Abstract number and arithmetic in preschool children

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Educated humans use language to express abstract number, applying the same number words to seven apples, whistles, or sins. Is language or education the source of numerical abstraction? Claims to the contrary must present evidence for numerical knowledge that applies to disparate entities, in people who have received no formal mathematics instruction and cannot express such knowledge in words. Here we show that preschool children can compare and add large sets of elements without counting, both within a single visual-spatial modality (arrays of dots) and across two modalities and formats (dot arrays and tone sequences). In two experiments, children viewed animations and either compared one visible array of dots to a second array or added two successive dot arrays and compared the sum to a third array. In further experiments, a dot array was replaced by a sequence of sounds, so that participants had to integrate quantity information presented aurally and visually. Children performed all tasks successfully, without resorting to guessing strategies or responding to continuous variables. Their accuracy varied with the ratio of the two quantities: a signature of large, approximate number representations in adult humans and animals. Addition was as accurate as comparison, even though children showed no relevant knowledge when presented with symbolic versions of the addition tasks. Abstract knowledge of number and addition therefore precedes, and may guide, language-based instruction in mathematics.

cognition | development | numeracy | quantitative skills

The capacity to represent number precisely and to perform exact arithmetic is unique to enculturated, educated humans, and its development depends in part on verbal counting and arithmetic instruction (1–3). Nevertheless, nonhuman primates and preverbal human infants possess two forms of numerical representation. First, they represent the exact number of objects in a scene, up to a set size limit of three or four (4, 5). Second, although the ability to represent exact number is restricted to very small sets, infants and primates can represent and compare the approximate cardinal values of large sets of objects or events, with accuracy decreasing as the ratio of the compared numerosities approaches 1 (6, 7) and increasing over development (8, 9) and training (10, 11).

What operations do these number representations support? Infants and monkeys track small numbers of objects as they move (12, 13), collect and bind information about object properties (14), compute the effects of adding one object to a set (15, 16), and compare two sets on the basis of number or continuous amount (4, 5). Despite considerable research (see refs. 3, 17, and 18 for reviews), less is known about the operations supported by large, approximate number representations. Untrained monkeys compare large numbers of food objects (19), but their choices might be based on total continuous amount of food rather than number (see refs. 20 and 21 for discussion). Trained pigeons have been claimed to subtract one sequence of events from another (22), but this claim has been disputed (23). Finally, human adults with minimal symbolic number knowledge, and infants and preschool children with no mathematics instruction, compare, add, and subtract large sets of elements in visual arrays (7, 24–27), but neither group has been tested in cross-modal tasks. Here we investigate whether 5-year-old children, with no school experience or relevant symbolic number knowledge, can perform arithmetic operations using abstract number representations permitting comparisons across modalities and formats.

Experiment 1 tested children's ability to compare two arrays of dots that differed by a ratio of 0.57, 0.67, or 0.8 (4:7, 4:6, or 4:5). Experiment 2 tested children's ability to add two arrays of dots and to compare their sum with a third array, when the sum and the comparison array differed by one of the same three ratios. Experiment 3, paralleling experiment 1, tested children's ability to compare the number of dots in a visual array with the number of tones in an auditory sequence. Experiment 4 tested children's ability to add two arrays of dots and to compare their sum with a sequence of tones. Experiment 5 investigated whether the same children were capable of performing approximate addition on verbally presented symbolic numerosities.

Experiment 1: Visual Comparison

Experiment 1 investigated whether 5-year-old children can compare two large sets of dots on the basis of numerosity and whether the accuracy of their comparisons depends on the ratio of the two set sizes.

Method. Participants were 16 preschool children (age range from 5 years 0 months to 5 years 11 months; mean 5 years 3 months) who were tested individually and videotaped throughout. Displays were presented on a PowerMac G4 computer (Apple Computer, Cupertino, CA) with a GS790 color monitor (ViewSonic, Los Angeles). The experimenter introduced the activity as a computer game with dots, in which the child would guess "whether there [were] more blue dots or more red dots." Children were tested with arrays of 10–58 equal-sized blue or red dots presented too briefly for counting. Dots of two sizes (2 or 3 mm) appeared within virtual rectangular enclosures of two sizes ($\approx 9 \times 6$ cm or 7×5 cm). Four practice problems acclimated children to the procedure, followed immediately by 24 problems in which a set of blue dots moved behind an occluder, then a set of red dots moved across the screen next to the occluder, accompanied by a narration (Fig. 1*a*). One-half of the problems were presented with a moving occluder and one-half with moving blue dots, in alternating order. In moving-occluder problems (total duration ≈9,450 ms), an array of blue dots appeared on the lower left side (≈1,300 ms), an occluder appeared on the lower right (≈1,300 ms) and moved leftward to cover the dots (≈1,450 ms), a pause ensued (≈1,300 ms), and then an array of red dots appeared on the upper right (≈1,300 ms) and moved downward to rest (≈2,150 ms). In moving-dots problems (total duration ≈10,250 ms), the occluder appeared on the lower left (≈1,300 ms), an array of blue dots appeared above it (≈1,300 ms) and moved downward behind it (≈1,450 ms), and after a pause (≈1,300 ms), the array of red dots appeared as before. After the animation (with the dots no longer visible), children were asked whether there were more blue dots or more red dots. Children were given mildly positive feedback for all responses, regardless of accuracy.

On the test problems, the numerosities of the sets differed by

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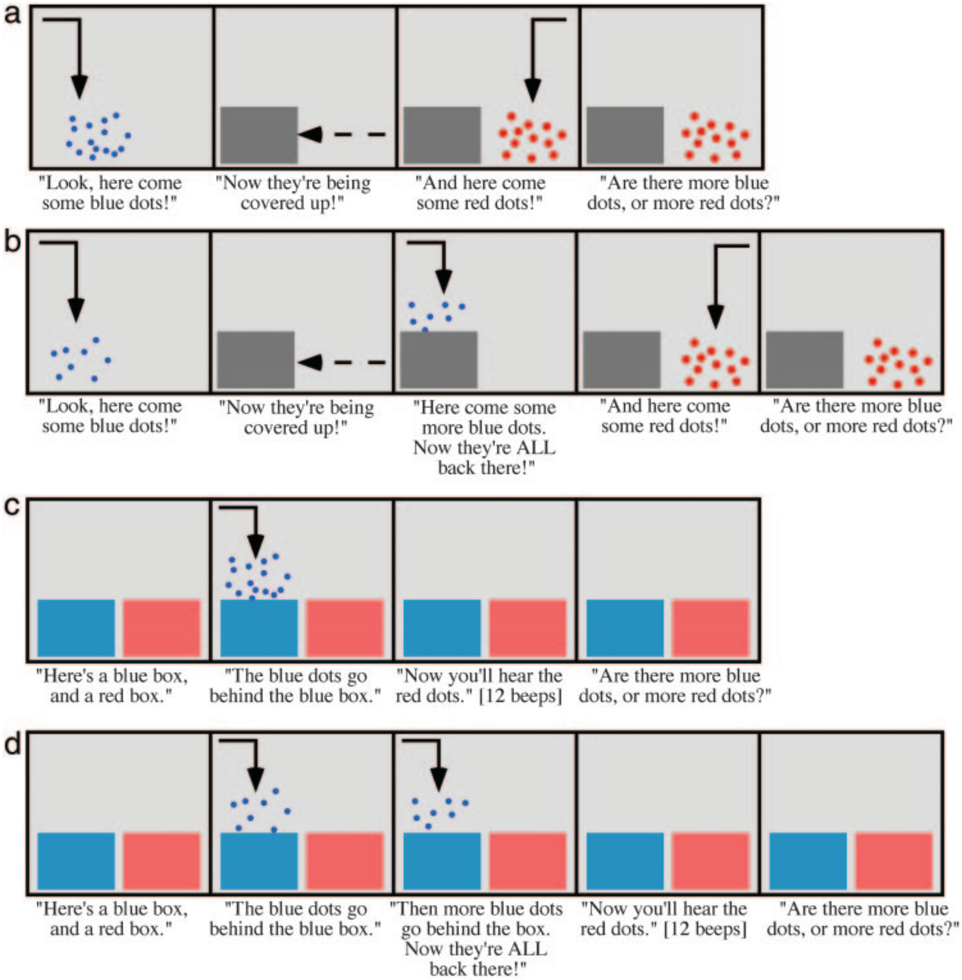

Fig. 1. Schematic depictions of procedures and narrative for the four nonsymbolic tasks. (*a*) Comparison of visual arrays. (*b*) Addition and comparison of visual arrays. (*c*) Comparison of visual arrays and auditory sequences. (*d*) Addition and comparison of visual arrays and auditory sequences.

ratios of 0.57, 0.67, or 0.8; the red-dot array was more numerous on half of the problems at each ratio. On half of the trials, dot size, total contour length, summed dot area, and density were negatively correlated with number (and, therefore, the size of the virtual enclosing rectangle was positively correlated with number); on the remaining trials, the correlations were reversed. Therefore, if children based their judgments on continuous quantities, performance would be at chance overall and would vary systematically across these subsets of trials.

Results. Across all problems, the children performed well above chance [67%, $t(15) = 6.454$, $P < 0.001$], and performance varied systematically as a function of the ratio of the two numerosities (Fig. 2*a*). A 2 (response: blue larger vs. red larger) by 3 (ratio) repeated measures ANOVA revealed a significant effect of ratio, $F(2, 30) = 12.239$, $P < 0.0005$, and a linear contrast analysis revealed that performance declined as the ratio approached 1, $F(1, 15) = 10.868$, $P < 0.005$. There was no effect of response and no interaction.

Accuracy was higher when the continuous variables of dot size, total contour length, summed area, and density were negatively correlated with number, and therefore when the size of the enclosing virtual rectangle was positively correlated with number [$t(15) = 2.68$, $P = 0.02$], although this effect cannot account for children's overall performance. Analyses of selected subsets of trials revealed that children did not adopt a strategy of basing responses on the presence of red or blue arrays that were extremely large or small. Performance was above chance for the trials that could not be answered correctly through the use of these strategies [$t(15) = 5.399$, $P < 0.0001$]. Fig. 3*a* depicts accuracy for trials containing extremely large or small sets vs. trials containing no such sets.

Discussion. Experiment 1 provided evidence that 5-year-old children can compare two dot arrays on the basis of numerosity. Controls within the experiment ensured that children's performance depended on the two numerosities, not on correlated continuous variables or guessing strategies. Performance depended on the ratio of the two sets, a signature of the nonsymbolic system of large-number representation found in infants, nonhuman animals, and human adults (6–11, 28). Accordingly, the next experiment investigated whether 5-year-old children can transform such representations by the operation of addition.

Experiment 2: Visual Addition

Experiment 2 investigated whether preschool children can add two successively presented arrays of dots and compare their sum with a third dot array on the basis of number.

Method. Seventeen children (age range from 5 years 0 months to 5 years 9 months; mean 5 years 4 months) were presented with

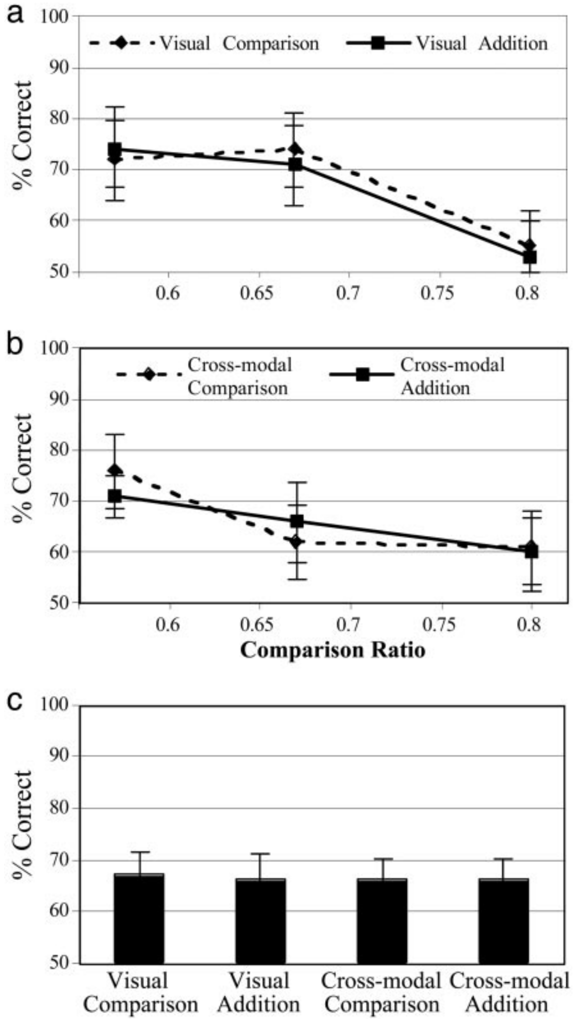

Fig. 2. Accuracy data for the four nonsymbolic tasks. The error bars represent 95% confidence intervals. (*a*) Accuracy scores for experiments 1 and 2 are plotted against the ratio of the numerosities to be compared (0.57 or 4:7, 0.67 or 4:6, and 0.8 or 4:5). (*b*) Accuracy scores for experiments 3 and 4 are plotted against the ratio of the numerosities to be compared. (*c*) Overall accuracy for all four experiments.

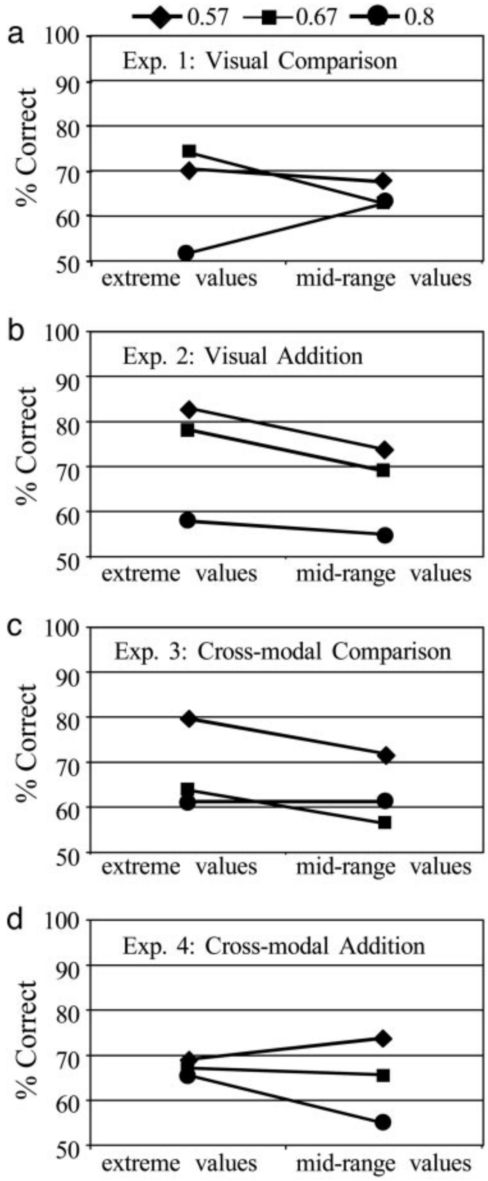

Fig. 3. Accuracy at each comparison ratio for trials including an extreme value (a set near the low end or the high end of the range of numerosities used) and for trials including only mid-range values for experiments 1–4.

animated dot arrays instantiating addition problems (Fig. 1*b*), following the method and procedure of experiment 1. After four practice comparison trials, children received four practice addition trials followed by 24 test trials, each constructed so as to parallel one of the test trials in experiment 1 (Fig. 1*b*). For each trial (with total duration of ≈13,650 ms), a blue-dot array from the comparison task was divided into two unequal subsets, each less numerous than the comparison array, varying in magnitude from 5 to 31. One subset of blue dots appeared on the lower left (≈1,300 ms), the occluder appeared on the right (≈1,300 ms) and moved leftward to cover it (≈1,450 ms), and after a pause (≈650 ms), the second subset of blue dots appeared on the upper left (≈1,300 ms) and moved behind the occluder (≈2,250 ms). The array of red dots appeared ≈1,300 ms later in the upper right (≈1,300 ms) and moved downward (≈2,150 ms), remaining in its final position for ≈650 ms. These events combined the procedures of the moving-occluder and moving-dots comparison trials from experiment 1. Because the red-dot array was more numerous than either blue-dot array but less numerous than the sum of the blue-dot arrays on half of the trials, children could not succeed at this task simply by comparing visible arrays. Because each addition problem was constructed from a comparison problem by dividing the blue-dot array in that problem into two subsets, all of the controls for continuous variables and range-based guessing strategies that were used in experiment 1 applied to the present study as well. Therefore, children could not succeed by assessing continuous variables or by following a strategy of choosing an array as more (or less) numerous if it was unusually large (or small).

Results. Children performed reliably above chance on the addition task [66%, $t(16) = 6.227$, $P < 0.0001$], and their performance was reliably affected by the ratio of the sum to the comparison array (see Fig. 2*a*). A 2 (response: sum more or less) by 3 (ratio) repeated measures ANOVA revealed a significant effect of ratio, $F(2, 32) = 11.07$, $P < 0.0005$, and a significant linear trend of decreased accuracy as the ratio approached 1, $F(1, 16) = 18.501$, $P < 0.001$. There was also a main effect of response [$F(1, 16) = 8.797$, $P < 0.01$]; children showed a tendency to choose the sum as the larger quantity. Nevertheless, children performed well above chance both on trials for which the sum was more numerous than the comparison array and on trials for which it was less numerous, [$t(16) = 6.37$, $P < 0.001$; and $t(16) = 1.82$, $P < 0.01$, respectively], indicating that they did not simply compare the red-dot array to a single blue-dot array or judge that two arrays were more numerous than one.

Children again performed better on the subset of problems for which dot size, total contour length, summed area, and density were negatively correlated with numerosity, [$t(16) = 5.23$, $P < 0.001$], indicating an influence of total array size on performance, but their success again cannot be explained by these variables. Children did not base their responses on the range of values presented in the two types of arrays (e.g., choosing the red-dot array when it was particularly large), because performance was above chance for the subset of trials that could not be answered correctly by using range-based strategies [$t(16) = 3.172$, $P < 0.003$]. Fig. 3*b* depicts accuracy for trials containing extremely large or small sets vs. trials containing no such sets.

Performance in experiment 2 was compared with that of experiment 1 by a 2 (operation: comparison vs. addition) by 3 (ratio) mixed-factor ANOVA. This analysis revealed only a significant effect of ratio, [$F(2, 62) = 14.544$, $P < 0.0001$], with a significant linear trend of declining performance as the ratio of the sum to the comparison number approached 1, [$F(1, 31) = 17.272$, $P < 0.0001$]. There was no main effect of operation and no interaction: children performed the addition task as accurately as the comparison task.

Discussion. Experiment 2 provides evidence that 5-year-old children can add two arrays of dots and compare their sum with a third dot array, even though only one array was visible at a time. Because their successful performance varied with the ratio of the sum to the comparison numerosity and could not be explained either by responses to nonnumerical variables or by guessing strategies, the findings suggest that children's performance depends on an abstract representation of number. The next experiments tested that suggestion more directly by investigating children's comparison and addition of sets in two modalities and formats: visual arrays and auditory sequences.

Experiment 3: Cross-Modal Comparison

Adults can compare sets of elements presented in different modes and formats as easily as sets of elements presented in a single mode and format (7, 25), but their success may depend on years of experience with symbolic numbers and arithmetic. Accordingly, experiment 3 tested preschool children's ability to compare visual arrays of dots with auditory sequences of tones.

Method. Sixteen children (age range from 4 years 9 months to 5 year 9 months; mean 5 years 4 months) completed a version of the experiment 1 comparison task in which the red dots were replaced by a series of tones presented too rapidly for counting (either 30- or 50-ms tones, presented respectively at 93- or 50-ms intervals). During a brief practice session, children saw two demonstrations of the blue dots moving behind the blue box, followed by two demonstrations of unoccluded dots paired with a sound sequence of equal number, followed by two demonstrations in which the sound sequence occurred after the red dots were occluded. Then the task was introduced with a set of four practice trials; both the red and the blue dots were revealed to provide feedback on the child's response. The final two practice trials were easy problems with no uncovering of the dots after the child's guess and no feedback. The 24 experimental trials followed (Fig. 1*c*), with two interspersed easy trials (not analyzed) to check on motivation and attention. The problems were the same as those presented for visual comparison and addition.

Results. Children performed reliably above chance on the two-modality comparison task [66%, $t(15) = 7.409$, $P < 0.0001$]. A 2 (response: more blue dots or more red dots) by 3 (ratio) repeated measures ANOVA again revealed a main effect of ratio [$F(2, 30) = 5.685$, $P < 0.01$; see Fig. 2*b*], and a significant linear trend of declining performance as the ratio differences approached 1 [$F(1, 15) = 9.288$, $P < 0.01$]. There was no main effect of response and no interaction: performance levels were the same for the two types of continuous-quantity trials [$t(15) < 1$, $P > 0.05$]. Children again did not rely on strategies based on numerical range information because performance was above chance for the subset of trials that could not be answered correctly by using such strategies [$t(15) = 5.965$, $P < 0.0001$]. Fig. 3*c* depicts accuracy for trials containing extremely large or small sets vs. trials containing no such sets.

Experiment 3 was compared with experiment 1 with a mixed-factor 2 (format: visual vs. cross-modal) by 2 (response) by 3 (ratio) ANOVA, with the first factor between subjects. There was a significant main effect of ratio [$F(2, 60) = 12.34$, $P < 0.0005$], but no main effects of response or format: participants' responses were as accurate for cross-modal comparison as for visual comparison. Nevertheless, there was a mildly significant interaction of format and ratio [$F(2, 60) = 4.518$, $P < 0.05$], indicating a steeper decline in cross-modal performance from the largest to the middle ratio.

Discussion. Experiment 3 provides evidence that preschool children, like adults, possess approximate number representations that are not dependent on the modality or format of the stimuli to be enumerated. In the next study, we extended this two-modality paradigm to an addition task analogous to that of experiment 2.

Experiment 4: Cross-Modal Addition

Experiment 4 tested whether preschool children can add two successively presented visual arrays of dots and compare their sum with a sequence of tones.

Method. Sixteen children (age range from 5 years 0 months to 5 years 11 months; mean 5 years 5 months) were given the experiment 2 addition task with the comparison sequences of tones used in experiment 3. Children received the initial practice trials of experiment 3, followed by the practice addition trials of experiment 2 with occluded red dots and tones presented as in experiment 3. On each trial, a blue occluder and a red occluder appeared on the screen. One group of blue dots moved behind the blue occluder, followed by a second group of blue dots. Then, as in experiment 3, participants heard a rapid, uncountable sequence of tones, each one representing a red dot "hiding" behind the red occluder. With no dots visible, children decided whether there were more blue dots or more hidden red dots (Fig. 1*d*). Addition problems were otherwise the same as in experiment 2, with two easy problems (not analyzed) interspersed as a check on motivation.

Results. Children performed reliably above chance on this task [66%, $t(15) = 7.595$, $P < 0.0001$; Fig. 2*b*]. A 2 (response) by 3 (ratio) repeated measures ANOVA showed that the main effect of ratio did not reach significance in this experiment [$F(2, 30) < 3$, $P > 0.05$], but there was a significant linear trend of decreased accuracy as the ratio approached 1 [$F(1, 15) = 5.14$, $P < 0.04$]. There was a significant main effect of response [$F(1, 15) = 4.621$, $P < 0.05$], with children tending to choose the sum as larger, but this tendency again did not account for their successful performance. As in experiment 3, performance levels were the same for the two continuous-quantity trial types [$t(15) < 1$, $P > 0.05$]. Children's performance again was above chance for the subset of trials that could not be answered correctly by using strategies

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based on numerical range information [all $t(15) < 4.213$, $P < 0.0004$]; Fig. 3*d* depicts accuracy for trials containing extremely large or small sets vs. trials containing no such sets.

Children responded as accurately in experiment 4 (cross-modal addition) as in experiment 2 (visual addition). A 2 (format) by 2 (response) by 3 (ratio) mixed-factor ANOVA revealed no effect of format. There were main effects of response [$F(1, 31) = 12.50$, $P < 0.001$] and ratio [$F(2, 62) = 11.92$, $P < 0.0.0005$], with accuracy higher when the sum was larger and accuracy decreasing as the ratio approached 1 [linear trend, $F(1, 31) = 21.49$, $P < 0.0005$]. Moreover, children responded as accurately at cross-modal addition (experiment 4) as at cross-modal comparison (experiment 3): A similar ANOVA comparing experiments 3 and 4 revealed no effect of operation (comparison vs. addition; $F(1, 30) < 1$, $P > 0.05$), the same main effects of response [$F(1, 30) = 6.24$, $P < 0.05$] and ratio [$F(2, 60) = 7.49$, $P < 0.0.001$], and the same linear trend of decreasing accuracy as ratios approached 1 [$F(1, 30) = 14.14$, $P < 0.001$].

Fig. 2*c* shows overall accuracy for all four studies plotted together. A 2 by 2 (modality by operation) mixed-factor ANOVA comparing performance across all four studies revealed no effect of either factor. Children performed as accurately in cross-modal as in visual tasks, and they performed the addition tasks as well as the comparison tasks.

Discussion. Experiment 4 provides previously undescribed evidence for abstract, large-number addition in preschool children. Children were able to add the numbers of elements in two visual arrays and compare their sum to a sequence of tones. Because the children had received no formal arithmetic instruction, this ability likely depended on nonsymbolic number knowledge. The last experiment tested this possibility more directly by investigating children's verbal knowledge of the addition facts tested in experiments 2 and 4.

Experiment 5: Symbolic Arithmetic

Could the successes of experiments 1–4 be based in children's knowledge of symbolic arithmetic? Although none of the participants in these experiments had received formal arithmetic instruction, many children learn simple arithmetic facts spontaneously, before they begin school (29, 30). Such children clearly would fail to solve exact arithmetic problems involving numerosities as large as those presented here, but their ability to give approximate answers to large-number problems, to our knowledge, has never been tested. Therefore, it is possible that children estimated the symbolic numerosity of each dot array that was presented and performed rough symbolic addition on these estimates. In an initial attempt to assess the plausibility of this alternative, the last experiment tested participants' knowledge of approximate symbolic addition.

Method. Thirty-three children (age range from 5 years 11 months to 5 years 9 months, mean 5 years 5 months) participated in this test after participating in experiment 1 or 2. The test consisted of one familiar control problem ($2 + 2 = 4$ vs. 8 or $5 + 5 = 10$ vs. 6) and three experimental problems by using numbers from selected nonsymbolic addition problems, one at each ratio (e.g., $16 + 17 = 33$ vs. 58 or $27 + 31 = 58$ vs. 33). First, the experimenter presented an addition problem (e.g., "If your mom gave you 27 marshmallows, and then she gave you 31 more, how many would you have?"), and the child was encouraged to generate an answer. Then the experimenter asked the child to choose between the correct answer and the answer corresponding to the comparison numerosity in the nonsymbolic problem from which it was derived (e.g., "Would it be more like 58 or 33?"). The order of presentation of the problems and of the correct answer vs. the foil were counterbalanced. If children's performance in experiment 2 was based on knowledge of symbolic arithmetic, then they should solve these two-choice verbal problems above chance, and their performance should depend on the ratio of the sum to the comparison array.

Results. Although children correctly answered the control problems on most trials (73%), they never guessed the answer to the test problems at either the 4:7 or 2:3 ratio and rarely guessed the correct answer to the 4:5 problem (12% of trials). Performance on control and test problems differed reliably [paired-samples $t(32) = 9.0$, $P < 0.001$]. On the two-choice follow-up test, children chose the correct answer at a rate significantly higher than chance for the control problems (88%, binomial $P < 0.001$) but not for the test problems at any of the three ratios (all $P > 0.2$). Performance on the control and test problems again differed reliably, paired-samples [$t(32) = 4.67$, $P < 0.001$]. Finally, a one-way repeated, measures ANOVA revealed no effect of ratio on children's two-choice responses, [$F(2, 64) = 1.62$, $P > 0.2$], and a linear trend opposite in direction to that predicted by the ratio signature.

Discussion. Children's successful performance on the control problems provides evidence that they understood the verbal problems and were motivated to solve them. In contrast, children's failure on the unfamiliar, large-number problems suggests that they had little knowledge of symbolic arithmetic. Moreover, children's performance on the symbolic problems did not show the ratio signature of large number representations found in studies of adults, infants, and nonhuman animals as well as in experiments 1 and 2. Children's poor performance and the absence of a ratio signature provide evidence that different processes underlie children's performance of nonsymbolic and symbolic arithmetic in these experiments. In particular, the ability to perform nonsymbolic addition does not depend on knowledge of the symbolic arithmetic facts instantiated in the nonsymbolic problems.

General Discussion

The present findings reveal that 5-year-old children can compare and add numerical quantities. Children base their responses on number rather than on continuous quantities that typically are correlated with number, and they focus their comparisons on the arrays that are presented within a problem rather than on guessing strategies based on the range of values presented across the set of problems. Children also readily solve numerical tasks that require the integration of quantity information presented in different modalities: accuracy for addition and comparison with two modalities was as high as accuracy for addition and comparison with a single modality. Children's nonsymbolic addition and comparison performance shows the ratio signature of large approximate number representations, adding to the evidence that children use the same system of representation found in human infants (8, 9, 24, 31), nonhuman animals (e.g., 10, 32), and adults (6, 7).

Previous studies have found that adults were successful at nonsymbolic comparison and addition tasks, both within and across modalities (6, 7), but adults' symbolic number knowledge might have contributed to this ability. However, 5-year-old children are able to perform very similar tasks in the absence of such knowledge. A comparison of performance on tasks of symbolic and nonsymbolic addition, using the same numerical values and the same children, reveals a striking difference: Although children performed the nonsymbolic addition problems well above chance (experiment 2), they performed at chance on symbolic versions of these problems (experiment 5). In accord with evidence from studies of human infants and nonhuman primates using purely visual arrays (24, 33), these findings provide evidence that abstract approximate number representations can enter into arithmetic operations in the absence of knowledge of the relevant symbolic arithmetic facts. Children compare and add quantities presented in distinct modalities before they begin formal arithmetic instruction. This is a surprising finding, given that many school-age children have considerable difficulty learning symbolic arithmetic. Our findings offer the promise that new strategies in elementary mathematics education might be devised: strategies that harness children's preexisting arithmetic intuitions to foster the acquisition of symbolic number knowledge.

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