PAPER

Kindergarten children's sensitivity to geometry in maps

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Abstract

Geometrical concepts are critical to a host of human cognitive achievements, from maps to measurement to mathematics, and both the development of these concepts, and their variation by gender, have long been studied. Most studies of geometrical reasoning, however, present children with materials containing both geometric and non-geometric information, and with tasks that are open to multiple solution strategies. Here we present kindergarten children with a task requiring a focus on geometry: navigation in a small-scale space by a purely geometric map. Children spontaneously extracted and used relationships of both distance and angle in the maps, without prior demonstration, instruction, or feedback, but they failed to use the sense information that distinguishes an array from its mirror image. Children of both genders showed a common profile of performance, with boys showing no advantage on this task. These findings provide evidence that some map-reading abilities arise prior to formal instruction, are common to both genders, and are used spontaneously to guide children's spatial behavior.

Introduction

Animals from ants to primates have mechanisms for navigating in relation to the geometry of the surrounding surface layout (e.g. Gouteux, Thinus-Blanc & Vauclair, 2001; Lever, Wills, Cacucci, Burgess & O'Keefe, 2002; Wystrach & Beugnon, 2009), but humans are unique in our capacity to navigate by symbolic, geometric maps (DeLoache, 2004; Golledge, 2008; Landau & Lakusta, 2009). Map-guided navigation is exhibited by adults from diverse cultures, with or without formal schooling (Dehaene, Izard, Pica & Spelke, 2006), as well as by children (e.g. Davies & Uttal, 2007; Liben & Downs, 1989), including 4-year-old children tested without training or feedback (Shusterman, Lee & Spelke, 2008; Vasilyeva & Bowers, 2006). It is a culmination of symbolic abilities that begin in infancy and undergo a series of dramatic changes over the preschool years (DeLoache, 2004; Newcombe & Huttenlocher, 2000). Nevertheless, the rich developmental literature on map-based navigation does not clarify the nature of the geometric information that children spontaneously extract from maps. Moreover, the developmental literature does not clarify the origins of the often-reported sex difference in map use, whereby males are more apt than females to navigate by purely geometric information (Halpern, Benbow, Geary, Gur, Hyde & Gernsbacher, 2007).

Here we report an experiment on 101 children, aged 5 to 6 years, using a very simple task in which purely geometric maps consisting of two connected line segments specify a unique location in a small-scale 3D layout. By varying the lengths of these lines, their angle of intersection, and the direction at which one line meets the other, we test for children's ability to extract and use the three fundamental relationships of Euclidean geometry that are preserved in standard maps of small environments: relationships of *distance* (or *length*)*, angle* and *sense* (i.e. the directional information that distinguishes forms facing leftward vs. rightward) (Figure 1a). By testing reasonably large numbers of children on sensitivity to each of these dimensions, we assess interrelationships between children's abilities to detect these different geometric properties, and we probe for the existence and nature of sex differences in sensitivity to these properties.

To use a purely geometric map, one must detect and compare geometric information in two quite different types of arrays. Most maps are two-dimensional pictures that can be held and moved by the navigator, who stands outside them. In contrast, the environment that they depict is a three-dimensional surface layout that is stable and surrounds the navigator, who moves within it. Because the map is movable, it usually differs from the environment that it symbolizes not only in size and dimensionality but also in orientation. Successful use of a purely geometric map therefore requires either (a) that the map be physically or mentally rotated into correspondence with the navigable layout, or (b) that geometric properties of the map and the layout be extracted and compared in an orientation- and scale-invariant manner.

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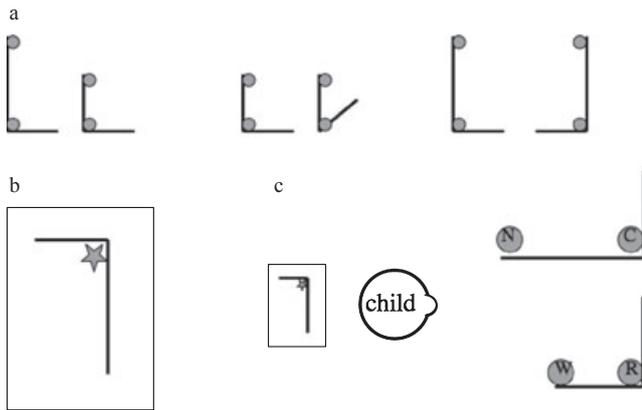
Figure 1 *Displays for the map task. (a) Schematic overhead view of the arrays used to test for sensitivity to distance (left), angle (center), and sense (right) in purely geometric maps. (b) A sample map specifying the structure and location at which the child is to place an object; across trials, the map varied in orientation as well as in side length, angle, or the direction of the shorter side relative to the longer side. In all cases, the map presented the same distance, angle, and sense relationships as one of the two structures, and it differed from the other structure with respect to just one of these relationships. (c) A sample scoring of children's placements on a distance trial, at the correct location (C), the incorrect location on the correct structure (N), the correct location on the incorrect structure (R), and the incorrect location on the incorrect structure (W); the map and child's position are not drawn to scale.*

The task of navigating by a purely geometric map may pose a further problem for young children. Developmental research provides evidence that children extract different geometric properties from visual objects and forms, on the one hand, and from extended spatial layouts on the other (Landau & Lakusta, 2009; Spelke, Lee & Izard, 2010). When children are presented with a two-dimensional visual form, they tend to focus on its distinctive angular and distance relationships (Gibson, Gibson, Pick & Osser, 1962; Izard & Spelke, 2009). In contrast, children are far less sensitive to the sense relationships that distinguish a 2D form from its mirror image, leading to confusions among letters (e.g. b vs. d) and letter-like forms (Gibson *et al.*, 1962; Izard & Spelke, 2009). Even infants distinguish visual forms by their length relationships (Newcombe, Huttenlocher & Learmonth, 1999) and angular relationships (Lourenco & Huttenlocher, 2008; Schwartz & Day, 1979), but they are less apt to distinguish forms by their sense relationships (Lourenco & Huttenlocher, 2008). Recent research, moreover, suggests a sex difference in infants' capacity for mirror image discrimination in rotated displays, with male infants outperforming females on such discrimination tasks (Moore & Johnson, 2008; Quinn & Liben, 2008). At all ages and for both genders, however, visual form analysis privileges angle and distance over sense (Izard & Spelke, 2009).

When infants and preschool children navigate through a large-scale spatial layout, they extract a different set of

shape properties. Sensitivity to the shape of the surrounding layout is shown most clearly in studies in which navigating children are disoriented, and therefore must rely on remembered properties of their surroundings in order to reorient themselves and relocate hidden objects. Like other animals including insects, birds and mammals (e.g. Cheng, 1986; Sovrano, Bisazza & Vallortigara, 2003; Wystrach & Beugnon, 2009), children reorient spontaneously and reliably by detecting relationships of *distance* and *sense*. For example, if an object is hidden in a corner of a rectangular room bounded by a *long* wall on the *left*, disoriented children confine their search to the two corners with these two geometric properties (Hermer & Spelke, 1996; see Cheng & Newcombe, 2005, for review). In contrast, children show little sensitivity to *angle* in reorientation tasks. If an object is hidden in a rhombic room with corners of markedly different angular size, disoriented 2–3-year-old children search the four corners equally, irrespective of angle (Hupbach & Nadel, 2005). By 4 years of age, children confine their search to the two corners of the appropriate size (Hupbach & Nadel, 2005), but further studies suggest that young children's search is not guided by angle but by *aspect ratio*: children reorient themselves by the differing distances of a rhombic room's corners from the center of the space, not by the differing angles at those corners (S.A. Lee & E.S. Spelke, unpublished data). Navigation therefore privileges distance and sense over angle, at least in the room-sized environments used in reorientation tasks.

Thus, children who explore a 2D form spontaneously extract the geometric properties of distance and angle but not sense, whereas children who explore a small but navigable 3D spatial layout spontaneously extract the geometric relationships of distance and sense but not angle. To use a purely geometric map effectively, however, one must extract all three properties from each of these types of arrays. Developmental studies of map use may serve, therefore, to probe how children come to accomplish this task.

To probe the geometric information that children extract from maps, it is necessary to devise a map task that probes sensitivity to distance, angle, and sense separately. Previous research using purely geometric maps (Davies & Uttal, 2007; Dehaene *et al.*, 2006; Shusterman *et al.*, 2008) met this requirement only to a limited degree. In Dehaene *et al.*'s studies, adults and children were presented with three objects arranged in a triangle, and a map presenting a smaller, geometrically similar triangle on which a target location was indicated. On a small subset of trials, the triangle was isosceles and the target location was chosen such that only directional information distinguished it from a second location. Adults made many errors but performed above chance on these trials, providing evidence for some sensitivity to direction in maps. On all the remaining trials, however, distance, angle and sense all specified the hiding location. For example, half the trials presented the objects in a right triangular array with three sides of different

lengths. Adults and children performed well on these trials (Dehaene *et al.*, 2006; Shusterman *et al.*, 2008), but their performance does not reveal which geometric properties guided their search. When the object was hidden at the most distant corner of the triangle, for example, participants may have encoded this distance information, or they may instead have encoded angle information (because the most distant corner is also the corner with the smallest angle) or directional information (because the most distant corner is also the corner directly to the left or right of the triangle's right angle). Distance, angle and sense relationships are similarly intertwined in studies presenting maps of more complex environments (e.g. Uttal, 1996). Thus, adults and 6–12-year-old children without formal education can navigate by purely geometric maps, but the geometrical information guiding their navigation has not been fully specified.

Research by Shusterman *et al.* (2008) conducted with 4-year-old US children further addressed this limitation by presenting children with three containers in linear arrangements as well as the triangular arrangements of Dehaene *et al.* (2006). On linear trials, three containers appeared at unequal distances along a single straight line. While the child stood with his or her back to this array, the experimenter presented a 2D map of the array composed of three circles in a line that preserved these distance relationships but that was many times smaller and oriented differently with respect to the array on each trial. The experimenter pointed to a single circle on the map, and instructed children to place a toy on top of the corresponding object. Because the circles appeared in a line, no angle information distinguished the correct location from the two alternatives. Because the orientation of this line varied arbitrarily with respect to the 3D array, no directional information distinguished among the locations either. Children reliably placed the toy on the corresponding object, showing spontaneous use of distance in the map. These findings accord with those of other map tasks using a continuous rectangular space: 4-year-old children were found to use the distance of a point from the edges of a surrounding rectangle in a map to indicate the distance of an object from the edges of a larger rectangular arena (Huttenlocher, Newcombe & Vasilyeva, 1999; Vasilyeva & Huttenlocher, 2004; Vasilyeva & Bowers, 2006).

Further findings from Shusterman *et al.* (2008) suggest that 4-year-old children may have failed to use angle or sense information in this task. When the objects were placed in a right triangle, such that angle, distance, and sense all specified the correct location, children performed no better than they had performed with distance alone. Nevertheless, the evidence against children's use of angle or sense is open to alternative interpretations. In particular, children may be sensitive to all three types of geometric information but may rely primarily on distance when multiple sources of information are available.

For the present research, we designed a new map task to assess whether children can use distance, angle, and sense relations under more closely comparable conditions. On each trial, children were shown two 3D L-shaped structures, each composed of two extended surfaces (Figure 2). The two structures appeared side by side and were identical in all respects except one. On distance trials, one L consisted of two surfaces of equal length, whereas the other had surfaces whose lengths differed. On angle trials, the two structures had corners whose angular size differed. On sense trials, the two sides of the L differed either in length or in color,[1] and the two L structures were mirror images of one another (Figure 1a).

On every trial, children stood facing away from this 3D array and were shown a map, presented at an arbitrary orientation with respect to the array and at a fixed position from which the subject could not see the map and the depicted environment simultaneously. The map depicted just one of the two L-shaped structures, and it indicated a single location on the L: either at its corner or at the end of one side (Figure 1b). Children were shown, in initial practice trials, that the map could depict either the structure on the left or that on the right, and that the position and orientation of the map varied randomly with respect to the array. Only the distance, angle, or sense information in the map therefore served to indicate which structure it depicted.
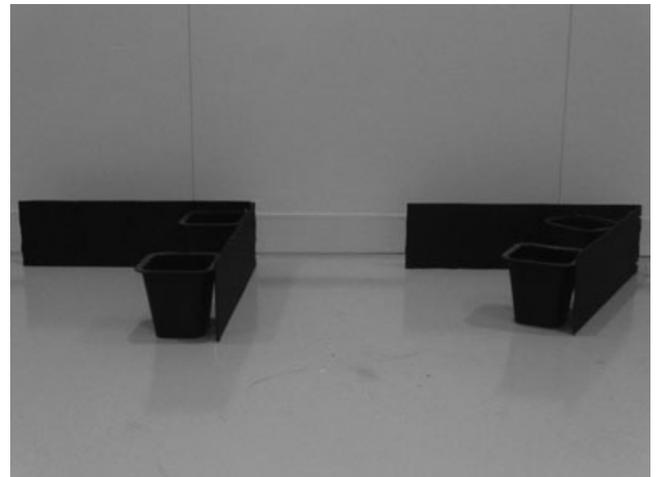

Figure 2 *Example array, as seen from the perspective of the child. The array tests for sensitivity to angle by presenting a right angle (left) and an acute angle (right).*

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[1] Although differences in distance and angle can be presented in figures that are otherwise indistinguishable, sense necessarily is a relation between two distinguishable entities: for any entities x and y, we can distinguish arrays in which x is left of y from arrays in which x is right of y only if x and y are themselves perceptually distinct. In designing our displays for the sense trials we were unsure as to whether children would most readily distinguish between the sides of the L on the basis of a geometric property – length – or on the basis of a non-geometric property – color. Accordingly, we presented children with trials of both types.

As in Shusterman *et al.*'s (2008) studies, children were told to place an object at the location in the 3D layout that corresponded to the indicated location on the map. After the location was indicated on the map, the map was removed, so that children could never see the map and the array that it depicted simultaneously. To limit children's placements and to facilitate coding of their responses, four containers were placed in the layout, at the corner and at one end of each of the L-shaped structures. Children's responses were scored as correct (C) if they placed the object in the correct position on the correct structure, as near (N) if they placed the object in the incorrect position of the correct structure, as reversed (R) if they placed the object in the correct position of the incorrect structure, and as wrong (W) if they placed the object in the incorrect position on the incorrect structure (Figure 1c).

We reasoned that if children were attentive to the task instructions, remembered the location on the map, were motivated to perform the task, and detected the basic topological distinction between the corner and the end of a structure, then they should confine their placements to corner locations when a corner was indicated on the map and to end locations otherwise. In all conditions, therefore, placement at C and R should exceed placement at N and W. Moreover, if children spontaneously extracted distance information from the map, as the findings of Shusterman *et al.* (2008) suggest, then children should tend to place the object at C more than at R on distance trials, where these two locations were distinguished by their length relations. The critical questions concerned children's performance on the angle and sense trials. If children can use both angle and sense information, then their placements should predominate at C on those trials as well. In contrast, if 6-year-old children use neither angle nor sense relations in maps, as might be the case for younger children, then they should place objects at C and R with equal frequency on those trials.

To study children's sensitivity to these three sources of information in detail, we presented this task to a fairly large sample of kindergarten children. We focused on children of this age because they are just at the start of instruction in mathematics. We used a larger sample than past experiments on geometric map use in children, in order both to increase the sensitivity of this test and to test for possible relationships between sensitivity to different geometric properties: for example, do the children who show the greatest sensitivity to distance relations also show greater sensitivity to angle or sense relations?

By testing a fairly large sample of children, we also hoped to shed light on the existence and nature of sex differences in map use at this age. Considerable evidence suggests that males show greater abilities to navigate by geometric information than do females (for reviews see Geary, 1996; Halpern *et al.*, 2007). If this suggestion is correct, then boys may show an advantage on the present geometric map task relative to girls. On the other hand, sex differences in spatial reasoning typically are shown in navigation tasks that present multiple sources of information, including both geometric relationships and non-geometric landmarks, and that therefore can be solved by a variety of different strategies (see Davies & Uttal, 2007; Spelke, 2005). When navigating in a purely geometric environment lacking landmarks, girls are sometimes found to use geometric information as effectively, or even more effectively, than boys do (Spelke & Ellison, 2009; Lourenco, Huttenlocher & Fabian, under review). If this pattern extends to map-based navigation, then girls may perform as well as boys on the present task.

Method

Participants

The map test was administered to 101 children (46 girls; mean age = 5.86 years, range 5.27 to 6.58 years) recruited from the kindergarten classes of a public elementary school in a middle-class community of suburban New York. Children were tested in the hallway outside their classroom.

Displays

On each trial, the navigable layout consisted of two structures in the form of an L, presented at the same orientation and separated from one another by 30.5 cm. Each structure was made of two pieces of foam core, 17.8 cm high and meeting in an L at an angle of 60, 90, or 120 degrees (Figure 1). On all the angle trials and the two-colored sense trials, the two sides of the L-shaped structure were 61.0 cm long. On the distance trials, one of the L-shaped structures had these dimensions and the other structure consisted of one side that was twice as long (121.9 cm). In all the structures for the distance and angle trials and for half the sense trials, the two pieces of foam core were black; for the remaining sense trials, one piece was black and one was red. On the two-colored sense trials, both the red and black sides were 61.0 cm long. Attached to each structure were two black square buckets, 16.5 cm high. One bucket was attached to the internal corner of the L; the other bucket was attached to the inward facing end of the longer or red side of the L.

For each trial, participants were presented with a 2D map printed on 21.6 cm by 27.9 cm laminated paper that depicted a single L with a red star either immediately in front of its corner or in front of its longer end (Figure 1). The map was geometrically equivalent to one of the two 3D structures at 1/12 the scale. Black lines were used to depict the structures on angle, distance, and all-black sense trials; black and red lines depicted the structures on two-colored sense trials and on practice trials; all lines, including the star, appeared on a white background and were easily visible. The particular structure depicted by the L on the map (left structure vs. right structure with respect to the child), the particular position depicted by

the star (corner vs. end), and the orientation of the map relative to the array (0, 90, 180, or 270 degrees) varied across trials. A toy frog served as the object that children placed in the array.

Displays for the practice trials were similar to the above test displays except that they included no informative geometric information. Practice arrays consisted of one bucket attached to the center of a single black surface (trial 1), and two buckets attached to the center of two surfaces of contrasting colors (red vs. black: trials 2 and 3). The corresponding maps for these arrays consisted of a black or red line with a star on one side of the line adjacent to its center.

Design

Each child was given three blocks of six trials testing use of distance, angle, and sense (18 trials in total). Sense trials with walls of different lengths and with walls of different colors were blocked (three trials each) and presented in counterbalanced order across participants. The order of trial blocks also was fully counterbalanced across children, yielding six different presentation orders. Within each block of trials, the position of the 3D array depicted on the map (left or right) and the position of the star on the map (at a corner vs. end) were counterbalanced both within and across children: in each block, children were tested once at each of the four positions and a second time at two positions; the order of trials varied quasi-randomly so as to discourage strategies of searching primarily to locations that were not previously tested. Children were assigned to presentation orders so as to equate as nearly as possible the numbers of children tested in each order ($N = 17$ in five orders, $N = 16$ in the sixth order).

Target location and structure orientation were counterbalanced across presentation orders, as were the angles of the two displays on distance trials and direction trials (60°, 90°, or 120°). On angle trials, the two contrasting angles were 60° and 90° for three orders and 90° and 120° for the remaining three orders.

Procedure

The experiment began with three practice trials that introduced the map task but presented no informative geometric information. Children stood with their backs to the 3D display at a distance of approximately 2 m, and they were presented with a map consisting of a single line with a red star near its center. The experimenter, standing in front of the child holding a toy frog, said, 'This picture tells us where Froggy wants to hide. What do you see in this picture? Do you see the red star? Froggy's favorite hiding place is where the red star is. Can you point to the star? Great, can you put Froggy in his favorite hiding place?' Children were then encouraged to turn around, where they saw one bucket attached to a flat black surface. Children were allowed to put the toy into the bucket

and were directed to choose the bucket as Froggy's hiding place if they did not do so spontaneously. On the second and third practice trials, the 3D array consisted of two buckets, each attached to a single flat surface of contrasting colors (red vs. black), and the map presented either one red or one black line with the red star near its center. On each of these three practice trials, the map appeared in a different orientation relative to the array (0, 90, and 180 degrees). Children were given informative feedback on these trials and their placement errors were corrected. Errors were rare (< 9% of practice trials). After the last practice trial, the test trials began, following the same procedure but with neutral to positive feedback on every trial, regardless of the child's response. In each block of trials with a single array, children saw the map before they were allowed to see the array on the first test trial. Because the array did not change within a block of trials, children were acquainted with the 3D array before the later map trials within each block. Short breaks were taken between trial blocks to allow the experimenter to change the 3D arrays.

Results

Overall, children performed well on this task, placing the object in the geometrically correct location on 43.56% of trials (chance = 25%, $t(100) = 10.04$, $p < .001$). Children successfully used the maps to locate the object not only when the map and the array that it depicted appeared at the same orientation (43.60% correct, $t(100) = 6.98$, $p < .001$) but also when they appeared at orientations that were separated by 90, 180, or 270 degrees (respectively, 41.93%, 45.79% and 44.11% correct, all ts $(100) > 6.5$, $p < .001$; Table 1). Children also showed considerable variability in performance (range, 11.11% to 77.78%, $SD = 18.59$), but gender did not account for this variability: performance by boys and girls was closely similar both on average (40.30% and 47.46% correct overall for boys and girls, respectively) and in range ($SD = 17.96$ and 18.76, respectively).

Our primary focus, however, is not on children's overall performance but on their patterns of performance when the critical information on the map was conveyed by distance, by angle, and by sense. To make these comparisons, we first tested whether the particular displays used to convey each of these three relationships

Table 1 *Performance on map-based navigation by trial type and map orientation (% correct)*

Orientation	Trial type		
	Distance	Angle	Direction
0° rotation	54.46	39.60	36.14
90° rotation	52.48	44.06	26.73
180° rotation	41.58	54.95	40.59
270° rotation	45.05	56.44	31.68

were responded to in similar ways. Then we tested whether children were equally attuned to the map, able to remember the target location, and motivated to perform the task during the tests for sensitivity to each of the different geometric properties. Three sets of preliminary analyses tested for stimulus effects, and a fourth set of preliminary analyses tested for performance effects that are not specific to sensitivity to geometry. After presenting these preliminary findings, we turn to our central question concerning children's use of geometry in maps.

Preliminary analyses of stimulus effects

Because the present study used L-shaped arrays, distance information can be tested only by presenting Ls at a particular angle and sense configuration. On the distance trials, the two Ls differed in their length relationships but presented forms that were equal in angle and sense relationships. For different children, however, the angle presented in the two Ls was either acute or obtuse (all Ls presented the same sense relationship, with the longer side to the left of the vertex). To test whether sensitivity to distance relationships differed for displays presenting these different angles, a preliminary analysis compared performance on the distance trials for the participants who were tested with structures with acute vs. obtuse angles. Performance did not differ across these two conditions, $F(1, 97) < 1$, and there was no interaction with sex, $F(1, 97) < 1$.

On the angle trials, we tested two different angular contrasts: half the children viewed a right angle and a second angle that was acute (45°) and the others viewed a right angle and a second angle that was obtuse (135°). To test whether children performed equivalently when the right angle was compared to a second angle that was smaller vs. larger, a second preliminary analysis compared performance on the angle trials for the participants who were tested with the right vs. acute angles vs. the right vs. obtuse angles. Performance did not differ across these two conditions, $F(1, 97) = 1.91$, *ns*, and there was no interaction with sex, $F(1, 97) < 1$.

On all the sense trials, children were presented with Ls of fixed distance relationships that met at right angles. Because the directional information that distinguishes a form from its mirror image can be presented either in the context of geometric or non-geometric information, two types of sense trials were presented. On half the sense trials, the directional information appeared in displays whose sides differed in length (forward vs. backward black L with longer vertical side); on the remaining sense trials, it appeared in displays whose sides differed in color (forward vs. backward L with equal-length but differently colored sides). To test whether children performed equivalently on these two types of sense trials, a 2 (display: length vs. color) by 2 (sex) ANOVA was conducted on all the data from the sense trials. This analysis revealed no difference between performance with the two types of displays ($F < 1$), no difference between performance by boys and girls ($F(1, 99) = 1.73$, *ns*), and no interaction ($F < 1$). Separate *t*-tests then tested for effects of the display variable on each of three measures of performance: selection of the appropriate target location (corner vs. side), selection of the correct target object (forward vs. backward L), and selection of the correct location and object (C). Performance with the two arrays did not differ by any of these measures, all *t*s(100) < 1.

In summary, these preliminary analyses suggest that children responded to distance distinctions, angle distinctions, and sense distinctions in a similar manner across the different displays used in this experiment. Accordingly, the principal analyses collapsed over the above display variables and compared performance on the six distance trials, six angle trials, and six sense trials. Our last preliminary analyses were undertaken to assess whether children's performance showed equal attention to the maps, memory for the target location, and motivation across these three types of trials.

Preliminary analyses of performance effects

To determine whether children were equally attuned to the task of map-reading, equally able to remember the target location, and equally motivated to perform the task across the trials testing for sensitivity to distance, angle, and sense, we assessed children's performance at choosing the correct location on an L (corner vs. end) regardless of whether or not the L that they chose had the correct distance, angle, or sense properties. Children succeeded across all three arrays, choosing the correct location on distance trials ($t(100) = 12.1$, $p < .001$), angle trials ($t(100) = 8.2$, $p < .001$) and sense trials ($t(100) = 6.9$, $p < .001$) (Figure 3a). A 2 (Gender) by 3 (Trial type: distance, angle, sense) by 6 (Order) ANOVA revealed a small but significant effect of Trial type, $F(2, 88) = 4.096$, $p = .02$: performance on the distance trials (mean = 4.50) exceeded performance on the sense trials (mean = 4.06; Bonferroni-corrected $p = .02$). The ANOVA also revealed a significant effect of Gender, $F(1, 89) = 9.302$, $p = .003$: performance on the map task was higher for girls (mean = 4.61) than for boys (mean = 3.94), although performance was well above chance for children of both sexes (see Table 2). Finally, the ANOVA revealed a significant effect of Order, $F(5, 89) = 3.146$, $p = .012$. Although no pairwise comparisons were significant with post-hoc correction, children tended to perform best in the orders in which the sense trials occurred in the middle positions, such that the study began and ended with either distance or angle trials. There were no significant interactions.

In summary, children's attention and general task performance was not fully uniform across the three different types of arrays: they may have been somewhat more attentive or motivated on distance trials than on angle or sense trials. Moreover, girls may have been somewhat more attentive or motivated than boys, across the experiment. These findings bear on our interpretation
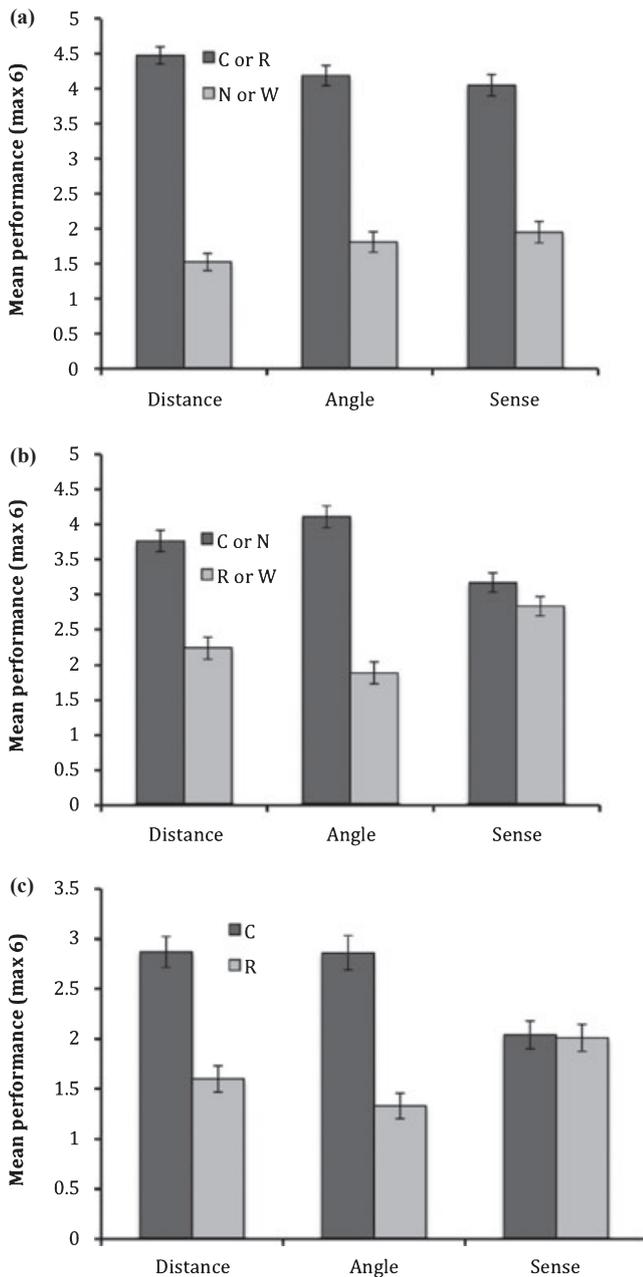
Figure 3 *Children's performance on the distance, angle, and direction trials of the map task. (a) Children's placements at the correct location (C or R) vs. the incorrect location (N or W) of either structure; (b) children's placements at either location of the correct structure (C or N) vs. the incorrect structure (R or W); (c) children's placements at the correct location of the correct structure (C) vs. the corresponding location of the incorrect structure (R).*

of our principal findings, and we return to them as we present those findings. Our principal analyses concern children's use of distance, angle, and sense relations in the map. They are revealed in two measures of performance that we will describe in turn: children's rates of placing the object on the correct structure, and children's rates of placing the object in the correct location on that structure.

Selection of the correct structure

We tested whether children used the map to confine their placements to the geometrically equivalent 3D structure by searching its two locations (C and N) more than the two locations on the geometrically incongruent structure (R and W). By this measure, children succeeded on distance trials ($t(100) = 4.98$, $p < .001$) and on angle trials ($t(100) = 7.08$, $p < .001$) but not on sense trials ($t(100) = 1.23$, $p = .223$; Figure 3b). The 2 (Gender) by 3 (Trial type: distance, angle or sense) by 6 (Group) ANOVA revealed only a significant effect of Trial type, $F(2, 88) = 7.366$, $p = .001$. Follow-up Bonferroni-corrected comparisons revealed that performance on angle trials (mean = 4.12) significantly exceeded performance on sense trials (mean = 3.25; $p = .001$). Performance on distance trials was intermediate (mean = 3.76) and did not differ from either of the other trial types. Although gender was not a significant variable in this analysis, further tests nevertheless tested the performance of boys and girls separately. Both sexes performed well above chance on distance and angle trials, and neither sex performed above chance on sense trials (Table 2).

Selection of the correct structure and location

Most importantly, we measured how often children placed the object at the correct location on the correct structure (at C), relative to the corresponding location on the incorrect structure (R). Children chose the correct spatial structure and location more than the incorrect structure but correct location on distance trials ($t(100) = 4.9$, $p < .001$) and on angle trials ($t(100) = 5.8$, $p < .001$) but not on sense trials ($t(100) = .13$, $p = .896$). The 2 (Gender) by 3 (Trial type) by 6 (Group) ANOVA on the proportion of correct location searches that were at the correct location and structure revealed a significant effect of Trial type, $F(2, 87) = 5.041$, $p = .008$ (Figure 3c). Post-hoc Bonferroni-corrected tests revealed that performance on distance trials (mean = .636) and on angle trials (mean = .673) each exceeded performance on sense trials (mean = .529; respective $p = .026$ and .009). Performance on distance and angle trials did not differ. There was no effect of Gender $F(1, 88) < 1$ or Group ($F(5, 88) < 1$, on children's performance. No interactions were significant.

Effects of map rotation on performance

A final set of analyses compared children's performance when the map appeared at different orientations relative to the array. A 2 (Gender) × 3 (Trial Type: distance, angle, sense) × 4 (map rotation: 0, 90, 180, or 270 degrees from the array) performed on the proportion of correct responses revealed no main effect of Rotation, but there was a Rotation by Trial type interaction, $F(6, 94) = 4.100$, $p = .001$. Separate ANOVAs conducted for each type of trial revealed no effect of rotation for

Table 2 *Performance of boys and girls on map-based navigation by trial type*

Measure		Trial type		
		Distance	Angle	Direction
Selection of correct	Boys	$t = 7.81$, $p < .001$	$t = 4.70$, $p < .001$	$t = 3.87$, $p < .001$
location compared	Girls	$t = 9.52$, $p < .001$	$t = 7.30$, $p < .001$	$t = 6.06$, $p < .001$
to chance				
Selection of correct	Boys	$t = 3.53$, $p = .001$	$t = 3.91$, $p < .001$	$t = 1.00$, *ns*
structure compared	Girls	$t = 3.58$, $p = .001$	$t = 6.57$, $p < .001$	$t < 1$
to chance				
Selection of correct	Boys	$t = 3.40$, $p = .001$	$t = 3.34$, $p = .002$	$t < 1$
structure & location	Girls	$t = 3.58$, $p = .001$	$t = 5.05$, $p < .001$	$t < 1$
compared to				
correct location on				
incorrect structure				

Degrees of freedom: Boys = 54; Girls = 45.

distance trials, $F(3, 98) = 2.05$, *ns*, or sense trials, $F(3, 98) = 2.21$, *ns*, but an effect for angle trials, $F(3, 98) = 4.64$, $p = .004$: Performance was higher on the angle trials with rotations of 180 deg (mean = 55.0%) and 270 deg (mean = 56.4%) than with rotations of 0 deg (mean = 39.6%) and 90 deg (mean = 44.1%). There were no main effects or interactions involving Gender in any of these analyses.

In summary, children appeared to encode the distance information that was presented in the map in an orientation-invariant manner, although they showed some effect of map orientation on their use of angle information. Girls and boys again showed converging performance profiles (Figure 4). Because each child received only 1–2 trials at each orientation of each type of array, however, we cannot exclude the possibility of subtle gender differences in the extraction of particular kinds of geometric information in maps at varying orientations.
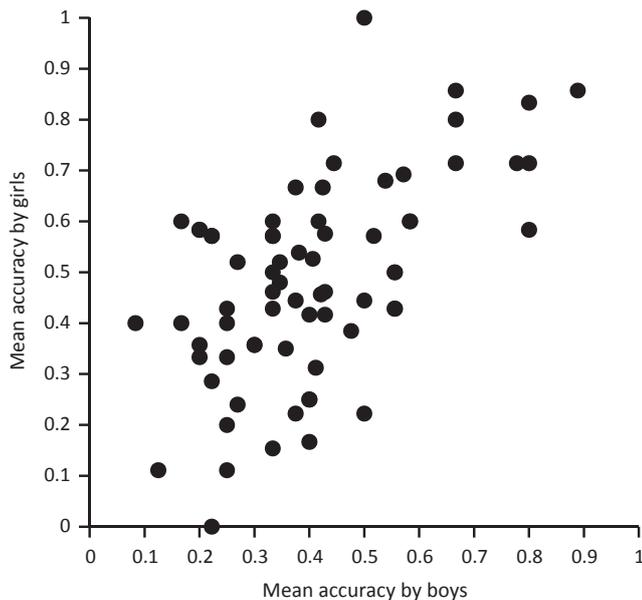
Finally, some children clearly performed better on the map task than others. In the next section, we ask whether children who are particularly sensitive to one geometrical property are also more sensitive to the other properties.

Interrelations among performance with distance, angle, and sense

To address this question, we tested for relationships between performance on the different measures and types of trials. For this analysis, we calculated for each child separately the proportion of correct responses for distance, angle and sense trials. Performance on the distance trials correlated strongly with performance on the angle trials ($r = .462$, $p < .001$) and to a lesser extent with performance on the sense trials ($r = .269$, $p = .006$). In contrast, performance on angle and sense trials was not significantly correlated ($r = .136$). As an initial attempt to investigate whether the correlation between distance and angle trials, and between distance and sense trials, reflects individual differences in sensitivity to these two Euclidean geometric properties, or more general differences in map reading, memory or motivation, the correlation between correct performance (C vs. R) on distance trials and angle trials was tested further with performance on the corner vs. end placement measure (C and R vs. N and W) as a covariate. We reasoned that correct performance on this covariate measure required that children understand the task, attend to the location that was indicated on the map, remember that location as they moved through the 3D layout, and use this remembered information to guide their behavior, but it did not require that they respond specifically to distance, angle, or sense relations. To ensure independence between this covariate and the two measures of interest, only the trials that did not enter into the correlations were used for the latter measure (for the two respective correlations, these were the sense trials and the angle trials). The correlation between distance and angle performance remained strong with the covariate ($r = .368$, $p < .001$). Thus, children who performed well on the distance trials also

Figure 4 *Scatter plot comparing performance by boys and girls across trials.*

tended to perform well on the angle trials, even after controlling for individual differences in memory for the object locations and in motivation to reproduce those locations by placing the object appropriately at a corner vs. an end of the structure. In contrast, the correlation between distance and sense was no longer significant with the covariate ($r = .188$, $p = .061$). The correlation between performance on distance and sense trials may stem from more general differences in memory or motivation, rather than from a specific relationship between processing of these two geometric relationships. In contrast, distance and angle appear to be specifically related for this group of children.

Because boys and girls are sometimes thought to solve spatial tasks by different strategies, a further set of analyses tested for relationships among sensitivity to distance, angle, and sense for each sex separately. The correlation between distance and angle was present both for girls ($r = .543$, $p < .001$) and for boys ($r = .398$, $p = .003$) and it remained significant for girls and marginally significant for boys when performance on the correct location measure, computed only over trials that did not enter into the correlation, was entered as a covariate ($r = .248$, $p = .07$ for boys; $r = .504$, $p < .001$ for girls). There was no gender difference in the magnitude of this correlation (Fisher's Z transformation $Z = .907$). For girls, there was no correlation between performance on either of these types of trials and performance on sense trials (both rs $< .2$, ns). For boys, in contrast, performance on sense trials correlated reliably with performance on distance trials ($r = .348$, $p = .009$) and this correlation remained significant when performance on the correct location measure of the remaining trials was entered as a covariate ($r = .273$, $p = .045$). Boys' performance on sense trials also correlated marginally with performance on angle trials ($r = .253$, $p = .063$), but this correlation disappeared when the covariate was entered. For boys, therefore, the correlation between distance and sense appears to reflect processes specific to map-based navigation and not more general differences in motivation, memory or other performance factors. Nevertheless, none of the correlations or partial correlations found in the sample of boys differs from the corresponding correlations or partial correlations in the sample of girls (i.e. the gender difference is not significant for any pair of correlations, Fisher's Z test).

Discussion

The present experiment provides evidence for three cognitive abilities in kindergarten children. First, such children readily come to see a map as a visual symbol that specifies locations in a distinct, 3D environment. The children in the present studies were given minimal training at using the map: just three practice trials. Most children performed correctly on the very first practice trial, suggesting that they already are prepared to treat

maps as representations of visual arrays. This finding accords with past studies of preschool children's understanding of spatial symbols (DeLoache, 1987), and suggests that this basic symbolic ability has developed to an impressive degree by the time that formal schooling begins.

Second, 5- and 6-year-old children spontaneously extract geometric information from a map. Their ability to do so is particularly striking, because the practice trials that children received involved landmark information (i.e. the presence or color of a line) but no informative geometry. Because the map task involved placing an object at a designated location, moreover, children received no informative feedback on their performance. Their successful performance in this task therefore suggests that children process geometric information readily and effectively in navigation tasks.

Third, 5- and 6-year-old children show impressive abilities to relate the geometric information that they extract from visual forms (the map) to the geometric information they extract from the 3D layouts presented in this experiment (the environment). Although the maps were very simple and the layouts were small, children's spontaneous linking of these two types of arrays is striking, because the map and the environment were never visible at the same time, and the map differed from the environment in scale (it was 1/12 the size), orientation (on ¾ of the trials, it was rotated 90, 180, or 270 degrees with respect to the environment), dimensionality (the 2D map depicted a 3D layout), and perspective (the map depicted the layout in aerial view; the layout was visible from a side view). Moreover, 2D visual forms and 3D navigable layouts typically are encoded in markedly different cortical regions (respectively, the lateral occipital and inferotemporal cortex for visual forms, and the parietal cortex, parahippocampal cortex, and the hippocampus for navigable layouts). These regions preferentially extract different kinds of information (angle and distance for visual forms, distance and direction for navigable layouts: see Spelke *et al.*, 2010, for discussion). By 5–6 years of age, children are well on the path to coordinating these two types of representation. This coordination might both reflect and contribute to children's intuitive grasp of abstract Euclidean geometry (see Spelke *et al.*, 2010).

Nevertheless, the integration of visual forms and 3D layouts does not appear to be complete at 5–6 years of age. Although children reliably navigated by distance and angle information, they failed to navigate by sense information in the map and therefore confused the correct target structure with its mirror image. Children's failure to navigate by direction in the present symbolic task contrasts with their highly successful use of sense relations in nonsymbolic tests of reorientation (Cheng & Newcombe, 2005; Hermer & Spelke, 1996). In the present study, children failed to use a map to distinguish a 3D structure with a longer wall on the left from a structure with a longer wall on the right when the two structures

were otherwise the same. In contrast, much younger children show exactly this ability in a reorientation task. This contrast suggests that the unification of representations of visual forms and of large-scale spatial layouts is not complete at 6 years of age.

Distance and angle as integrated geometric properties

The present experiment is the first, to our knowledge, to test separately for children's use of distance, angle, and sense relations in maps, and to test sufficient numbers of children to begin to investigate interrelationships among sensitivity to these different Euclidean properties. We found that abilities to navigate by distance and by angle were highly correlated across this sample of children: children who navigated effectively by one of these sources of information also tended to navigate effectively by the other. This correlation remained when children's tendency to select a location of the correct type (i.e. the corner vs. the end of a structure) was controlled. Because selection of the right type of location reflects children's understanding of the map task, their memory for the location on the map, and their motivation to place the object in the corresponding location in the structure, these findings suggest that the relation between children's use of distance and angle is not explained by more general differences in cognition or motivation. Instead, the relation between children's use of distance and angle information may shed light on the geometric representations that guide children's map-based navigation. It is possible that children form holistic representations of the geometric displays, such that distance and angle are encoded and retrieved together.

Because children performed at chance on sense trials, one would not expect performance on those trials to be correlated with performance on distance or angle trials, and indeed it is not in the sample as a whole. Nevertheless, one correlational analysis hints at a nascent effect that may merit further study. Performance on sense trials showed no correlation with performance on distance or angle trials for girls, but it tended to correlate with performance on those trials for boys. Although this gender difference was not significant, and in one case appeared to reflect motivational or general cognitive differences rather than a specific linkage between two types of geometric information, the emerging correlation between boys' performance on distance and sense trials hints that some of the children in the sample were beginning to consider direction in relation to distance. It would be interesting to test for this possible developmental change by repeating the experiment with older children.

Developmental changes in geometric map use

Our findings contrast with previous findings from studies of younger children, who showed sensitivity to distance relations but little sensitivity to angle relations in a similar map task (Shusterman *et al.*, 2008). Thus, sensi-

tivity to angle may develop between 4 and 6 years of age. Nevertheless, this conclusion must be qualified because of the important differences between the displays that tested for map-based navigation at 4 and 6 years. The present studies presented 3D surfaces and maps consisting of connected lines, whereas the studies of Shusterman *et al.* (2008) presented arrays of objects and maps consisting of spatially separated circles depicting each object. Moreover, angular differences provided the only information that distinguished the target structure from its distractor on angle trials in the present studies: the structure of the present task therefore required that children focus on angle in order to place the object correctly. In contrast, the arrays used in the research of Shusterman *et al.* (2008) presented angle information only in the context of distance information. It is possible that younger children will use angle information if it is conveyed by an array of intersecting lines, and if it is the only diagnostic information available. Further research using the present displays could distinguish these possibilities.

Our findings also complement previous findings with adults (Dehaene *et al.*, 2006). Adults in both the US and the Amazon performed much better when a purely geometric map specified an object location by distance, angle and sense together than when it specified the location by sense relations alone. Nevertheless, performance on the basis of sense information was above chance in both adult samples, whereas it was at chance in the present experiment with 6-year-old children. This contrast suggests that the ability to use sense relations in maps may develop after 6 years of age. Again, however, it is possible that procedural differences, rather than age differences, account for the differing sensitivity shown in these experiments.

Sex differences in geometric map use

Our findings bear on debates concerning sex differences in sensitivity to geometry. Many experiments report such sex differences, with males outperforming females on tests of sensitivity to the geometrical structure of maps, navigable layouts, and visual forms (for reviews, see Halpern *et al.*, 2007; Voyer, Voyer & Bryden, 1995). Superior performance by males has been reported not only in studies of adults, but also in studies of preschool children (e.g. Levine, Huttenlocher, Taylor & Langrock, 1999) and even infants (Moore & Johnson, 2008; Quinn & Liben, 2008). These findings often are interpreted as reflecting gender-specific cognitive strategies, with females focusing more on learning routes through the environment and on processing and remembering object features, and males focusing more on layout geometry.

Our findings modulate this picture, and suggest that girls and boys show very similar profiles of performance on geometric map tasks. Children of both sexes responded reliably to distance and angle information, and neither sex responded reliably to the sense information

that distinguishes an array from its mirror image. For both sexes, moreover, sensitivity to distance and angle were correlated. Thus, geometric map abilities appear to be highly convergent across males and females near the start of formal education.

Why do we find such convergence in the present study compared to past research? The most obvious difference between the present map task and those of its predecessors is that we present children with purely geometric maps and environments: displays lacking any distinctive landmarks that could serve as beacons for navigation. In contrast, most studies of map-based navigation, both in adults and in children, present maps that convey a mixture of geometric and non-geometric information, especially information about landmark objects. When presented with both geometric and landmark information, it is possible that girls attend preferentially to the landmarks whereas boys attend preferentially to layout geometry. When presented only with geometric information, in contrast, both sexes may attend to and use that information.

Research by Lourenco et al. (under review) supports these suggestions. They presented toddlers with an environment with both landmark information (a distinctively colored wall) and geometric information (a distinctive room shape), and then they probed for encoding of each type of information by means of a reorientation task. The primary finding was that girls tended to outperform boys at learning the landmarks. Crucially, however, memory for landmarks and memory for layout geometry were inversely related for both sexes: the more a child attended to landmarks, the less able he or she was to navigate by geometry. Thus, when both sources of information were available, girls' greater memory for the landmarks led to a suppression of geometry-based navigation.

Together, Lourenco et al.'s (under review) findings and our own suggest that the reported male advantage in geometry-based navigation reflects a sex difference in strategies for encoding landmarks but not sex differences in the ability to navigate by geometry. When landmarks are absent, girls encode and use geometric information at least as well as boys do. Because the present studies were conducted with 5–6-year-old children from middle-class backgrounds, and because gender and social class can have interactive effects on young children's developing spatial skills (Levine, Vasilyeva, Lourenco, Newcombe & Huttenlocher, 2005), it remains to be seen whether the same ability patterns obtain at other ages and in other populations.

Beyond simple forms and small-scale layouts

Although the present studies showed that children have considerable abilities to read geometric maps at 6 years of age, they are limited in three respects. First, most real maps present a complex mix of geometric and landmark information, whereas the maps in the present studies presented only the simplest geometric forms. Second,

most maps require that navigators represent and respond to continuous gradations in length and angle (e.g. Uttal, 1996). In the present study, in contrast, children were presented only with dichotomous distinctions of distance, angle and sense: the distinction between two sides that were or were not equal in length, that met at a corner that was or was not right-angled, and that presented the same or opposite sense relations. Because sense is inherently dichotomous, the use of dichotomous distinctions of distance and angle allowed us to place the three geometric properties on a level playing field, but it greatly reduced the complexity of the map task. Finally, most maps are used when children or adults must navigate a terrain that extends beyond their present field of view, whereas the map task in the present study required only that children move through, and place an object within, a small-scale surface layout (Davies & Uttal, 2007). Although children were not allowed to see the map and the layout at the same time, they could inspect all the possible target locations in the layout at once, from a single viewpoint.

For these reasons, children's successful performance on the distance and angle trials of the present study likely represents only an early step in the development of their map reading skills. Later in development, children must come to incorporate not only sense information into their map reading, but also non-geometric information (such as street names and visually distinctive landmarks) and continuous variations in distance and angle (see Liben & Downs, 1989). Moreover, children must come to form integrated representations of layouts that are too large to apprehend from any single stationpoint (see Davies & Uttal, 2007). We hope the present task will be useful in further studies of these important developments.

Beyond maps

The geometric properties that are preserved in maps of small-scale environments – *distance, angle,* and *sense* – have a significance that extends beyond the domain of map-based navigation. These three properties suffice to specify all of the objects of Euclidean plane geometry, for they are preserved over all rotations and translations of the Euclidean plane. Thus, skills for analyzing Euclidean distance, angle and sense may be important not only for map-based navigation but also for the development of abstract geometrical reasoning.

Despite the importance of geometry in all math and science curricula, many students find formal geometry to be difficult and even aversive; their difficulties with geometric reasoning can impede their progress in mathematics and science (Clements & Battista, 1992). Nevertheless, the present findings provide evidence that certain skills of geometric analysis are present near the start of formal education. Children draw on these abilities spontaneously, without instruction or feedback, in symbolic navigation tasks. Indeed, the children

in these studies found the navigation tasks to be engaging, challenging, and fun. These observations raise the possibility that educational curricula, building on children's capacities for geometric analysis in search tasks, might serve to enhance the development of skills at the center of their studies of mathematics and related fields.

Acknowledgements

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