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Brief article

Preschool children master the logic of number word meanings

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Abstract

Although children take over a year to learn the meanings of the first three number words, they eventually master the logic of counting and the meanings of all the words in their count list. Here, we ask whether children's knowledge applies to number words beyond those they have mastered: Does a child who can only count to 20 infer that number words above 'twenty' refer to exact cardinal values? Three experiments provide evidence for this understanding in preschool children. Before beginning formal education or gaining counting skill, children possess a productive symbolic system for representing number.

Keywords: Preschool children; Number word; Children

1. Introduction

What do children understand about large-number words before they can count to large numbers or master the base-ten counting system? Do preschool children who cannot count beyond 40 understand that words such as 'eighty-six' refer to exact cardinal values, changing their application when items are added to or removed from a set?

Although most 3-year-old children can recite the ordered list of number words at least to 6, children take many months to learn the meanings of these words (Wynn, 1990, 1992). Prior to this learning, children show inconsistent understanding of the logic of number words: They judge that each word picks out a specific, unique and exact cardinal value in

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some tasks (Sarnecka & Gelman, 2004) but not others (Condry, Cayton, & Spelke, 2002; Sarnecka & Gelman, 2004). Young children either fail to understand that their counting words denote unique, specific numerosities, or their understanding is fragile and task-dependent.

By the end of the fourth year, most children have mastered the meanings of the smallest counting words (Wynn, 1990, 1992), but their count list typically is limited to 20 or fewer items. Over the next year, children learn to count to higher numbers, and they map number words in their expanded count list to non-symbolic, approximate numerosities (Siegler & Opfer, 2003). What do such children understand about number words beyond those to which they can count?

In one series of experiments (Lipton & Spelke, in press), we have shown that preschool children map the number words within their counting range onto nonsymbolic numerosities, but that they show no such mapping for number words beyond that range. Children first were given an estimation task, in which they were shown arrays of 20–120 dots and gave verbal estimates of the number of dots in each array. Verbal estimates were linearly related to the presented numerosities for adults and for children who could count to 100. In contrast, preschool children who could not count beyond 60 showed this linear relation only for numbers within their counting range and produced number words at random for larger numerosities. Children next were given a comprehension task, in which they were shown two dot arrays, one twice as large as the other, and were asked to pick the array with a given verbally presented number of elements. Performance was above chance within children's counting range but at chance beyond that range. Finally, children were shown two dot arrays (one with twice as many elements as the other), were told how many dots were in one array, and were asked how many dots were in the other array. When tested with numbers within their counting range, children nearly always produced a number word in the correct direction: a smaller word when the to-be-estimated set was smaller than the named set, and a larger word when it was larger. In contrast, children produced larger and smaller words randomly when tested with numbers outside their counting range.

These findings provide evidence that children begin to map number words onto nonsymbolic representations of numerosity only when they become able to count to those words reliably. It does not follow, however, that unskilled counters know nothing about the meanings of words beyond their counting range. Here we ask whether these children understand the logic of such number words.

Adults understand that any counting number, such as '1,814,' refers to an exact cardinal value: its application changes as items are added or removed. This knowledge is productive: It applies not only to number words whose meanings have been verified by experience (e.g. by counting a set, removing one number, and counting it again), but also to number words whose meanings have never been directly verified. Adults, however, have mastered the recursive, place-value system that would allow them to count reliably to any large number in principle, even if they have not done so in practice. Adults also can use addition and subtraction to solve large-number problems. Children, in contrast, learn to count small sets of objects well before they master recursive counting rules or symbolic arithmetic. As children become proficient at counting small sets of objects, do they possess the same, productive understanding of the logic of number words?

The present experiments test for this understanding against two alternatives. First, because infants' preverbal representations of large numerosities are imprecise (Lipton & Spelke, 2003; Xu & Spelke, 2000), children may infer that words such as 'eighty-six' refer to sets with approximately 86 members. We test, therefore, whether children who are told that a large set contains 86 items will continue to apply the term 'eighty-six' to the set if a single item is removed. Second, some linguists have proposed that number words place a lower bound on cardinal values: When a speaker claims that a set of six items contains 'five items,' her statement is strictly true, though in many contexts it is pragmatically inappropriate (Grice, 1957). Although this proposal has been disputed (Koenig, 1991), it is difficult to evaluate in adults, who possess a complex mix of semantic and pragmatic knowledge. Studies of children provide evidence against Grice's proposal (Huang, Snedeker, & Spelke, 2004; Musolino, 2004; Papafragou & Musolino, 2003), but their findings are mixed and may reflect effects of experience with number words, since children were tested with words for small numbers. We investigate whether children who cannot count to 86 have a lower-bound interpretation of the corresponding number word by asking whether they continue to apply 'eighty-six' to a labeled set after one object is added to the set.

2. Experiment 1

Experiment 1 investigates whether children understand that round number words beyond their counting range (e.g. 'eighty') change their application when one or more elements are removed from a set to which the words are applied.

2.1. Method

Ten preschool children (mean 5–2; range 5–0 to 5–5) were recruited from the Cambridge, MA community. Two children were eliminated from the sample because they performed without error on our counting assessment (see below), leaving eight children in the experiment.

The experiment consisted of a counting assessment, a task assessing the mapping of number words to approximate numerosities, and an assessment of understanding the logic of number word application, in that order. For the counting assessment, the experimenter began counting and participants were asked to continue counting after the experimenter stopped. For example, the experimenter said '24, 25, 26' and children continued counting from 26. Sequences were chosen to assess whether the child could add one to the units place and whether he or she could make the decade change correctly. For each child, we determined the highest number to which he or she counted reliably. For example, if a child responded '58, 59, 40,…' we then presented '35, 36, 37,…' and '46, 47,…' to determine if he or she could count to 40 reliably. No feedback was given and children were encouraged to guess.

The mapping task followed that of Lipton and Spelke (in press). On each of six trials, children were shown two cards displaying sets of rectangles. One card, the target, presented either 20, 40, 60, 80, 100, or 120 rectangles; the other card presented either half or twice as many elements. On half the trials, the two cards had rectangles of the same element size such that summed area and numerosity were confounded. On the other trials, the rectangles differed in size such that the total summed area was equated across the sets. The experimenter placed the two cards side-by-side and asked the child to point (e.g.) to the card with 'forty rectangles'. Children were given feedback after each trial.

The logic task investigated whether children infer that words for large numbers refer to specific cardinal values. Children were shown one set of objects (bears, frogs, balls, etc.) in a clear container on each of nine trials. They were told, correctly, that there were N objects in the container, for N = 6, 7, 9, 20, 40, 60, 80, 100, or 120. For each N, children were asked whether there were still N objects in the container after four manipulations: stirring, removing one object, removing about half the objects, and returning the latter subset to restore the original numerosity. These manipulations occurred in random order with the restriction that removing half the objects was always followed by restoring those objects. After the trial in which one item was removed, that item was replaced before the next trial. Therefore, children were asked 'Are there N objects?' four times for each numerosity, and for half the questions the correct answer was 'yes'. Children were given neutral feedback.

2.2. Results

Four of the children counted to 20 reliably, and four counted to 40 reliably. In each case, mistakes tended to occur at decade changes. We analyzed the data on subsequent tasks separately for numbers within and outside of each child's counting range.

Overall, performance on the mapping task was 48% correct (chance = 50%) and did not differ when the two sets were equated for element size vs. summed area, $F(1, 7) = 2.04$, $P > 0.10$. Children were more likely to select the more numerous set than the less numerous set, yielding better performance when the target set was larger than the distractor, $F(1,7) = 29.2$, $P < 0.01$. Children performed above chance on the problems in which the target numerosity was within their count range, $F(1,7) = 21.0$, $P < 0.01$, and at chance when the target numerosity was outside that range, $F(1,7) = 3.72$, $P > 0.05$. Performance on the two sets of problems differed reliably, $F(1,7) = 16.3$, $P < 0.01$.

The findings of the logic task are presented in Fig. 1. Children performed above chance on the no-change, subtract-one, subtract-half, and restore-half questions, all $t(7)'s > 10$, $P < 0.01$. Children's performance on each question was reliably above chance on the subset of problems presenting numerosities outside their counting range, all $t(7)'s > 5$, $P < 0.01$. A repeated measures ANOVA comparing correct performance on each task with numbers within versus outside the count range showed that correct performance did not differ for number words inside vs. outside children's counting range, $F(1,7) = 4.70$, $P > 0.05$.

2.3. Discussion

Although children had little knowledge of the specific meanings of the number words tested in this experiment, they reasoned appropriately about the application of these words. When presented with a large set, children judged that if one item was removed from

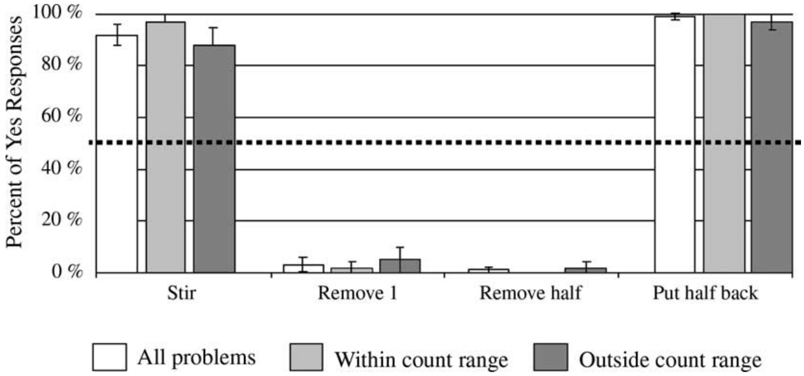

Fig. 1. Percent of 'Yes' responses in the logic task of Experiment 1, when subjects are asked 'Are there N items?' following initial labeling of a set with that target number word and then manipulating the set.

the set, the number word that previously designated the set would no longer apply. Because removal of a single item from a large set does not result in a discriminably different numerosity, this finding provides evidence that preschool children do not treat words for large numbers as specifying approximate cardinal values. When presented with a large set from which half the items were removed and then returned, children judged that the number word once again applied to the set. This finding provides evidence that children remembered the words and set transformations, and they treated successive subtraction and addition of a subset as restoring the original cardinal value. Finally, children understood that if the objects in the set were shaken, stirred, or moved, the number word continued to apply to the set.

Although these findings suggest that preschool children share adults' productive mastery of the logic of number word meanings, it is possible that children construe the meanings of number words in a different way. When children see a set labeled with a number word, they may understand the word to refer to 'all the objects'. When objects are removed, therefore, children may judge that the number word no longer applies because the set is not complete. Experiment 2 tested for this alternative interpretation of number word meanings.

3. Experiment 2

Experiment 2 extended the findings of Experiment 1 in two ways. First, whereas Experiment 1 used only round number words, Experiment 2 tested children's understanding of number words that are not multiples of 10. Second, Experiment 2 used an altered reasoning task in which a number word is applied to a set, one individual is removed, and a different individual is restored to the set. If children take large number words to refer to all members of a designated set, they should fail to apply a number word to a set that is altered in this manner, because one of the original members is missing. In contrast, if children take large number words to refer to exact cardinal values, then they should apply each word to a set transformed by the operations of subtracting one object and adding another.

3.1. Method

Of 10 preschool children (mean 5–1; range 5–0 to 5–4) recruited from the Cambridge, MA community, 2 were eliminated from the sample because they performed without error on our counting assessment.

The experiment consisted of a counting assessment as in Experiment 1, followed by a test of children's understanding of the logic of large number words. For the test, children received trials in which the experimenter presented a set of objects in a clear container, stated the number of objects, had the child repeat the number, and then performed manipulations on the set. Children received one no-change block and one change block of trials for each of six test numerosities presenting 26, 42, 61, 84, 109, and 127 objects. On the no-change blocks, the experimenter stirred, shook, or moved the set of items and asked the child twice 'Are there N objects?', once using the correct number word and once using a different large number word. For example, the experimenter presented 61 frogs, stirred the set, and asked 'Are there 75 frogs? Are there 61 frogs?' The order of questions was counterbalanced; for half the trials, the incorrect number word designated a larger numerosity.

On the change blocks, the experimenter removed one object from the set, placed it on the table (subtract-one trial), and asked, 'Are there N objects in the bucket?' Then she took a different object of the same kind from under the table, placed it in the bucket, and asked again 'Are there N objects in the bucket?' The order of no-change and change blocks was quasi-random such that there were no more than two blocks in a row of the same type. In each block, half the correct answers were 'no'. Children were given neutral feedback.

3.2. Results

Two children counted only to 10, three children counted reliably to 20 but not higher, two children counted reliably to 30 but not higher, and one child counted reliably to 40 but not higher. For five children, therefore, all tested number words lay beyond their counting range; for the remaining three children, one tested number word lay within their counting range.

Fig. 2 presents children's performance on the task assessing their understanding that number words specify exact cardinal values. Overall, children performed above chance on all four questions, all ts(7) > 3.8, $P < 0.01$. Performance on numbers outside of each child's count range was above chance for all four questions: $t(7)' = 3.46$, $P < 0.05$ for the no-change same number questions; all other ts$(7)' > 3.91$, $P < 0.01$.

3.3. Discussion

Before children can count to high numbers, they know that large-number words apply to sets uniquely: if a set has 'sixty-one' members, it does not also have 'seventy-five' members. Moreover, children know that a large-number word continues to apply to a set if

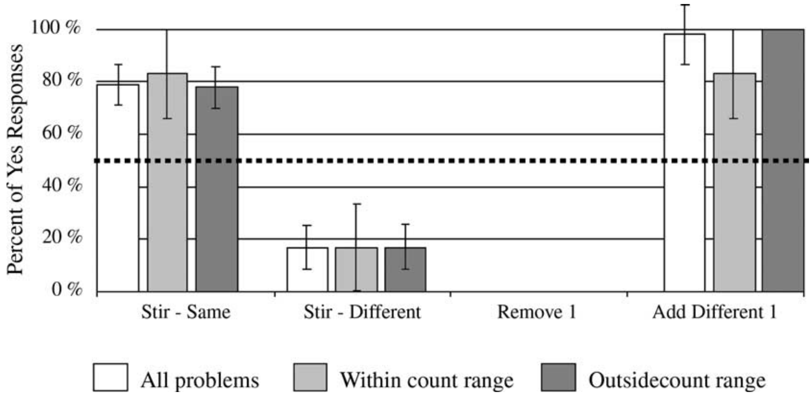

Fig. 2. Percent of 'Yes' responses in the logic task of Experiment 2, when subjects are asked 'Are there N items?' ('stir-same,' 'remove 1,' and 'add different 1' or asked 'Are there X items?' ('stir-different') following initial labeling with the target number word and then manipulating the set.

items in the set are moved but not added or subtracted, that it no longer applies if a single item is removed from the set, and that it again applies if a single, different item is restored to the set. Experiment 2 therefore, provides evidence for productive knowledge of number words outside of children's counting range.

4. Experiment 3

Experiments 1 and 2 provide evidence that children understand that a large number word ceases to apply to a set after a single item is removed. Do children also infer that a large number word ceases to apply to a set when a single item is added to the set? If number words define only a lower bound of numerosity, as Grice (1957) has suggested, then children might judge that a number word continues to apply to a set when its cardinal value increases. Experiment 3 tested this possibility. After informing children (e.g.) that a jar of marbles contained 'sixty-one marbles', removing one marble, and restoring a different marble, the experimenter returned the original marble to the jar, and asked 'Are there sixty-one marbles in the jar?' If children give a lower-bound interpretation to large number words, then they should respond 'yes' to this question; if they interpret each word as *exactly N*, then they should respond 'no'.

4.1. Method

Of the ten preschool children (mean 5–1; range 5–0 to 5–6) recruited from the Cambridge, MA community, two were eliminated because they performed without error on our counting assessment.

The counting assessment was as in Experiments 1 and 2. The test of children's understanding of the logic of number word application was identical to that of

Experiment 2, except for one additional question on the change trials. After the subtract-one and restore-one trials, the original object was added back to the set and children were asked ‘Are there N objects?’

4.2. Results

On the counting assessment, two children counted reliably to 10, two to 20, one each to 30 and to 40, and two to 50.

Fig. 3 presents the principal findings of this experiment. Both overall and on the subset of problems testing numbers outside each child’s counting range, children performed above chance on the first four questions, all $ts(7) > 5$, $P < 0.01$. When the original object was returned to the set, yielding a cardinal value one higher than the originally named set, children judged correctly that the original number word did not apply, both overall and for the subset of trials presenting number words outside children’s counting range, respective $ts(7)' = 2.6$ and 2.5, $P < 0.05$.

4.3. Discussion

Experiment 3 provides evidence that children infer that number word beyond their counting range apply to exact cardinal values. If a child who cannot count beyond 50 is told that a jar contains ‘seventy-six’ marbles, she judges (a) that this number word no longer applies when one marble is removed, (b) that the word again applies when a different marble is added, and-most dramatically-(c) the number word does not apply when the original marble is restored to the jar. For preschool children, ‘seventy-six’ means *exactly 76*, not ‘about 76’ or ‘at least 76’. Because children make this judgment for numbers outside their count range, their knowledge evidently is productive.

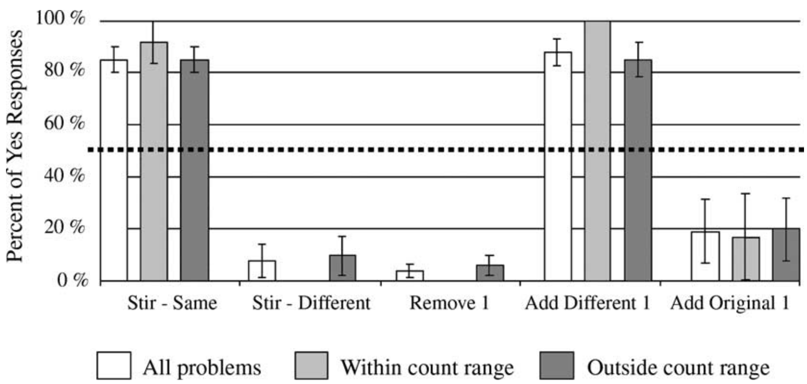

Fig. 3. “Percent of ‘Yes’ responses in the logic task of Experiment 3, when subjects are asked ‘Are there N items?’ (‘stir-same’, ‘remove 1’, ‘add different 1’, and ‘add original 1’) or asked ‘Are there X items?’ (‘stir-different’) following initial labeling with the target number word and then manipulating the set.”.

5. General discussion

Three experiments provide evidence that five-year-old children take large number words to apply to specific, unique cardinal values. Children understand that the number word that correctly labels a large set of items changes when items are removed from or added to the set, but remains the same when items are rearranged. Moreover, children understand that if an item is removed from a set and a different item is added to the set, the original number word labeling the set will apply. Importantly, children possess this knowledge not only for number words to which they can count reliably, but also for number words beyond their counting range. Before children begin school or master decimal notation, they understand the logic of number word meanings.

Our findings provide some evidence against Grice's view that number word meanings specify a lower bound on numerosity, and that pragmatic concerns lead speakers to choose the largest number word of those applying to a set. Because the children in the present study could not count to the words on which they were tested and showed no evidence of mapping these words to non-symbolic numerosities (Lipton & Spelke, in press), it is unlikely they have mastered the pragmatic use of these words. Nevertheless, children take each word to specify an exact cardinal value.

The present findings do not shed light on the origins of children's productive knowledge of number words. They are consistent both with the thesis that humans have an innate system of knowledge of natural number from which they construct an understanding of number words and verbal counting (Dehaene, 1997; Gelman & Gallistel, 1978; Pica, Lemer, Izard, & Dehaene, 2004; Wynn, 1990, 1992), and with the thesis that natural number concepts are constructed over the preschool years as children master the logic of the verbal counting routine (Carey, 2001; Gordon, 2004; Spelke, 2000). On either view, children appear to begin mathematics education with a powerful system for representing and reasoning about number.

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