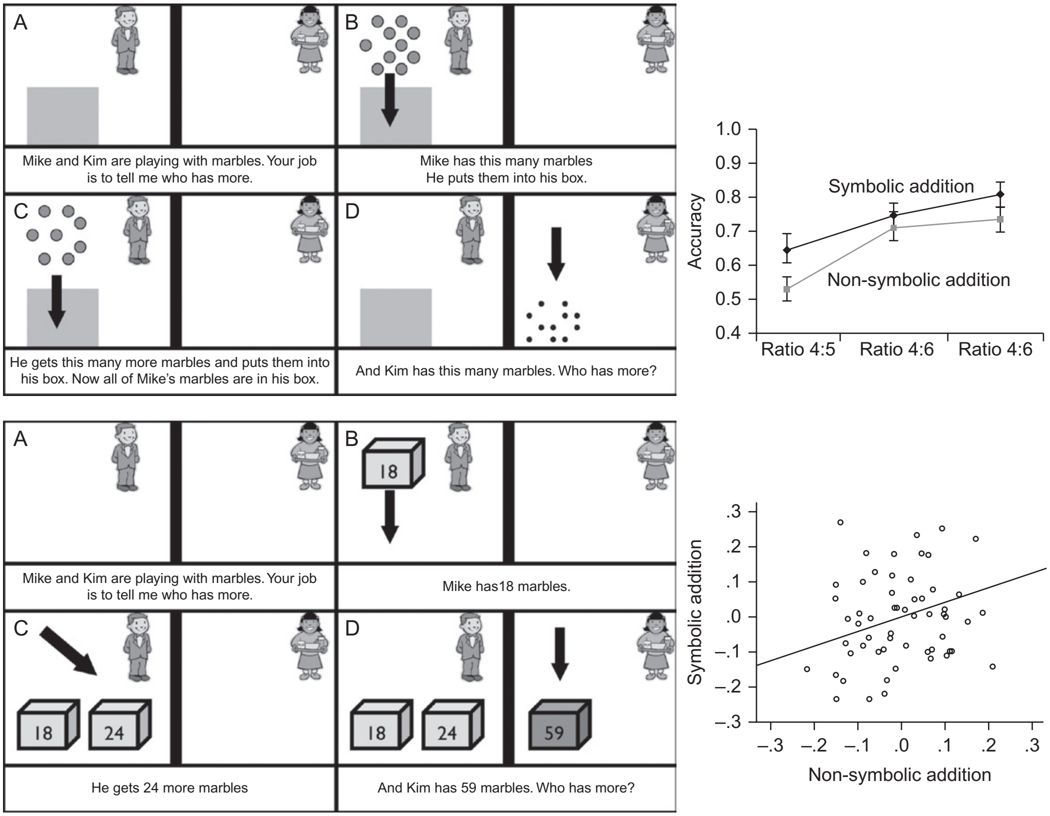
Armed with these findings, I believe it is now possible to address a vexed question: how can one determine the *content* of a mental representation as it is found in the mind of an infant, an animal, or a member of a culturally remote community? The research described above provides evidence that infants discriminate, compare, and combine arrays that educated adults would describe with numbers and arithmetic, but this evidence does not suffice to ensure that the arrays evoke number representations or arithmetic operations in infants. For example, consider infants' perception not of number but of surface lightness. Research on visual development provides evidence that infants, like adults, perceive edges by detecting abrupt luminance changes across visual arrays. Psychophysicists use mathematics to describe the mechanisms that detect these changes. We would not conclude, however, that infants who see contrast borders form representations of number: they perceive edges, not numbers. The fact that we can use mathematics to describe infants' or animals' responses to arrays of dots or sequences of sounds does not in itself imply that infants or animals represent number, because mathematics is a powerful tool for characterizing all the mental representations formed by any creature. How can psychologists determine if representations of these arrays have numerical content?

The research reviewed above suggests an answer to this question. Tests of the signatures of the system found in infants and animals reveal that the ANS is shared in large part by older children and adults. School children and adults, however, also have *symbolic* systems for representing number: including number words, Arabic notation, number lines, and

BOX 18.1

RELATIONSHIPS BETWEEN NON-SYMBOLIC AND SYMBOLIC NUMERICAL COGNITION

When preschool children first master number words and counting, they draw spontaneously on the ANS to solve new symbolic problems. Asked to add two symbolic numbers and to compare the results to a third number that differs from the sum (Box 18.1 Fig. 1, bottom left), children who have been taught no symbolic arithmetic perform above chance, as they do when they are presented instead with numbers instantiated non-symbolically, as arrays of dots (Box 18.1 Fig. 1, top left). Moreover, children's performance



BOX 18.1 FIGURE 1 Example problems from tests of non-symbolic and symbolic addition (left). The accuracy of five-year-old children was affected by ratio on both tasks (top right), and performance on the two tasks was correlated (bottom right). Gilmore et al., 2007, 2010, and in review, reprinted with permission from Nature, and reprinted from Cognition, 115/3, Gilmore, C. K., McCarthy, S. E., & Spelke, E. S., Non-symbolic arithmetic abilities and mathematics achievement in the first year of formal schooling, 394–406, 2010, with permission from Elsevier

BOX 18.1 *(cont'd)*

shows critical signatures of their performance with non-symbolic numbers, such as the ratio limit ([29]; Box 18.1 Fig. 1, top right). The children who perform these problems best also perform better on non-symbolic problems, controlling for IQ and literacy ([31]; Box 18.1 Fig. 1, bottom right). Similarly, when young children, or adults lacking formal education, are introduced to a number line and are asked to place symbolic numbers on the line, their placements reveal the compressed pattern observed with non-symbolic numerical arrays [32,33]. Thus the ANS provides usable information that guides children's performance on symbolic number tasks.

When school children learn mathematics, moreover, individual differences in their non-symbolic numerical performance correlate with individual differences in school mathematics achievement. In adolescents, the precision of the ANS retrodicts symbolic mathematics performance at seven years of age, controlling for IQ and performance in other school subjects [34]. Studies of adults with varying levels of schooling suggest that non-symbolic numerical representations are activated, exercised, and sharpened during learning and performance of symbolic mathematics (Piazza, Chapter 17 in this volume). Moreover, children who perform better on a non-symbolic addition task at the start of schooling go on to higher achievement in mathematics at the end of the first school year, controlling for IQ and literacy [30]. Symbolic and non-symbolic numerical representations therefore appear to have bidirectional effects on each other, although these effects are not shown on all measures of non-symbolic performance [35,36], and definitive claims about causal relationships must await the findings of training experiments.

Further evidence for a relation between ANS representations and symbolic numerical abilities comes from studies in neuropsychology and cognitive neuroscience. Adults activate the same brain areas when they compare or operate on non-symbolic and symbolic numbers (see [28]; Piazza, Chapter 17 in this volume). When non-symbolic numerical abilities are impaired by brain injury or transcranial magnetic stimulation, adults show corresponding impairments on symbolic numerical tasks [37,38]. Most important, activation to dot arrays of a given number produces adaptation of neural responses to symbolic arrays and the reverse [39], and fine-grained cortical responses to particular numbers of dots can be predicted from cortical responses to the corresponding symbolic numbers [40].

All these findings provide evidence that ANS representations have numerical content for adults and children. Because these ANS representations are shared by infants and animals, it follows that infants and animals have representations with some numerical content as well.

other symbolic devices. Thus, we can ask whether the ANS found in infants or animals has numerical content by investigating whether, and how, ANS representations relate to the symbolic numerical representations that are unique to human children and adults. Three types of findings provide evidence for a close relationship between the ANS and symbolic numerical abilities (Box 18.1). These findings of course do not imply that infants or animals have the full array of numerical abilities found in educated adults. Nevertheless, they

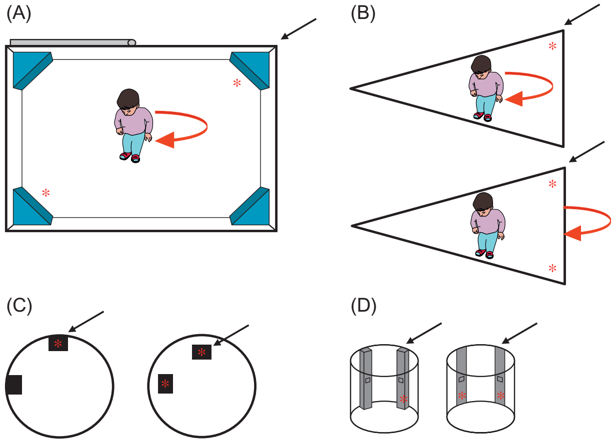


FIGURE 18.2 Displays and findings from studies of children's search (A) in a rectangular room (after [42]), (B) in an isosceles triangular room after disorientation of the child (top) or the room (bottom; after [44]), (C) in a circular room with two columns against the wall (left) or offset from it (right; after [43], or (D) in a circular room in which the columns (left) were replaced by flat stripes (right; after [43]). Arrows indicate the location of a hidden object, and asterisks indicate the location(s) at which children searched for the object. Rooms are depicted from above (A–C) or from the side (D).

provide evidence that the system by which infants and animals discriminate, compare, and combine arrays of discrete dots or sounds contains numerically relevant information.

In summary, research provides evidence for a core system of number. The system is present in newborn infants and in other animals [41], and therefore is not learned through experience counting, communicating about, or even manipulating sets of objects. The system, moreover, is a foundation for learning of symbolic mathematics. Nevertheless, the system has two striking limits. First, it is imprecise and therefore fails to support representations such as *exactly seven*. Second, the system fails to operate under conditions in which objects are presented individually and can be attended to and tracked over time and occlusion. For both reasons, the ANS fails to represent explicitly the individual entities that comprise the set whose approximate cardinal value it registers, and it fails to capture the fundamental operation of *adding one individual* to a set.

A CORE SYSTEM OF GEOMETRY

Geometry is the measurement of the earth. True to this meaning, the first core system of geometry is a system by which navigating humans and animals compute their own positions, and those of significant objects, by measuring properties of the surrounding terrain. An experiment by Lee and Spelke [42] serves to introduce this system. Children (aged three to four years) were brought into a closed rectangular room with four corner panels (Fig. 18.2A). Because the room was uniformly colored and contained no distinctive, asymmetrically placed objects, only the relative lengths of the walls broke its four-fold symmetry, and no information distinguished any direction from its diagonal opposite. While children watched, a sticker was hidden at one corner, and then children turned with eyes closed until they were disoriented. Finally children were allowed to open their eyes and search for the sticker. Children used the shape of the room to confine their search to two corners: the correct location and the opposite location with the same distance and directional relationships (e.g., the corner to the *left* of one of the two *more distant* walls). This finding provides evidence that children were sensitive to these properties of the shape of their surroundings.[2]

Although full disorientation rarely occurs during human navigation, experiments using reorientation tasks are valuable, because they reveal the information about the environment that navigators encode and rely on automatically (since children do not expect to be disoriented). As a consequence, a rich array of experiments has investigated children's reorientation in diverse environments. Reorientation by room geometry is highly robust across age (from infancy to adulthood), across variations in room size and shape, and across variations in the nature and presence of landmarks (see [43], for review). As in the case of core number representations, however, the most interesting findings from studies of children's reorientation concern not the existence of geometry-guided navigation but the signature limits on this capacity. These limits again provide clues to the nature of the representations that guide children's navigation, and they allow investigators to track these representations across species, ages, cultures, tasks, and brain systems.

The first signature concerns the task-specificity of geometry-guided navigation: the system serves to locate the child in relation to her surroundings, but it does not directly specify the relative locations of movable objects. Elegant experiments by Lourenco and Huttenlocher [44] (see also [45,46]) reveal this limit. Children searched for an object hidden in one of the three corners of a triangular enclosure. In one condition, the child was disoriented while the enclosure remained stationary (Fig. 18.2B, top). In a second condition, the child remained oriented, with eyes closed, while the enclosure was moved around her (Fig. 18.2B, bottom). Both conditions ended with the same perceptible environment and behavioral instruction, but they presented different cognitive problems: determining one's own position in a stable environment or relocating a displaced object. As in past research, children used the distances and directions of the triangular walls to reorient themselves, but they used only distance, not direction, to locate the displaced object in the rotated room. Searching for a displaced object is not simply more difficult, however: when an object is hidden at a distinctive landmark, children are better able to use the landmark when they are oriented and it moves than when they are disoriented and it is stable [46]. Children use the distances and directions of surfaces to specify their own location but not as direct landmarks to the locations of hidden objects.

A second signature of this core geometry system concerns the kind of layout information that it accepts: children reorient by the distances and directions of extended surfaces but not by the distances and directions of freestanding objects, even large ones. A recent study by Lee and Spelke [43] (see also [47,48]) illustrates this limit. In a series of experiments, children were

[2] One question that is frequently raised in studies of spatial representation concerns the coordinate system within which information is represented: do navigating children represent the shape of their surroundings allocentrically (for example, as a rectangular room with two long and two short walls) or egocentrically (for example, as an array of surfaces standing at particular distances and directions from their current station point)? Although a great deal has been learned about the representations that guide navigation in children and animals, this question has proved to be very difficult to answer in this domain, as it is in the domain of visual form analysis discussed below. For this reason, I do not address questions concerning spatial reference frames and coordinate systems in this chapter.

disoriented in a cylindrical environment with two large, stable columns that contrasted with the walls of the cylinder in brightness and color, positioned so that they both stood on one side of the room, separated by 90 degrees. When the columns stood flush against the cylindrical wall of the room (Fig. 18.2C, left), children used them to reorient themselves and locate a hidden object, both when that object was hidden directly at one of the columns and when it was hidden elsewhere. When the columns were offset slightly from the walls, however, children failed to use their positions to locate the hidden object. This failure did not stem from a failure to attend to or remember the relation of the object to the columns: if the object was hidden directly at one of the two columns, children searched only at the columns, showing that they appreciated their relevance to the task (Fig. 18.2C, right). Children failed, however, to confine their search to the column with the correct directional relationship to the child: e.g., the freestanding column *on the left*. Columns only specified the child's position when they were placed flush against the walls, and therefore contributed to the shape of the room.

A third signature concerns the dimensionality of the information that children use to track their own positions: children reorient by the distances and directions of extended 3D surfaces but not by the distances and directions of extended 2D patterns. Another experiment by Lee and Spelke [43] reveals this limit (see also [48–50]). Children were tested in the same cylindrical environment as in the above studies. Instead of viewing two 3D columns against the wall (Fig. 18.2D, left), however, children were presented with two 2D patches on the walls (Fig. 18.2D, right): patches of the same angular size as the columns, made of the same material and contrasting dramatically from the surrounding walls in brightness, texture and color. When an object was hidden at one of these patches, children confined their search to the two patches, showing again that they detected them and appreciated their relevance for the task. As in the case of freestanding objects, however, children searched equally at the correct patch (e.g., the patch on the right) and the incorrect patch (on the left). Although the patches were clearly detectable, they did not alter the shape of the cylindrical environment so as to break its symmetry. Accordingly, the geometric navigation system did not analyze their distance and directional relationships to specify the position of the child (Box 18.2).

BOX 18.2

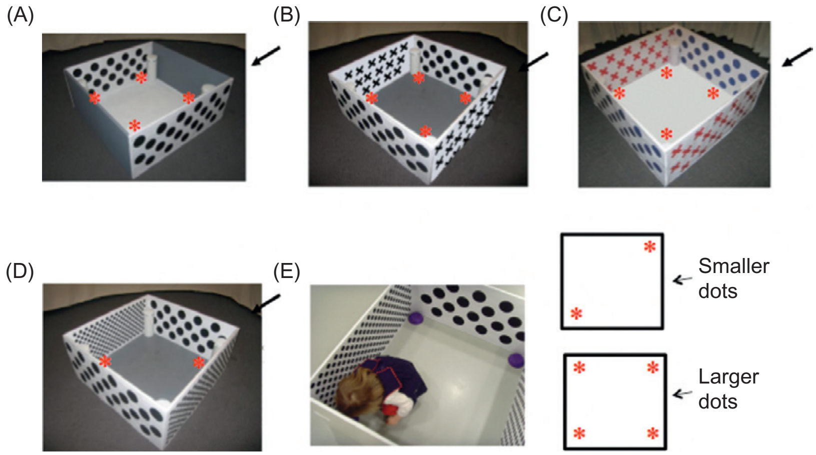
REORIENTATION DEPENDS ON 3D BUT NOT 2D GEOMETRY

Research by Lourenco and Huttenlocher highlights the obliviousness of navigating children to 2D geometrical forms that could specify their own position [51,52]. Children were disoriented in a square room whose opposing walls displayed distinctive 2D patterns (for example, crosses *vs* discs: see Box 18.2 Fig. 1A–C). The patterns differed only in shape, but children did not reorient by this geometric information. In contrast, children confined their search to the two directionally consistent corners whenever the opposite walls were covered with forms of the same shape but of differing size and density (Box 18.2 Fig. 1D). Children therefore reoriented by a contrast between large and small discs, but not by a contrast between discs and crosses or between discs and a blank wall.

What accounts for this pattern? When equidistant surfaces are covered by forms differing in size and density, the surface with larger forms appears closer to the

BOX 18.2 (cont'd)

viewer. Depth perception based on *relative size* begins in infancy [54], and it could lead children to perceive the square room as slightly rectangular, triggering the reorientation system. This interpretation leads to two predictions. First, children should reorient in uniformly colored environments that are only very slightly rectangular (because the effect of relative size on depth is subtle). Second, relative size should interact predictably with other depth cues to enhance or diminish children's reorientation. Lee *et al.* [55] confirmed both these predictions. In an unpatterned room, children successfully reoriented by a subtly rectangular shape (sides differing in an 8:9 ratio). When large and small discs were added to these walls, children reoriented successfully when larger discs adorned the closer walls, but not when they adorned the more distant walls (Box 18.2 Fig. 1E). Flat geometrical patterns therefore serve as a depth cue, allowing children to reorient by a subtle, perceived difference in surface distance. Such patterns do not serve as independent information guiding children's reorientation, however, for children fail to reorient by them when the depth effect is cancelled.



BOX 18. 2 FIGURE 1 Displays and findings from studies of the effects, on children's reorientation, of (A) the presence of a pattern, (B) the shapes of the pattern elements, (C) the shapes and colors of the pattern elements, or (D) the size and density of the pattern elements (Huttenlocher & Lourenco, 2007, and Lourenco et al., 2009, reprinted with permission from John Wiley and Sons, reprinted from Journal of Experimental Child Psychology, 104, Lourenco, S., Addy, D., & Huttenlocher, J., Location representation in enclosed spaces: What types of information afford young children an advantage? 313–325, 2009, with permission from Elsevier, and reprinted with permission from S. Lourenco, 2010). Rooms are shown from the side, arrows indicate the location of the hidden object, and asterisks indicate the locations at which children searched. In (E), pattern elements of different size and density were placed on the walls of a slightly elongated rectangle; children's performance (depicted from above) depended on the pairings of patterns to wall lengths (after [53]).

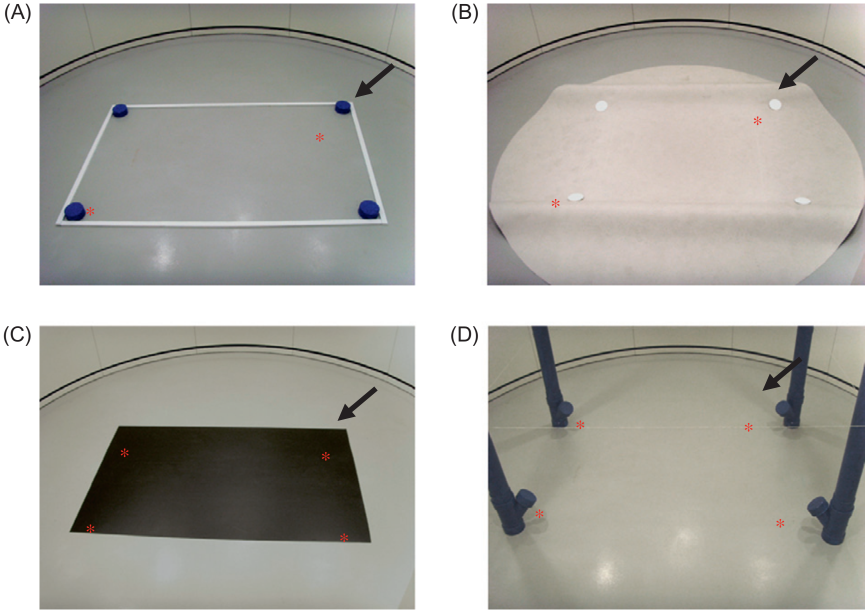


FIGURE 18.3 Displays and findings from studies of reorientation by subtle perturbations to the 3D layout on the ground surface (A and B), by a salient 2D figure on the ground surface (C), or by freestanding columns connected by a frame that was offset from the ground and constrained children's motion (D). Rooms are shown from the side; arrows indicate the location of the hidden object, and asterisks indicate the locations at which children searched (after [55]).

A fourth signature of the geometric navigation system concerns the primacy of the ground surface and its borders as information for one's own position: children reorient effectively not only in enclosed environments with a distinctive shape, but in environments where the only distinctive shape is provided by a tiny rectangular frame or arrangement of bumps on the floor (Fig. 18.3A and B). In contrast, children fail to reorient by salient 2D contrast borders or by large freestanding columns connected by a raised barrier that similarly constrains their motion but does not contact the ground (Fig. 18.3C and D) [57]). The system's high sensitivity to subtle perturbations of the ground surface, coupled with its insensitivity to much larger vertical landmarks, provides evidence against a popular theory whereby reorientation depends on processes for matching brightness contrast borders in 2D panoramic images of the layout [56–58] (the experiments described in Box 18.2 provide further evidence against this theory). Image-matching processes contribute to a wealth of navigation processes in animals (e.g., [59]) and they may aid in landmark guidance in humans [60], but they do not account for the process by which children locate themselves within the larger spatial layout.

There are other features of this system, related to its automaticity and robustness over variations in attention (see [61,62]) and motion [43], but I will focus on only one final signature: this system is sensitive to two fundamental properties of Euclidean geometry—*distance* and *direction*—but not to a third Euclidean property, *angle*. Children's insensitivity

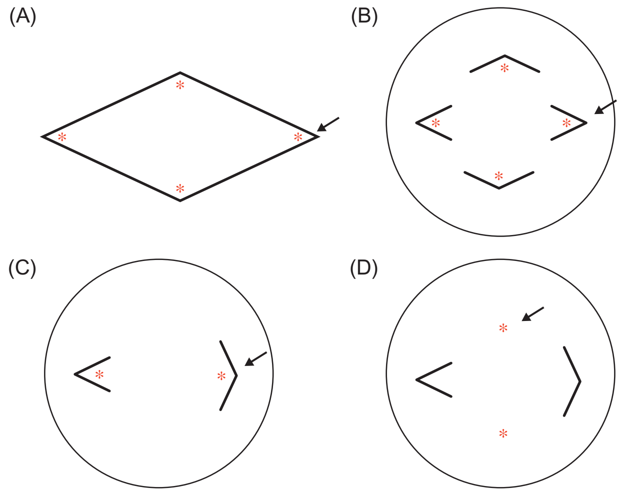


FIGURE 18.4 Displays and findings from studies of reorientation by length and by angle in (A) a connected rhomboid environment (after [63]), (B) a circular environment containing a fragmented rhombus with no informative aspect ratio, or (C and D) a circular environment containing two angles differing in size and four hiding containers (not shown) in a symmetrical arrangement (after [64]). Displays are depicted from above; arrows indicate the location of the hidden object and asterisks indicate the locations where children searched. In (C), the object was hidden at one of the two containers nested within the angles, and children divided their search between those two containers; in (D), the object was hidden at one of the two containers displaced from the angles, and children divided their search between those two containers.

to the angles at which walls meet at corners was first shown by Hupbach and Nadel [63], who reported that 2 to 3-year-old children failed to reorient by the distinctive shape of a rhomboid environment consisting of four walls that were equal in length but met at unequal angles (Fig. 18.4A). In a recent replication and extension of this research, we found that such children were strikingly insensitive to angle when the four angles of a rhombus were presented in an array with no informative aspect ratio (Fig. 18.4B), and they remained insensitive to angle when tested in the simplest environments [64]. In one experiment, two corners of markedly different angle (an obtuse angle of 120 degrees and an acute angle of 60 degrees) were placed opposite one another in a cylindrical room, each adjacent to one of four hiding containers (Fig. 18.4C and D). When an object was hidden in the container directly in front of the obtuse-angled corner, disoriented children searched only the two containers in front of corners, showing that they noticed the corners and appreciated their relevance as landmarks. Nevertheless, the children searched those two containers equally: they didn't use the angular difference between the corners to reorient themselves or to specify the object's unique location.

As in the case of the core number system, these five signatures allow investigators to test for this system of geometry in other animals, in human adults in diverse cultures and circumstances, in specific systems in the brain, and even into the genes. For brevity, I will discuss only the first and the last two of these tests. Studies of reorientation began with the classic studies of Cheng and Gallistel [65] and Cheng [66], conducted on rats. Reorientation

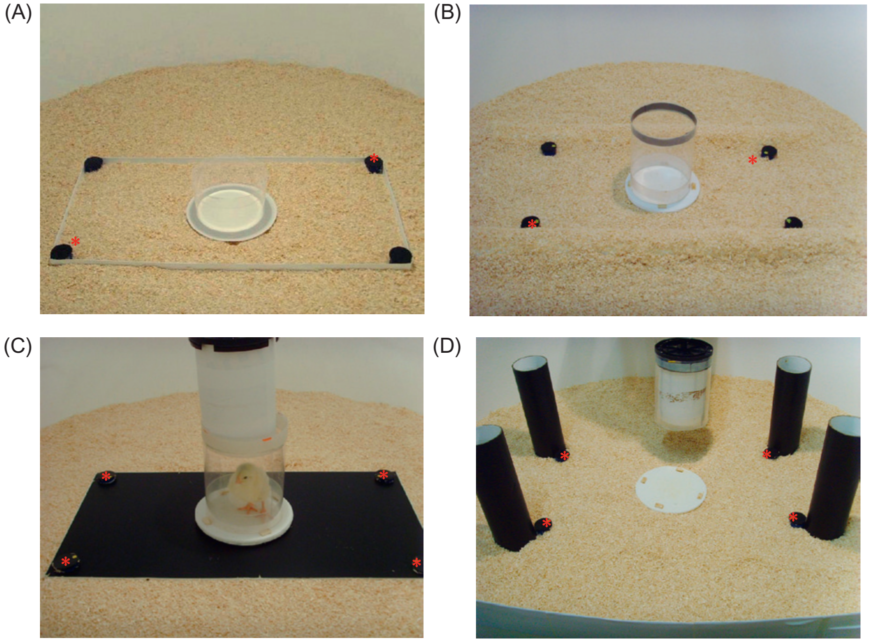


FIGURE 18.5 Displays and findings from studies of reorientation in chicks. Displays are depicted from the side; arrows indicate the location of the hidden object, and asterisks indicate the locations at which chicks searched. Chicks viewed the hiding of the object while confined in a transparent cylinder at the center of the room (depicted in A); for disorientation, a second cylinder with opaque walls was inserted in the first and chicks were turned (depicted in B). Then the opaque cylinder was lifted (C) and the cylinder was removed to allow the chick to search (D; after [68]).

by the shape of the environment has now been shown in a wide range of nonhuman animals, from primates to birds and even to ants [67] (see [61] for review). The literature is vast and complex, and studies that involve training give divergent findings, likely due to effects of training on attention to landmarks. Moreover, not all of the signatures have been tested in all animal species. Where untrained animals have been tested, however, they show the same signatures of reorientation found in children. Like children, for example, ants use 2D geometric patterns as beacons but reorient only by the 3D shape of the environment [67]. Moreover, newly hatched chicks reorient by the same patterns of subtle geometric information as children, and they too use 2D patterns and large freestanding objects as beacons but fail to reorient by them [68] (Fig. 18.5). Mice show the same reorientation performance as children in square rooms containing patterning information evoking the relative size depth cue [69]. Rats are strikingly insensitive to angle when tested in rooms whose shape is perturbed so as to change angle information while holding length relations constant [70].

In research using neurophysiological methods, the same signatures have been found in the brains of navigating animals, in areas whose activity specifies the animal's location, heading, or motion (see Burgess, Chapter 5 in this volume). The firing patterns of place cells in the hippocampus, and of grid cells and boundary cells in the nearby entorhinal cortex, are systematically affected by the distance and direction of the walls of the chamber [71,72], but markedly

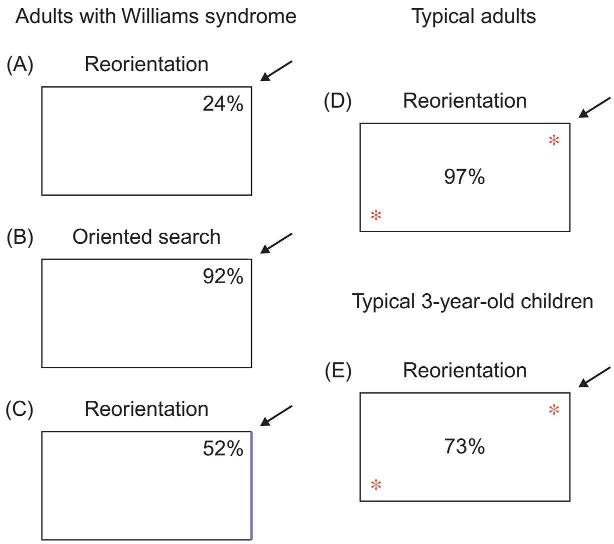


FIGURE 18.6 Displays and findings from studies of adults with WS who (A) were disoriented in a rectangular room with no landmarks, (B) remained oriented with eyes closed for a similar delay in the same room, or (C) were disoriented in a rectangular room with a single colored wall. For comparison, (D) and (E) show the performance of typical adults and young children in the same environment and test as (A; after [76]). Displays are depicted from above; arrows indicate the location of the hidden object. In A–C, numbers give the percentage search at each of the correct locations; in D and E, numbers give the percentage of search at either of the two geometrically correct locations (indicated by asterisks).

unaffected by the positions of freestanding objects [72] or the orientations of walls and angles of corners [73]. New research using neuroimaging methods provides indirect evidence for place and grid cells in humans as well, activated when humans learn object locations in a virtual environment in relation to extended surface boundaries ([74]; Burgess, Chapter 5 in this volume). Hippocampal activity associated with learning an environmental location in relation to an extended surface in the virtual layout is markedly impervious to effects of attention and interference, in contrast to activity in other brain structures associated with learning a location in relation to a freestanding landmark object [75]. These studies show a remarkable convergence across humans and rodents, and across behavioral and neurophysiological methods, in the core mechanisms for encoding the shape of the surrounding surface layout.

Finally, an exciting new line of research hints that the core system of geometry may have a specific genetic basis. Lakusta, Dessalegn and Landau studied adults with Williams Syndrome (WS), a developmental disability stemming from a genetic deletion that produces a variety of structural and cognitive abnormalities including an especially impaired capacity for spatial reasoning [76]. WS adults perform a wide variety of spatial tasks at roughly the level of typical three-year-old children, but the reorientation task shows a different pattern. Lakusta *et al.* [76] tested the reorientation performance of adults with WS in a homogeneously colored, rectangular room and found complete failure: in contrast to all the studies reviewed above, WS adults searched the four corners of the room equally (Fig. 18.6A). Their performance did not stem from a failure to remember the location of the hidden object, because they performed well after a delay over which they remained oriented (Fig. 18.6B). Their performance also did not stem from any debilitating effects of the disorientation procedure, because they performed fairly well when tested, after disorientation, in a rectangular room with one distinctively colored wall (Fig. 18.6C). The reorientation performance of the adults with WS contrasted both with that of typical adults (Fig. 18.6D) and with that of typically developing children (Fig. 18.6E). Indeed, the experiments revealed a striking double dissociation: whereas young children successfully navigated in accord with the shape of the environment and failed to navigate in accord with the colored wall, WS adults did the reverse. WS therefore seems to produce a specific deficit in the core system for navigating by layout geometry. Because WS is caused by a genetic deletion, and mouse models of WS have been developed [77], future experiments on mice can probe the mechanisms by which this cognitive system develops or goes awry.

These new findings raise a second vexed question in cognitive science: what are the effects of experience in a geometrically structured world, and of intrinsic, genetically specified developmental processes, on the emergence of this system of geometry? Questions concerning the innateness of knowledge systems are as thorny as questions concerning the content of those systems. In the case of human navigation by layout geometry, the debate has been particularly difficult to resolve, because children don't begin to navigate independently until the end of the first year. Children's system of geometry-guided navigation is shared by other animals, however, so studies of animals can probe its developmental foundations. Studies of controlled-reared chicks by Chiandetti and Vallortigara reveal that the system develops independently of any experience in a geometrically structured surface layout. Chicks reared in a geometrically uninformative, cylindrical environment reorient by the distances and directions of extended surfaces as consistently as do chicks reared in rectangular or asymmetrical environments [78], even on their first exposure to those surfaces (Vallortigara, Chapter 13 in this volume). The system for locating the self in relation to the distinctive shape of the surface layout therefore develops independently of any experience with layouts of distinctive shapes.

Research probing the innate foundations of human cognition is sometimes criticized as leading to an impasse: when a capacity is found to be innate, it is argued, there is no further research for developmental and comparative psychologists to do. Research on core geometry provides an illuminating counterexample to this argument. Since Plato, thinkers have wondered about the effects of experience on the development of knowledge of geometry: would a lifetime spent navigating in environments that systematically violate Euclidean relationships change our geometrical intuitions? With the controlled rearing methods of Vallortigara and others, these questions can be addressed, and the literature on effects of controlled rearing on navigation already is yielding intriguing findings. In particular, Brown *et al*. [79] and Twyman *et al.* [80] have investigated effects, on navigating fish and mice, of rearing in a geometrically structured environment with one or more salient landmarks. Like Chiandetti and Vallortigara [78], Brown *et al.* [79] find that reorientation in accord with the distances and directions of surfaces is independent of experience in a geometrically structured environment (Twyman *et al*. did not test for this effect). In contrast, experiments in both labs reveal that navigation by landmarks is affected by rearing experience: animals reared with a distinctively colored wall in a stable position are more likely to use that wall to guide their navigation. Research is therefore beginning to chart both the foundations and the malleability of cognitive mechanisms guiding navigation.

In summary, research provides evidence for a core system of geometry that emerges in human children soon after they begin to locomote independently, that is common to a broad range of animals, and that can develop independently of any prior experiences with the geometrical relationships that it analyzes. Like the core system of number, however, this system is limited. Although human adults use Euclidean geometry to characterize the shapes of freestanding objects and 2D forms arrayed at any orientation, this core system only applies to extended surfaces and privileges surfaces that border the ground over which we navigate. Whereas Euclidean geometry can be used for many purposes, this core system only serves to specify the position and heading of the navigator with respect to the surrounding environment. And whereas Euclidean transformations (rigid displacements) preserve distance, angle and direction, this core system is blind to angle and preserves only information about surface distances and directions. Core geometry for navigation cannot be the sole source of our Euclidean geometrical intuitions.

MORE CORE SYSTEMS

If the above two systems are not the sole sources of natural number and Euclidean geometry, what other sources do we draw on? Research on human cognitive development, animal cognition, cognition across cultures, and cognitive neuroscience provides evidence for two more core systems of number and geometry. The second number system, whose nature and limits are described by Feigenson (Chapter 2 in this volume), serves to represent sets of up to three to four numerically distinct individuals, as well as the operation of adding one individual to a set. The second geometry system, whose nature and limits are described by Izard (Chapter 19 in this volume), serves to represent the shapes of 2D visual forms and moveable objects, capturing the relations of length and angle that are invariant over changes in size.

Each of these systems is activated under conditions complementary to the conditions that activate the two core systems described above. Whereas core representations of numerical magnitudes are inhibited under conditions in which a small number of objects are attentively tracked, these are just the conditions that elicit activity in the second number system. And whereas core representations of layout geometry do not include the shapes of 2D patterns or freestanding objects, these are just the arrays that activate the system of visual form analysis. Moreover, each of these systems captures information that the other system lacks. In the case of number, the first core system captures cardinal information across a broad range of values, but only does so imprecisely, whereas the second core number system captures the exact number of individuals in an array, but only when those numbers are small. In the case of geometry, the core system for navigation represents distance and direction but not angle, whereas the system for form analysis represents distance and angle but not the directional information that distinguishes a form from its mirror image (Izard, Chapter 19 in this volume).

The contrasting properties of the two core systems of number and geometry are summarized in Fig. 18.7. This summary suggests that more powerful and abstract mathematical concepts could arise if the representations from the core systems could be productively combined. If children could systematically combine representations of sets and their cardinal

Core systems of number and geometry

(A)	2 vs. 3	4 vs. 8
Comparing sets by cardinal values	--	√
Tracking individuals, adding one	√	--

(B)	Distance	Angle	Direction
Navigating in 3D layouts	√	--	√
Recognizing 2D forms	√	√	--

FIGURE 18.7 Contrasting properties of, and limits on, the core systems of number and geometry. Checks indicate successful performance, and dashes indicate failures of performance, by (A) six-month-old infants whose numerical sensitivity is assessed with large or small numbers of objects, and (B) preschool children whose geometrical sensitivity is assessed with large-scale layouts or small-scale forms.

values with representations of numerically distinct individuals, formed by successive addition of one, they could overcome both the ratio limit on the representation of cardinal values and the set size limit on the representation of individuals, representing sets of any size with exact cardinal values. Similarly, if children could systematically combine the geometric properties they extract from large-scale navigable layouts and from small-scale forms, they could overcome the limits on the domains of application of these systems and increase the power of their geometrical analyses. By combining these systems, children might navigate by angle as well as distance by viewing the extended surface layout through the lens of visual form analysis, as painters do. Moreover, children might distinguish forms and objects from their mirror images by viewing those forms and objects through the lens of geometry-guided navigation, using real or mental rotation to view objects from changing perspectives. In the next two sections, I turn to evidence bearing on these possibilities.

CONSTRUCTING NATURAL NUMBER

Children appear to overcome the limits of the core number systems when they begin to use number words in natural language expressions and counting. For most children, counting begins to be mastered at about two years of age, when children learn the first 10 or so words of the counting list. Initially, these words have little numerical meaning beyond the fact that they are elicited by the presence of a collection of objects (a display that is likely to activate representations of approximate cardinal values) and they are accompanied by gestures of pointing to each object in turn (an activity that likely depends on attentive object tracking). At some point in the third year, most children learn that *one* designates a single

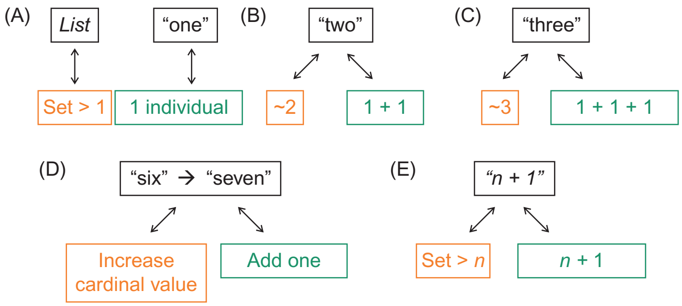


FIGURE 18.8 Successive steps in children's learning of counting (after [5,81,82]), and a proposed account of these steps as the progressive combination of information from the two core systems of number.

object and that all the other number words designate a plurality of objects. Over the next year, children learn that *two* designates exactly two objects, and that *three* designates exactly three objects. Some children also learn that *four* designates exactly four objects, but children then abandon this pattern of piecemeal learning and make two related inductions: every word in the counting list designates a set of individuals with a unique cardinal value, and each cardinal value can be constructed through progressive addition of one. Figure 18.8 suggests how children might learn this mapping by connecting each word in the counting list to representations from the two core number systems.

For most children, the language of number words and verbal counting appears to provide the critical system of symbols for combining the two core systems, and some evidence suggests that language may be necessary for this construction (Box 18.3). Once natural number concepts are constructed, however, does language continue to play a role in their use? Intuition suggests that language plays no role in mature mathematical reasoning (see [89]). Contrary to this intuition, however, three sources of evidence suggest that language serves throughout life as the medium by which representations from the two distinct core systems of number are combined (Box 18.4). I believe the role played by language is small (consistent with the intuitions of mathematicians), but crucial. All of the information supporting our numerical intuitions derives from the two core number systems, and these systems are fully independent of language. Nevertheless, the language of number words and quantified expressions may serve to link this information together. Absent language, human infants and other animals may have all the information they need to represent the natural numbers, but they may lack the means to assemble that information into a set of workable concepts.

CONSTRUCTING NATURAL GEOMETRY

Although the development of natural number has been subjected to intense investigation since the pioneering studies of Piaget [94] and Gelman and Gallistel [95], the development of Euclidean geometry has received less attention. Some experiments nevertheless provide clues to its development. Like counting, full Euclidean geometry develops in humans with or without formal education. Also like counting, its development requires a protracted process.

BOX 18.3

IS LANGUAGE NECESSARY FOR THE CONSTRUCTION OF NATURAL NUMBER?

The counting list learned by most children is composed of words with the grammatical properties of quantifiers [83], but the relation of language to number is debated. Is language merely a convenient source of symbols for combining information from the two number systems so as to enumerate entities exactly, or is it necessary for the construction of natural number concepts?

Children and adults in remote cultures, whose languages lack words for most numbers, tend to preserve approximate, but not exact, numerical equivalence when matching numbers larger than three [84,85] (cf. [86,87]). The interpretation of these studies is debated, however, in part because of the difficulty of disentangling effects of language and culture on cognitive capacities and predispositions.

Deaf adults who live in a numerate culture, but who have little or no exposure to a deaf community and, therefore, speak no conventional language, provide a different test of the role of language in the development of natural number and counting. Spaepen *et al.* [88] studied four adults living in remote areas of Nicaragua, who communicate with their hearing families and friends through a gestural system called *homesign* [88]. These homesigners received no formal education, but they hold jobs and use and exchange money, possibly both by recognizing the distinctive appearances of different bills and coins and by recognizing Arabic notation to some degree. When the homesigners communicate about number, they do not count or make tally marks. Although they use their fingers to convey numerical information, they do so with only approximate accuracy. Finally, they perform non-symbolic numerical matching tasks with approximate but not exact accuracy. These findings suggest a special role for language in the construction of natural number, but they leave many questions open. First, it is not clear which of the many aspects of language that are available to hearing people and to deaf speakers of sign language, but not to homesigners, are critical for the development of natural number concepts. Moreover, it is not clear whether language is necessary for the construction of natural number concepts or whether other symbolic systems, also not available to these homesigners, could support this construction.

Evidence for the spontaneous emergence of Euclidean geometry comes from three experiments performed on the Munduruku, who lack both formal education and experience with symbolic maps. First, Munduruku adults and older children were asked to navigate to a specific location in a simplified 3D layout (a triangular arrangement of containers) when the location was indicated by means of a Euclidean, 2D map (three small forms in the shape of a similar triangle, oriented variably with respect to the environment: Fig. 18.9A). Over a set of trials that varied the shape of the triangle, the Munduruku used distance, angular and sense relationships to specify the 3D location [98]. In further tests, moreover, the Munduruku used information on the map indicating landmark objects as well: their performance was enhanced when the location indicated on the map was a form with a distinctive color and shape that

BOX 18.4

LANGUAGE AND NUMERICAL REASONING IN EDUCATED ADULTS

Evidence for a role of language in mature numerical reasoning comes from experiments that compare educated adults' performance of approximate symbolic arithmetic (which could be supported by the ANS alone) to their performance of exact symbolic arithmetic (which goes beyond the limits of the ANS). First, educated adults who suffer language impairments often show impairments in exact numerical reasoning, despite preserved approximate numerical abilities [37]. Second, bilingual adults who are taught new number facts in one of their languages show a cost if they must produce exact number facts in the untrained language, relative to performance in the trained language [90,91]. Third, adults who perform exact, but not approximate, mental arithmetic respond more slowly when the numbers they must add require more time to pronounce, even though the numbers are presented in Arabic notation, not as words [92]. If language merely scaffolded the acquisition of natural number concepts and abilities, and then was replaceable by other symbol systems, one would not expect adults to translate Arabic symbols into words for purposes of exact computation.

Despite this evidence, it is clear that non-linguistic symbols contribute to numerical reasoning: arithmetic is far easier to perform with Arabic than with Roman numerals, and devices such as the abacus can greatly speed its execution [93]. The role of language in mature natural number concepts therefore continues to be debated.

conformed to the color and shape of a landmark in the 3D array. As in non-symbolic navigation tasks, representations of landmarks and representations of layout geometry were found to be distinct and possibly mutually inhibitory: Munduruku showed higher sensitivity to the geometric information in maps when landmark information was absent [96].

Second, Izard *et al.* [99] presented Munduruku adults and children with a computer-animated depiction of a large-scale spatial layout consisting of two locations, described as villages, on a textured surface that was either flat or curved. They were shown the directions of two paths leaving from each location; one was described as leading straight to the other visible village and the other was described as leading straight to a third, unseen village. Then participants were asked to determine both the location of the third village and the angle at which the two paths converged at that location. Across trials, the distance between the two visible villages and the angles of the paths varied. When the surface was planar, the Munduruku used both the distance between the villages and the angles formed by the paths that left them to specify the distance and angle of the third, unseen apex of the triangle. In particular, they produced angles whose size followed from the principle that the three angles of a planar triangle will sum to 180 degrees, in accord with Euclidean geometry, whereas the three angles of a triangle on a sphere will sum to a larger value [99] (see Izard, Chapter 19 in this volume).

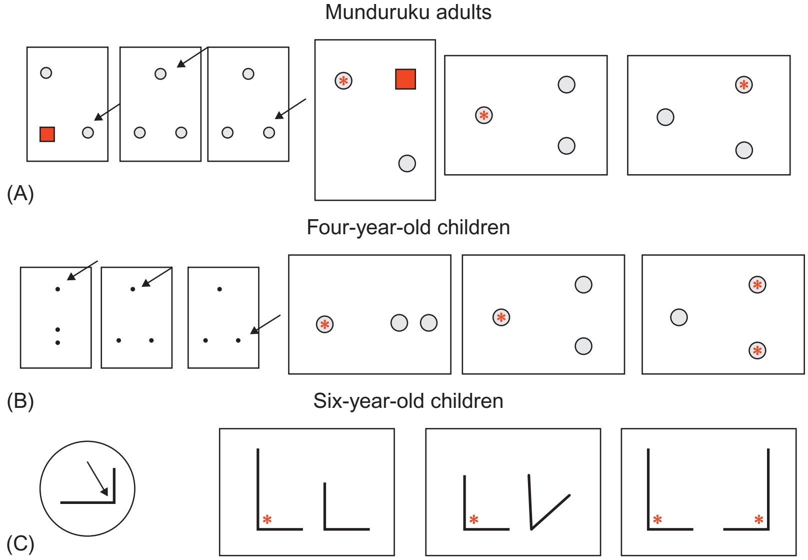


FIGURE 18.9 Geometric map tasks performed (A) by Munduruku adults and older children (after [96]), (B) by four-year-old U.S. children (after [97]), and (C) by six-year-old US children (after [98]). Arrows designate the target positions indicated on the maps (left); asterisks indicate the positions in the 3D arrays chosen by the participants (right). Across trials, the target location and the map orientation varied relative to the array. Maps were presented as participants faced away from the test arrays, so that a map and the array that it depicted were not simultaneously visible.

The third task presented the Munduruku with a series of questions about the behavior of abstract, dimensionless points and one-dimensional lines on a planar or spherical surface. For this task, the visual displays were zoomed in so as to remove all perceptual differences between the planar and spherical surfaces: all questions about the behavior of points and lines on the plane *vs* the sphere therefore were accompanied by identical displays. The intuitions of the Munduruku about points and lines accorded well with the principles of Euclidean geometry when they were asked to consider the points and lines as lying on the planar surface. When they were presented with two non-parallel, short line segments, for example, they judged that two segments, if extended, would cross on only one side; when given an single line segment and a point that was displaced from the line on which the segment lay, they judged that a line could be placed through the point such that it never crossed another line [99] (Izard, Chapter 19 in this volume). The Munduruku also modulated their judgments to some degree, but not fully, when asked to imagine the extensions of straight lines on the sphere: they judged that lines would cross on both sides of two visible segments, but they continued erroneously to judge that a line could be placed through a point such that it never crossed a second line. All these findings provide evidence that Euclidean geometry develops, by middle childhood, even in a culture lacking formal education, rulers, or maps.

Studies of younger children suggest, nevertheless, that Euclidean geometry develops gradually over childhood. Shusterman *et al.* [97] presented versions of the Munduruku map

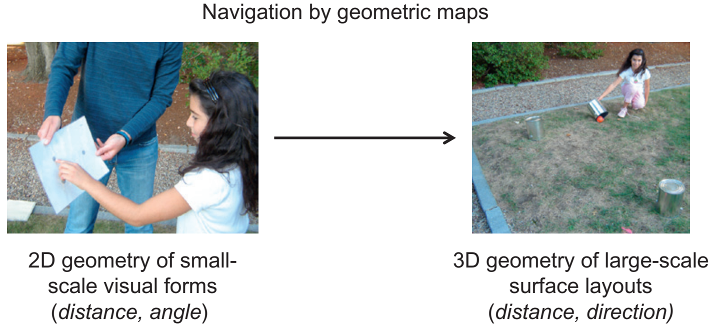


FIGURE 18.10 Navigation by geometric maps in relation to the two core systems of geometry.

task to four-year-old children in the US (Fig. 18.9B). Like the Munduruku, children performed well when the target location was distinctive in color and shape, and they proved more sensitive to geometry when no distinctive landmarks were presented (see also [100]). Analyses of performance across the different shapes suggested, however, that children were not sensitive to all the geometric relationships detected by the Munduruku adults and older children. When the map consisted of three collinear but unequally spaced points, children reliably used the relative distances of the points on the map to distinguish among the similarly spaced 3D objects. Children performed no better, however, when angle was added to distance information (in a triangular map), and they failed altogether to use sense information to distinguish the two similar corners of an isosceles triangle. At four years, children may be sensitive only to distance in purely geometric maps. Further experiments suggest a regular developmental progression in map understanding. At six years, children navigate by both distance and angle information in maps [98] (Fig. 18.9C), but they still fail to navigate by sense information, as adults do [96].

Studies using the tests developed for the Munduruku, provide further evidence that six-year-old US children have only limited command of Euclidean geometry [99]. On the triangle completion test with virtual villages on a flat surface, children's placement of the third corner of the triangle was reasonably accurate, but their estimation of the angle at which the two paths met at that corner was not: children's estimates were appropriately influenced by the distance between the two visible villages but not by the angles of the paths leading from them. On the intuitions task, moreover, six-year-old children performed poorly, and they failed to distinguish the properties of points and lines arrayed on planar or curved surfaces. Although Euclidean geometry develops in the absence of education or experience with maps, that development appears to be a long, protracted process that may begin with a focus on distance: the Euclidean property that is shared by the two core systems of geometry.

When children use a map, they display an ability to combine geometric information from two different systems: the core system for navigation that analyzes geometric information in the 3D layout in which they must place or find an object, and the core system for form analysis that analyzes information in the 2D image that serves as the map (Fig. 18.10). What allows children to make this link? Prior experience with maps evidently is not necessary, since it is unlikely that the Munduruku, or most of the young children in these studies, had ever used a map before. Both Munduruku adults and US children, however, can use language to specify spatial locations. In all the above studies, the experimenter used object names and spatial terms to connect each form on the map to an object in the 3D layout (e.g., "Big Bird wants to sit in this chair [pointing to a dot on the map]. Can you put him in his favorite chair [pointing in the direction of the 3D array]?"). The act of using the same spatial expression to refer both to a 2D point and to a 3D position may have served to link these representations for children [101].

It is possible, therefore, that the spatial expressions of natural language initially serve to connect geometric information in 2D forms to geometric information in 3D navigable arrays. If that suggestion is correct, then there is a parallel between the construction of natural number and natural geometry: both would depend first on language, and then on other symbol systems (Arabic notation, number lines, maps). Far more research is needed, however, to explore the process by which children integrate information from the two core systems of geometry, and to test the roles of language and of other symbol systems in that process. I end by considering one line of research exploring a small corner of this terrain: studies of the role of spatial language in integrating representations of the shape of the layout with representations of the positions of landmarks.

These experiments focused on children's reorientation in a rectangular environment with one landmark that broke the rectangle's symmetry: a single distinctively colored wall. Although young children can use a colored wall to directly mark the location of a hidden object, they typically fail to use such a wall to guide their reorientation [44,48,50,51,57]. Young children use wall colors as beacons, and wall lengths and directions to specify their own position and heading, but they fail to combine these sources of information. Studies of older children reveal a change, however, at about the age when children begin to master spatial expressions involving the terms *left* and *right*. At about six years, children begin to reorient in accord with the lengths, directions *and* colors of walls. The development of this ability coincides with the acquisition of spatial language [102] and is enhanced by spatial language training [103], but these findings do not reveal the role that language plays. Does language serve as a medium for combining information about the spatial layout with information about landmark objects?

Recent studies of adult speakers of Nicaraguan Sign Language (NSL) shed light on this question [104]. NSL is a new sign language that began to emerge in the 1970s, developed by children attending a new school for the deaf. It is now the primary language spoken by the school's graduates. Importantly, the first cohort of graduates entered the school with a variable array of homesign gestural systems, and the common language on which they converged lacks many of the grammatical devices of fully developed signed or spoken languages. These first-cohort speakers have no consistent means for expressing or interpreting spatial relationships such as *left of X* [105]. The second cohort of graduates entered the school at a later point in the development of NSL, and their language is richer and more lawful. Second-cohort speakers are more consistent in their use of expressions for left–right relationships, and they communicate these relationships more effectively [108]. Except for these language differences, however, members of the two cohorts are similar: all are adults who live in the same culture and communicate regularly with one another. They provide, therefore, an excellent population for studying whether differences in their spatial language lead to differences in performance on non-linguistic navigation tasks.

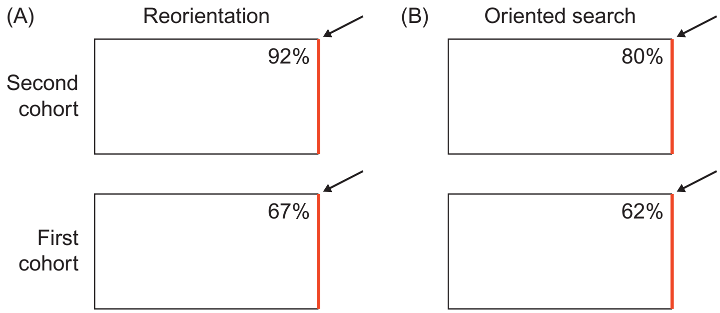


FIGURE 18.11 The performance of first- and second-cohort speakers of Nicaraguan sign language on (A) a test of reorientation in a rectangular environment with one colored wall, and (B) a test of oriented search for an object in a rectangular box with one colored side, following rotation of the box (after [104]). Arrows indicate the location of the hidden object: numbers indicate the percentage of first searches at the correct hiding location.

To address this question, Pyers *et al.* [103] presented first- and second-cohort NSL speakers with two spatial tasks: a reorientation task in a rectangular room with a single colored wall, and an oriented search task in which an object was hidden in a rectangular box with a single colored side that was then rotated on a table (Fig. 18.11). After completing both tasks, participants were asked to describe where the object was hidden, and their spatial expressions were coded. There were three principal findings. First, second-cohort signers performed markedly better than first-cohort signers on both spatial tasks, although performance was well above chance (25%) for each group. Second, across the entire sample, use of the colored wall for reorientation correlated with one specific aspect of spatial language: the consistency of signing of expressions involving the relations *left* and *right*. Third, across the sample, use of the colored side of the box to locate the hidden object correlated with a different aspect of spatial language: the consistency of the positioning of the colored wall within the signing space. Importantly, neither language variable consistently predicted performance on the opposite task. Thus, these correlations do not reflect individual differences in the overall proficiency of language or spatial cognition. They testify to more specific relationships between spatial language and spatial representation.

How might spatial expressions such as *left of the tree* serve to combine geometric and landmark information automatically and productively? These combinations may depend on three attainments achieved by speakers of any natural language [106]. First, speakers have learned a lexicon of words referring to entities in diverse cognitive domains including objects (*box, wall*), properties (*red*), numbers (*three*), and spatial relationships (*left, longer*). Second, speakers have induced a set of rules for combining these words to form expressions, and those rules are conditioned only by the grammatical properties of the words that they serve to combine, not by their content domains. Although *red* and *long* refer to properties in different cognitive domains, both are adjectives, and so for any grammatical expression that includes one (*left of the long wall*), there is a possible grammatical expression that includes the other (*left of the red wall*). Third, speakers who have learned the words and rules of a language can infer the meaning of an expression in the language the first time that they hear it, because the meanings of expressions follow from the meanings of their words and the rules for combining them. If one learns a new color term (say, *chromium*) and already

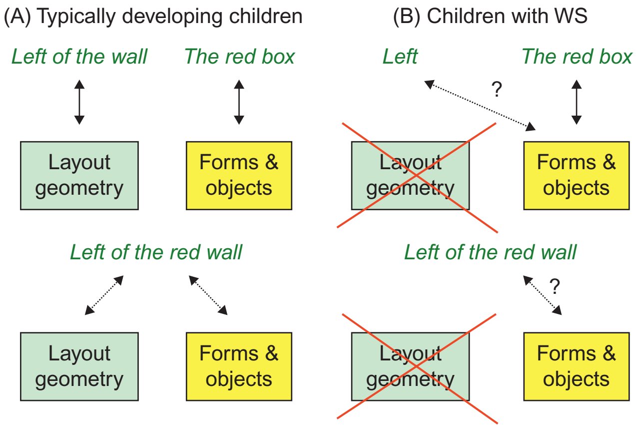


FIGURE 18.12 Schematic and simplified depiction of the possible learning (top) and use (bottom) of language to combine representations of layout geometry and landmark objects in (A) typical development and (B) Williams Syndrome.

knows the meaning of phrases like *the red tray*, one needs no further learning to know the meaning of phrases like *the chromium tray* [107].

With these three properties, language could serve as the medium in which information about object properties, and information about the shape of the surrounding layout, could be productively combined. With a cognitive system for representing objects, children can learn terms like *red* and *triangle* by mapping words and expressions to object representations. And with a separate cognitive system for representing distances and directions in the navigable environment, children can learn terms like *long* and *left* by mapping words and expressions to representations of the extended surface layout. The combinatorial machinery of natural language could then derive the meanings of expressions that combine these terms, and thereby serve as a medium in which information from these diverse representations is productively combined (Fig. 18.12A). On this account, as in the case of natural number, the information that guides adults' navigation resides entirely in core systems for representing objects and the surface layout; language serves only to link information from these distinct systems together.

Perhaps, however, language plays a different role. When children learn an expression like *left of the blue wall*, they may gain a means for encoding properties of the environment that bypasses core representations altogether.[3] Lakusta, Dessalegn and Landau's studies of adults with Williams Syndrome shed light on this possibility. Recall that WS adults appear to lack altogether the core system of geometry for navigation: after disorientation, they show no ability to distinguish among the corners of a rectangular room by representing the distances and directions of its walls. In contrast, however, WS adults have relatively proficient language, including some spatial language, and considerable abilities to use the distinctive color of a wall to specify the location of a hidden object. If language serves to

[3] I am grateful to Susan Carey for this suggestion (see also [5]).

bypass geometric representations, then these two abilities should be related to one another as they are for Nicaraguan signers: WS adults with more consistent spatial language should be more consistent in their search to the left or right of a colored wall. Contrary to this prediction, WS adults show no relation between the consistency of their spatial language and the consistency of their reorientation performance in a room with one colored wall. These findings support the view that language serves to combine core representations. In the absence of a core representation of layout geometry, spatial language cannot play this role (Fig. 18.12B), and it does not enhance navigation.

CONCLUSION AND PROSPECTS

Abstract concepts in general, and the concepts of natural number and geometry in particular, present a longstanding puzzle to the brain and cognitive sciences. Nevertheless, a rich array of studies in cognitive development, animal cognition, cognitive psychology, comparative cultural psychology, and cognitive neuroscience is providing clues that may lead to its solution. Four cognitive systems found in infants, animals, and human adults across widely varying cultures give rise to representations with true numerical or geometrical content. The systems develop on the basis of little or no experience in numerically or geometrically structured environments, and therefore are innate. Abstract concepts of natural number and Euclidean geometry build on these systems of core knowledge.

Nevertheless, each of these four systems is limited in its domain of application (none is fully abstract) and in the information that it makes available (none has the power of the system of integers or of Euclidean geometry). The limits on the two core number systems, and on the two core geometry systems, are complementary: richer systems of number and geometry could be constructed if representations from the different core systems could be productively combined. Research suggests that children begin to make these combinations in the preschool years. Thus, the most intuitive, abstract geometrical and numerical concepts that we possess as adults may not be given to us as infants; they may develop as children come to combine their core representations productively.

The processes that give rise to fundamental human conceptual integrations are only beginning to be explored. Some of the research reviewed in this chapter suggests that natural language plays a pivotal role in the development of abstract numerical and geometric concepts, and does so by serving as the primary medium for combining information productively across distinct systems of core knowledge. These suggestions raise many questions, however, concerning the aspects of language that play this role, and the ways in which language interfaces with non-linguistic conceptual representations. Research is also needed to probe whether other symbolic devices can substitute for language and serve to combine core representations productively. Finally, research into other abstract concepts, in domains such as morality, politics or economics, is needed to explore whether core systems and productive combinatorial abilities produce a broad range of abstract concepts or apply more narrowly to the concepts of mathematics. By addressing such questions, research in cognitive science promises to elucidate the mechanisms by which humans go beyond the core knowledge systems that we share with other animals and construct truly abstract knowledge systems that are unique in the living world.

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