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What Makes Us Smart? Core Knowledge and Natural Language

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10.1 Introduction

When we compare the sensory and motor capacities of humans to those of other primates, we discover extensive similarities. Human visual and auditory capacities closely resemble those of rhesus monkeys, for example, as do the neural mechanisms that subserve these capacities (e.g., Felleman and van Essen 1991). Human locomotion and other actions also depend on systems shared by many animals (e.g., Thelen 1984). These similarities strongly suggest that the psychology of humans is continuous with that of nonhuman animals and depends on a common set of mechanisms.

When we compare the cognitive achievements of humans to those of nonhuman primates, however, we see striking differences (table 10.1). All animals have to find and recognize food, for example, but only humans develop the art and science of cooking. Many juvenile animals engage in play fighting, but only humans organize their competitive play into structured games with elaborate rules. All animals need to understand something about the behavior of the material world in order to avoid falling off cliffs or stumbling into obstacles, but only humans systematize their knowledge as science and extend it to encompass the behavior of entities that are too far away or too small to perceive or act upon. As a final example, all social animals need to organize their societies, but only humans create systems of laws and political institutions to interpret and enforce them.

What is it about human cognition that makes us capable of these feats? In this chapter, I consider two possible answers to this question.

Table 10.1 Some unique feats of human cognition.

Cooking	Theater	Science	Science	
Music	Architecture	Politics		
Sports	Tool manufacture	Law		
Games	mes Mathematics Religion			

The first answer guided my research for 20 years, but I now believe that it is wrong. The second answer is just beginning to emerge from research conducted over the last decade, and I think it has a chance of being right. Both answers center on the concept of core knowledge, which I can best introduce by turning to the first answer.

10.2 What Makes Us Smart? Uniquely Human, Corc Knowledge Systems

According to the first answer, the cognitive capacities of any animal depend on early-developing, domain-specific systems of knowledge. Just as infant animals have specialized perceptual systems for detecting particular kinds of sensory information and specialized motor systems guiding particular kinds of actions, infant animals have specialized, task-specific cognitive systems: systems for representing material objects, navigating through the spatial layout, recognizing and interacting with other animals, and the like. These specialized systems provide the core of all mature cognitive abilities, and so whatever is unique to human cognition depends on unique features of our early-developing, core knowledge systems. At the root of our capacities to construct and learn physics, for example, may be a distinctive core system for representing material objects and their motions; at the root of human mathematics may be uniquely human core systems for representing space and number; and at the root of human politics, law, and games may be distinctive systems for representing people and their social arrangements.

This thesis supports a particular research agenda: to understand what is special about human cognition, we should study core knowledge systems as they emerge in infants and young children. Such studies have been conducted over the last 30 years, and they indeed suggest that human infants are equipped with core knowledge systems. Nevertheless, the systems found in young infants do not appear to distinguish us from many nonhuman animals.

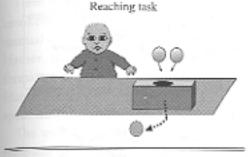
10.2.1 Object Mechanics

Consider, for example, the core system for representing material objects. Research over the last two decades provides evidence that infants have a system for perceiving objects and their motions, for filling in the surfaces and boundaries of an object that is partly hidden, and for representing the continued existence of an object that moves fully out of view. Evidence for these abilities comes from studies using both reaching methods and preferential looking methods (see Spelke 1998, for review). An experiment by Wynn (1992a) serves as an example of the latter.

Wynn (1992a; figure 10.1) presented 5-month-old infants with a puppet stage on which she placed a single puppet. Then a screen was introduced, concealing the puppet, and a second puppet appeared from the side of the display and disappeared behind the screen. Finally, the screen was lowered to reveal one or two puppers on the stage, and infants' looking time at these displays was measured and compared. If infants failed to represent the existence and the distinctness of the two puppets behind the screen, then the outcome display presenting one puppet should have looked more familiar to them, because they had only ever seen a single pupper on the stage at a time. Because infants tend to look longer at displays that are more novel, infants therefore should have looked longer at the display of two puppets. In contrast, if infants represented the continued existence of the first puppet behind the screen, the distinct identity of the second puppet when it was introduced from the side, and the continued existence of the second puppet behind the screen, then the outcome display presenting only a single puppet should have looked more novel to them, because it suggested that one of the puppets had mysteriously disappeared. Infants indeed looked longer at the one-Puppet outcome, providing evidence that they perceived and represented two puppets in this event.

Wynn's experiment has enjoyed many replications and extensions (see Wynn 1998, for review). Notably, it has been replicated in studies that control for infants' representations of the features and spatial locations

Figure 10.1 Schematic depiction of displays for a study of infants' representations of persisting, numerically distinct objects using a preferential looking method. (After Wynn 1992a.)



Locomotor choice task



Schematic depiction of displays for studies of object representation using reaching and locomotor choice methods. (After Feigenson, Carcy, and Hauser 2002; Van de Walle, Carey, and Prevor 2001.)

of the objects (respectively, Simon, Hespos, and Rochat 1995; Koechlin, Dehacne, and Mchler 1998): infants look longer at arrays presenting the wrong number of objects, even when the shapes, colors, and spatial locations of the objects in both displays are new. Wynn's findings also have been replicated with older infants in experiments using two different methods, each focusing on a different response system: manual search in a single, opaque box containing one or two objects, and locomotor choice between two such boxes (Feigenson, Carey, and Hauser 2002; Van de Walle, Carey, and Prevor 2001; figure 10.2). In studies using the latter method, for example, infants who have just begun to locomote independently are shown two cookies placed in succession into one opaque box and one cookie placed into a second box, and then they are allowed to crawl toward one or the other box. Infants were found

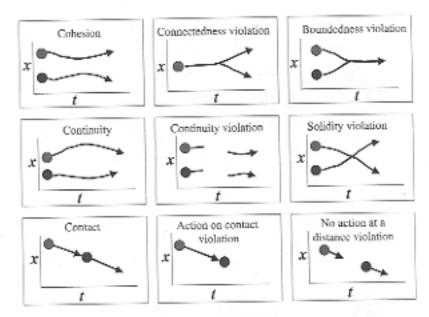


Figure 10.3 Principles of object representation in human infancy. (After Spelke 1990.)

to crawl preferentially to the box with the greater number of cookies (Feigenson, Carey, and Hauser 2002). These converging findings from three paradigms suggest that infants have robust abilities to represent the persistence and the distinctness of hidden objects.

Summarizing these and other studies, I have proposed that human infants represent objects in accord with three spatiotemporal constraints on object motion (figure 10.3). Infants represent objects as cohesive bodies that maintain both their connectedness and their boundaries as they move, as continuous bodies that move only on connected, unobstructed paths, and as bodies that interact if and only if they come into contact. Despite some controversy in the field, I believe these conclusions are well supported (Spelke 1998). Nevertheless, there is no reason to think that the core system for representing objects, centering on the constraints of cohesion, continuity, and contact, is unique to humans. Representational abilities that equal or exceed those of human infants have been found in a variety of nonhuman animals, including both adult monkeys and newly hatched chicks.

Hauser has presented Wynn's task to adult free-ranging rhesus monkeys, using all three methods used with infants: preferential looking, manual search, and locomotor choice (Hauser, Carey, and Hauser 2000; Hauser, MacNeilage, and Ware 1996). With all three methods, the performance of adult monkeys equaled or exceeded that of human infants. Humans evidently are not the only creatures to represent objects as spatiotemporally continuous bodies.

The monkeys in Hauser's experiments were adults, but capacities to represent objects have been found in infant animals as well. Indeed, they have been found in chicks who are only 1 day old. Investigators in two laboratories have used an imprinting method in order to present newly hatched chicks with some of the object representation tasks used with human infants (e.g., Lea, Slater, and Ryan 1996; Regolin, Vallortigara, and Zanforlin 1995). As is well known, chicks who spend their first day of life in isolation with a single moving inanimate object will tend to approach that object in preference to other objects in any stressful situation. In a variety of studies, this approach pattern has been used to assess chicks' representations of the hidden object. In one set of studies, for example, chicks who spent their first day of life with a center-occluded object were placed on their second day of life in an unfamiliar cage (a moderately stressful situation) with two versions of the object at opposite ends, in which the previously visible ends of the object either were connected or were separated by a visible gap. Chicks selectively approached the connected object, providing evidence that they, like human infants, had perceived the imprinted object to continue behind its occluder (Lea, Slater, and Ryan 1996; see also Regolin and Vallortigara 1995; figure 10.4). In further studies, chicks were presented with events in which the imprinted object became fully occluded. Even after an extended occlusion period, the chicks selectively searched for the occluded object, providing evidence that they represented its continued existence (Regolin and Vallortigara 1995).

These findings suggest that a wide range of vertebrates have earlydeveloping capacities to represent objects. The core system for representing objects found in human infants does not appear to be unique to us and so cannot in itself account for later-developing, uniquely human abilities to reason about the physical world.

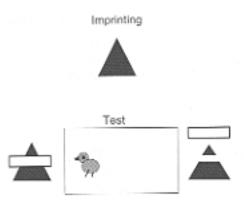


Figure 10.4 Schematic depiction of an experiment on object representation in 2-day-old chicks using an imprinting method. (After Regolin and Vallortigara 1995.)

10.2.2 Number Sense

Perhaps studies of object representations fail to reveal uniquely human capacities, because object representations are so close to perception and so fundamental to many animals. Our human capacities for science and technology, however, depend greatly on the development and use of mathematics. Moreover, formal mathematics is a uniquely human accomplishment. Perhaps a core system for representing number distinguishes human cognition from that of nonhuman animals and serves as the basis for the development of mathematics, technology, and science.

Research on normal human adults and on neurological patients provides evidence that representations of number and operations of arithmetic depend in part on "number sense": a sense of approximate numerical values and relationships (Dehaene 1997; Gallistel and Gelman 1992). The performance of this system is characterized by Weber's law: as numerosity increases, the variance in subjects' representations of numerosity increases proportionately, and therefore discriminability between distinct numerosities depends on their difference ratio. Does this number sense derive from a core cognitive system that is present in infants?

Recently, Fei Xu, Jennifer Lipton, and I have addressed this question through studies of 6-month-old infants' abilities to discriminate between large numerosities. In our first studies (Xu and Spelke 2000b), infants

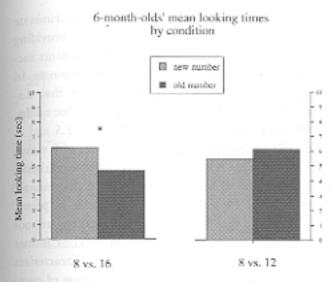


Figure 10.5 Looking times to displays presenting a novel number of dots, in experiments testing for discrimination of 8 from 16 or 12 dots. 3 indicates a significant difference. (After Xu and Spelke 2000b.)

were presented with visual arrays of dots on a succession of trials. On different trials, the dots appeared in different sizes and at different positions, but there were always 8 dots in the array for half the infants and 16 dots for the others. To control for display brightness and size, the dots in the more numerous arrays were half the size, on average, of those in the less numerous array and appeared at twice the density. Dot arrays were presented until infants' spontaneous looking time to the arrays declined to half its initial level. Then infants were presented with new arrays of 8 and 16 dots in alternation, equated for density and dot size. If infants responded to any continuous properties of the dot arrays, they should have looked equally at the two test numerosities, because those variables were equated either across the familiarization series or across the test series. In contrast, if infants responded to numerosity and discriminated the arrays with 8 versus 16 elements, they were expected to look longer at the array with the novel numerosity. That looking preference was obtained, providing evidence for numerosity discrimination at 6 months of age (figure 10.5).

In subsequent studies using this method, infants failed to discriminate between arrays of 8 versus 12 dots (Xu and Spelke 2000b), providing evidence that their sense of number is imprecise. Moreover, infants successfully discriminated 16 from 32 dots and failed to discriminate 16 from 24 dots (Xu and Spelke 2000a), providing evidence that discriminability accords with Weber's law for infants, as it does for adults, and that the critical Weber fraction for infants lies between 1.5 and 2. Finally, infants successfully discriminated between sequences of 8 versus 16 tones, presented with the same controls for the continuous variables of the duration and quantity of sound, and they failed to discriminate between sequences of 8 versus 12 tones (Lipton and Spelke in press). These findings provide evidence that numerosity representations are not limited to a particular sensory modality (visual or auditory) or format (spatial vs. temporal), and that the same Weber fraction characterizes discriminability across very different types of arrays. The sense of number found in adults therefore appears to be present and functional in 6-month-old infants.

Does a core sense of number account for our uniquely human capacity to develop formal mathematics? If it did, then no comparable evidence for number sense should be found in any nonhuman animals. In fact, however, capacities to discriminate between numerosities have been found in nearly every animal tested, from fish to pigeons to rats to primates (see Dehacne 1997 and Gallistel 1990 for reviews, and figure 10.6 for evidence from a representative experiment). Like human infants, animals are able to discriminate between different numerosities even when all potentially confounding continuous variables are controlled, they discriminate between numerosities for both spatial arrays and temporal sequences in a variety of sensory modalities, and their discrimination depends on the ratio difference between the numerosities in accord with Weber's law. Humans' early-developing number sense therefore fails to account, in itself, for our uniquely human talents for mathematics, measurement, and science.

10.2.3 Natural Geometry

Before abandoning my first account of what makes humans smart, I will consider one last version of this account, inspired by Descartes (1647).

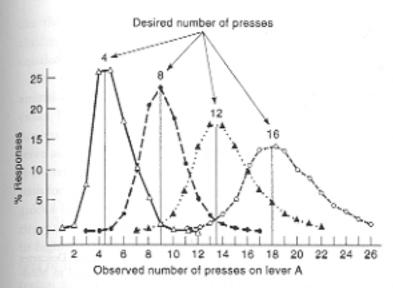


Figure 10.6
In this experiment, rats obtain food by pressing one lever (A) a predetermined number of times and then pressing a second lever. The number of presses on A matches approximately the required number, and responses become increasingly variable as the numbers get larger. (After Mechner 1958.)

Descartes famously proposed that humans are the only animals who are endowed with reason and that human reason is the source of all our distinctive cognitive achievements. Many of Descartes's examples of the use of reason come from the domain of geometry. Descartes invited us to consider the case of a blind man who holds two sticks that cross at a distance from himself (figure 10.7, top). Because the man is blind, he lacks any distal sense for apprehending the distance of the sticks' crossing point (c). Nevertheless, Descartes suggested, the man can use "natural geometry" to infer the location of this crossing point from knowledge of the distance and angular relation between his two hands at the points at which they grasp the sticks (a and b). Systematic use of Euclidean geometric principles not only allows the blind man to perceive objects at a distance, it also allows the development of the sciences of astronomy, optics, and physics (Descartes 1647). Perhaps, then, natural geometry is the core knowledge system that accounts for our uniquely human cognitive capacities.

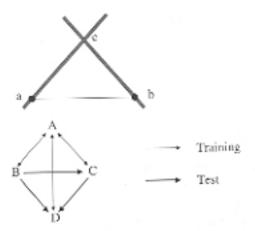


Figure 10.7

Top: schematic representation of the blind man's problem. (After Descartes 1647.) Bottom: schematic depiction of a task presented to blind and to blind-folded young children. (After Landau, Spelke, and Gleitman 1984.)

Almost 20 years ago, Barbara Landau, Henry Gleitman, and I attempted to test Descartes's conjecture by presenting a version of his triangulation problem to young blind and blindfolded children (Landau, Spelke, and Gleitman 1984; figure 10.7, bottom). Children were introduced into a room containing objects at four stable locations, and they were walked between the objects on specific paths. For example, a child might be walked from her mother seated in a chair (location A) to a table (location B), a box of toys (location C), and a mat (location D). Then the child was asked to move independently from one object to another on a path she had not previously taken (e.g., she might be asked to take a toy from the box and put it on the table, traversing the novel path from C to B). Note that the same principles of Euclidean geometry that allow solution of the blind man's stick problem should, in principle, allow solution of this triangle problem. Both blind and blindfolded children solved the problem reliably, providing evidence for Descartes's thesis that humans are endowed with natural geometry.

Does this endowment account for uniquely human reasoning abilities? Once again, studies of navigation in other animals are pertinent to this claim, and they provide resounding evidence against it. An exceedingly wide range of animals have been observed and tested in navigation tasks like the one Landau, Spelke, and Gleitman (1984) presented to young children. In every case, the performance of nonhuman animals has equaled or exceeded the performance of young children.

The most dramatic evidence for natural geometry in a nonhuman animal comes from studies of navigating desert ants (Wehner and Srinivasan 1981; figure 10.8). These ants leave their nest in the nearly featureless Tunisian desert in search of animals that may have died and can serve as food, wending a long and tortuous path from the nest until food is unpredictably encountered. At that point, the ants make a straight-line path for home: a path that differs from their outgoing journey and that is guided by no beacons or landmarks. If the ant is displaced to novel territory so that all potential landmarks are removed, its path continues to be highly accurate: within 2 degrees of the correct direction and 10 percent of the correct distance. This path is determined solely by the geometric relationships between the nest location and the distance and direction traveled during each step of the outgoing journey. Ants therefore have a "natural geometry" that appears to be at least equal to, if not superior to, that of humans.

To summarize, humans indeed have early-developing core knowledge systems, and these systems permit a range of highly intelligent behaviors and cognitive capacities including the capacity to represent hidden objects, to estimate numerosities, and to navigate through the spatial layout. In each case, however, nonhuman animals have been found to have capacities that equal or exceed those of human infants. The core knowledge systems that have been studied in human infants so far therefore do not account for uniquely human cognitive achievements. It remains possible, of course, that other core knowledge systems are unique to humans and account for unique aspects of our intelligence. In the absence of plausible candidate systems, however, I will turn instead to a different account of uniquely human cognitive capacities.

10.3 What Makes Us Smart? Uniquely Human Combinatorial Capacities

The suggestion I now explore begins with the thesis that humans and other animals are endowed with early-developing, core systems of knowl-

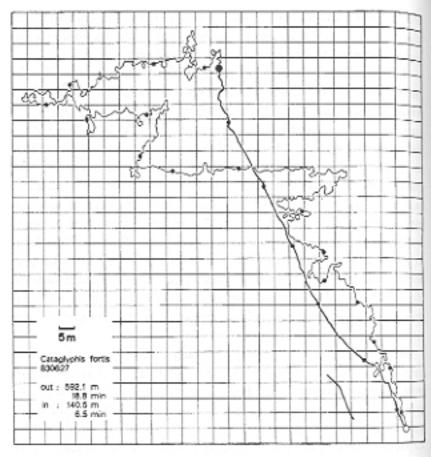


Figure 10.8 Path taken by a descrt ant during its outward (thin line) and homeward (thick line) journey in familiar territory. (After Wehner and Srinivasan 1981.) Very similar behavior was observed after a displacement that removed all local spatial edge but that these systems are limited in four respects. First, the systems are domain specific: each serves to represent only a subset of the entities in the child's surroundings. Second, the systems are task specific; the representations constructed by each system guide only a subset of the actions and cognitive processes in the child's repertoire. Third, the systems are relatively encapsulated: the internal workings of each system are largely impervious to other representations and cognitive processes. Fourth, the representations delivered by these systems are relatively isolated from one another: representations that are constructed by distinct systems do not readily combine together.

The core knowledge systems found in human infants exist throughout human life, and they serve to construct domain-specific, task-specific, encapsulated, and isolated representations for adults as they do for infants. With development, however, there emerges a new capacity to combine together distinct, core representations. This capacity depends on a system that has none of the limits of the core knowledge systems: it is neither domain nor task specific, for it allows representations to be combined across any conceptual domains that humans can represent and to be used for any tasks that we can understand and undertake. Its representations are neither encapsulated nor isolated, for they are available to any explicit cognitive process. This system is a specific acquired natural language, and the cognitive endowment that gives rise to it is indeed unique to humans: the human language faculty. Natural languages provide humans with a unique system for combining flexibly the representations they share with other animals. The resulting combinations are unique to humans and account for unique aspects of human intelligence.

To illustrate this suggestion, I will briefly describe the two lines of research that led to its emergence. First, I present a series of studies on children's developing navigation and spatial memory, conducted in collaboration with Linda Hermer-Vazquez, Ranxiao Frances Wang, and Stephane Gouteux. Then, I discuss a larger body of research on children's developing concepts of number undertaken by Susan Carey and myself, with numerous collaborators and students.

10,3.1 Space

Although animals are endowed with rich and exquisitely precise mechanisms for representing and navigating through the spatial layout, the

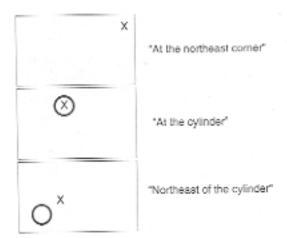


Figure 10.9 Schematic and simplified depiction of three tasks presented to rats. (After Biegler and Morris 1993, 1996.)

navigation of nonhuman animals sometimes shows interesting limits, In experiments by Biegler and Morris (1993, 1996; figure 10.9), for example, rats learned quite readily to locate food by searching in a particular geocentric position (e.g., the northeastern corner of the test chamber) or by searching near a particular landmark (e.g., in the vicinity of a white cylinder), but they had more difficulty learning to search in a particular geocentric relationship to a particular landmark (e.g., northeast of the white cylinder). Although rats evidently could represent that food was located "northeast of the room" or "at the cylinder," they could not readily combine these representations so as to represent that food was located "northeast of the cylinder."

A similar limit has appeared in experiments by Cheng and Gallistel (Cheng 1986; Gallistel 1990; Margules and Gallistel 1988). In their studies, rats were shown the location of food, then were disoriented, and finally were allowed to reorient themselves and search for the food. Rats readily reoriented themselves in accord with the shape of the room, but not in accord with the brightness of its walls, even though experiments dating back to Lashley show that rats can learn to respond selectively to white versus black walls directly. Although the rats' reorientation system evidently represented that food was located "at a corner with a long wall

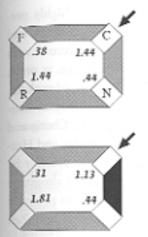


Figure 10.10 Tasks and performance of young children tested in a version of Cheng's (1986) reprientation task in which a toy was hidden and searching was measured at the correct location (C), the geometrically equivalent location (R), and the near and far, geometrically distinct locations (N and F). (After Hermer and Spelke 1984.)

on the left," it did not readily represent that food was located "at a corner with a white wall on the left." Like Biegler and Morris's studies, these studies suggest a limit to the combinatorial capacities of rats in navigation tasks.

Hermer-Vazquez and I sought to determine whether the same limit exists in children; to our surprise, we found that it did. In our studies, 1.5 to 2-year-old children were tested in a situation similar to Cheng's, in which they saw an object hidden in a corner of a rectangular room, they were disoriented, and then they searched for the object (Hermer and Spelke 1994, 1996; figure 10.10). Like Cheng's rats, children reoriented themselves in relation to the shape of the room but not in relation to the coloring of its walls. In subsequent experiments, children failed to reorient in accord with wall coloring even when it was made highly familiar (through experience over several sessions), when it was highly stable, When it was a successful direct cue for children in a task not involving reorientation, and when the distinctive wall coloring was presented in a cylindrical room with no geometrically distinctive shape (Gouteux and Spelke 2001; Wang, Hermer-Vazquez, and Spelke 1999).

Research in other laboratories confirms that children are highly predisposed to reorient in accord with the shape of their surroundings, and that under many circumstances children fail to reorient in accord with nongeometric information (Learmonth, Nadel, and Newcombe, in press; Learmonth, Newcombe, and Huttenlocher, in press; Stedron, Munakata, and O'Reilly 2000). Both rats and children do show sensitivity to nongeometric information in some circumstances, however (e.g., Cheng and Spetch 1998; Dudchenko et al. 1997; Learmonth, Newcombe, and Huttenlocher, in press; Stedron, Munakata, and O'Reilly 2000), possibly by means of a mechanism that circumvents geocentric navigation altogether and locates food by matching specific views of the environment to stored "snapshots" (e.g., Cartwright and Collett 1983; see Collett and Zeil 1998, for discussion).

In brief, both children and rats can learn to search to the left or right of a geometrically defined landmark, and they can learn to search directly at a nongeometrically defined landmark, but they do not readily combine these two sources of information so as to search left or right of a nongeometrically defined landmark. In contrast, human adults tested under similar circumstances show this ability quite readily (Gouteux and Spelke 2001; Hermer and Spelke 1994). What accounts for this difference?

Developmental research by Hermer-Vazquez, Moffett, and Munkholm (2001) suggested that the transition to more flexible navigation is closely related to the emergence of spatial language. In cross-sectional research, the transition was found to occur at about 6 years of age, around the time that children's language production shows mastery of spatial expressions involving left and right. Further studies of children at this transitional age revealed that performance on a productive language task with items involving the terms left and right was the best predictor of success on the reorientation task. Spatial language and flexible navigation therefore are correlated, but are they causally related?

In an initial attempt to address this question, Hermer-Vazquez, Spelke, and Katsnelson (1999; figure 10.11) returned to studies of human adults, using a dual-task method. If spatial language is causally involved in flexible navigation, we reasoned, then any task that interferes with subjects' productive use of language should interfere with their navigation

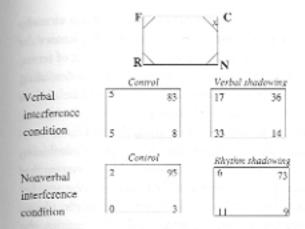


Figure 10.11

Tasks and performance of adults tested in the reorientation task used with children, under conditions of no interference (left) or of verbal or nonverbal interference (right). (After Hermer-Vazquez, Spelke, and Katsnelson 1999.)

as well. Accordingly, adults were tested in Hermer's reorientation task while performing one of two simultaneous interference tasks: a verbal shadowing task that interferes specifically with language production, or a nonverbal, rhythm shadowing task that is equally demanding of attentional and memory resources but does not involve language. Although rhythm shadowing caused a general impairment in performance, subjects in that condition continued to show a flexible pattern of reorientation in accord with both geometric and nongeometric information. In contrast, subjects in the verbal shadowing condition performed like young children and rats, reorienting in accord with the shape of the room but not in accord with its nongeometric properties. These findings provide preliminary evidence that language production is causally involved in flexible performance in this reorientation task.

Why might language make humans more flexible navigators? One possible answer relies on the combinatorial properties of language. Perhaps the most remarkable property of natural language is its compositionality: once a speaker knows the meanings of a set of words and the rules for combining those words together, she can represent the meanings of new combinations of those words the very first time that she hears them. The compositionality of natural languages explains how it is

possible for people to understand what they hear or read, when virtually every sentence they encounter is new to them. Once a speaker knows the syntactic rules of her native language and the meanings of a set of terms; she will understand the meanings of any well-formed expressions using those terms the first time that she hears them, and she will be able to produce new expressions appropriately without any further learning,

Although the compositional semantics of a natural language is intricate and not fully understood, one thing is clear: the rules for combining words in a sentence apply irrespective of the core knowledge system that constructs the representations to which each word refers. Once a speaker has learned the expression left of X and a set of terms for people, places, numbers, events, objects, collections, emotions, and other entities, she can replace X with expressions that refer to entities from any and all of these domains (e.g., left of the house where the happy old man cooked a 14-pound turkey for his family last Thanksgiving). Natural language therefore can serve as a medium for forming representations that transcend the limits of domain-specific, core knowledge systems.

More specifically, the navigation experiments of Cheng and Hermer suggest that humans and other animals have a core system for representing geometric properties of the spatial layout (in the terms of Cheng and Gallistel, a "geometric module"). Lest-right relationships are distinguished in this system: a rat or a child who has seen an object hidden left of a long wall scarches reliably to the left of that wall rather than to its right. Children therefore may learn the meaning of the term left by relating expressions involving that term to purely geometric representations of the environment. Studies of the visual system suggest further that children also have relatively modular systems for representing information about colors and other properties of objects, and these systems may permit children to learn the meanings of terms for colors such as blue and for environmental features such as wall. Once they have learned these terms, the combinatorial machinery of natural language allows children to formulate and understand expressions such as left of the blue wall with no further learning. This expression cannot be formulated readily outside of language, because it crosscuts the child's encapsulated core domains. Thanks to the language faculty, however, this expression

serves to represent this conjunction of information quickly and flexibly. Such use may underlie adults' flexible spatial performance.

10.3.2 Number

So far, I have suggested that natural language allows humans, and only humans, to represent combinations of information such as "left of the blue wall." Does language also allow humans to construct new systems of knowledge? Research on children's changing concepts of number is beginning to suggest that it may.

I have already described two lines of research providing evidence that human infants and other animals represent numerical information. First, experiments by Wynn and others reveal that infants and nonhuman primates can represent the numerical identity of each object in a scene, the numerical distinctness of distinct objects, and the effects of adding or subtracting one object. Second, experiments by Xu and others reveal that infants and many nonhuman animals can represent the approximate numerosity of a set of objects or events. These two capacities, however, appear to depend on distinct systems: human infants and adult nonhuman primates do not spontaneously combine them into a system of knowledge of natural number.

Evidence for the distinctness of core representations of small numbers of objects, on one hand, and of approximate numerical magnitudes, on the other hand, comes from four types of experimental findings. First, representations of numerically distinct objects show a set size limit of about 3 for infants (4 for adult humans and for nonhuman primates), whereas representations of approximate numerosities are independent of set size: infants and nonhuman primates can discriminate equally well between sets of 8 versus 16 and 16 versus 32, for example (Xu and Spelke 2000a,b). Second, representations of large approximate numerosities show a Weber fraction limit between 1.5 and 2 for 6-month-old infants, between 1.2 and 1.5 for 9-month-old infants, and about 1.15 for adult humans (e.g., Lipton and Spelke, in press; van Oeffelen and Vos 1982), whereas representations of numerically distinct objects do not: infants can discriminate 2 from 3 objects, even though the Weber fraction is below their threshold. These contrasting limits create a double

Table 10.2 Dissociations between human infants' representations of individuals and their numerical distinctness, and of sets and their cardinal values

	Individuals	Sets
a. Limits to discrimination		
Set size limit of 3-4	+	_
Weber limit of 1.5-2		+
b. Robustness over stimulus variations		
Variation in visibility	+	-
Variation in element size		+

dissociation between representations of small numbers of objects and representations of sets (table 10.2a).

A third finding that differentiates between representations of objects and sets concerns the effects of occlusion; representations of numerically distinct objects are robust over occlusion, whereas representations of approximate numerosities are not. Although human infants and monkeys who witness the successive introduction of individual objects into an opaque box can represent that a box with 3 objects has more objects than a box with 2, they fail to represent that a box with 8 objects has more objects than a box with 4, even though the ratio difference between these numerosities is above their Weber limit (Feigenson, Carey, and Hauser 2002; Hauser, Carey, and Hauser 2000).

A fourth difference concerns the effects of variations in properties of the items to be enumerated such as their size and spacing: representations of large approximate numerosities are robust over such variations, whereas representations of objects are not. Human infants discriminate 8 from 16 items on the basis of numerosity when item size, item density, filled area, and total area are varied-findings that provide evidence that they represent large numbers of items as forming a set with an approximate cardinal value. In contrast, infants fail to discriminate 1 item from 2 or 2 items from 3 on the basis of numerosity under these conditions (Clearfield and Mix 1999; Feigenson, Carey, and Spelke 2002; Xu and Spelke 2000a). This latter finding suggests that infants represent small numbers of objects as distinct individuals but not as forming a set, whose cardinal value can be compared to the cardinal values of sets composed of other, numerically distinct objects. The third and fourth findings coneritute a second double dissociation between representations of small numbers of objects and representations of large approximate numerosities (table 10.2b).

Considerable evidence therefore suggests that human infants are endowed with two distinct systems for representing numerosity. One system represents small numbers of persisting, numerically distinct individuals exactly and takes account of the operation of adding or removing one individual from the scene. It fails to represent the individuals as a set, however, and therefore does not permit infants to discriminate between different sets of individuals with respect to their cardinal values. A second system represents large numbers of objects or events as sets with cardinal values, and it allows for numerical comparison across sets. This system, however, fails to represent sets exactly, it fails to represent the members of these sets as persisting, numerically distinct individuals, and therefore it fails to capture the numerical operations of adding or subtracting one. Infants therefore represent both "individuals" and "sets," but they fail to combine these representations into representations of "sets of individuals."

The concept "set of individuals" is central to counting, simple arithmetic, and all natural number concepts. If infants lack this concept, they should have trouble understanding natural number terms such as two. Moreover, young children should miss the point of the verbal counting routine, even if they learn to mimic this routine. A rich body of research provides evidence that preschool children have both these problems (Fuson 1988; Griffin and Case 1996; Wynn 1990, 1992b).

Most children begin verbal counting in their second or third year of life. For months or years thereafter, however, they fail to understand the meaning of the routine or of the words that comprise it. Research by Wynn (1990, 1992b) provides evidence that children's understanding develops in four steps (table 10.3). At stage 1, when they first begin counting, children understand that one refers to "an object": if they are shown a picture of one fish and a picture of three fish and are asked for one fish, they point to the correct picture; if they are allowed to count an array of toy fish and then are asked to give the experimenter one fish, they offer exactly one object. At this stage, children also

Table 10.3

The development of children's understanding of number words and the counting routine, (After Wynn 1990, 1992b.)

Age	Understanding of number words and counting routing
2-2.5 years	One designates "an individual." Two, three,, six designate "a set."
2.5-3.25 years	One designates "an individual." Two designates "a set composed of an individual and another individual." Three, , six, designate "a set other than two."
3.25-3.5 years	One designates "an individual." Two designates "a set composed of an individual and another individual." Three designates "a set composed of an individual, another individual, and still another individual." Four, six, designate "a set other than two or three."
3.5-adult	Each number word designates "a set of individuals." The set designated by each number word contains "one more individual" than the set designated by the previous word in the counting routine.

understand that all other number words apply to arrays with more than one object. They never point to a picture of one object when asked to point to two fish or six fish, and they never produce just one object when asked for more than one.

Nevertheless, stage 1 children have very limited understandings of the meanings of the words in their counting routine. When they are shown pictures of two fish and of three fish and are asked to point to the picture with two fish, they point at random. Moreover, when they are allowed to count an array of objects and then are asked to give the experimenter one of the numbers of objects designated by a word in their own counting routine, they grab a handful of objects at random (Wynn 1990, 1992b). At this stage, children do not even understand that the applicability of specific number words changes when the numerosity of a set is changed by addition or subtraction: if children are allowed to count a pile of eight fish and then are told that the pile contains eight fish, they will continue to maintain that the pile has eight fish after four fish are removed (Condry, Spelke, and Xu 2000). For stage 1 children, one apnears to refer to "an individual" and all other number words appear to refer to "some individuals" (in the informal sense of "more than one").

After about nine months of counting experience, on average, Wynn's children work out the meaning of the word two. At this stage; children correctly point to or produce two objects when asked for two, and they point to or produce arrays of more than two objects when asked for any larger number. Three further months suffice for children to learn the meaning of three. Finally, children show comprehension of all the words in their counting routine, and they use counting when they are asked for larger numbers of objects. On average, it takes children about 1-1.5 years of experience with counting before they achieve this understanding.

Why does it take children so long to learn the meanings of words like two? I suggest that two is difficult to learn because it refers to a "set of individuals," and such a concept can only be represented by combining information across distinct core knowledge systems. Children readily learn part of the meaning of one by relating this word to representations constructed by their core system for representing objects: they learn that one applies just in case the array contains an object. Children also readily learn part of the meanings of the other number words by relating each word to representations of sets constructed by their core system of number sense: they learn that (e.g.) six applies just in case the array contains a set with an approximate cardinal value. To learn the full meaning of two, however, children must combine their representations of individuals and sets: they must learn that two applies just in case the array contains a set composed of an individual, of another, numerically distinct individual, and of no further individuals (figure 10.12). The lexical item two is learned slowly, on this view, because it must be mapped simultaneously to representations from two distinct core domains.

Children eventually are able to learn the meanings of two and three, because the sets of individuals to which these terms refer are within both the set size limit of their system for representing objects and the Weber traction limit of their system for representing sets. Larger numbers, however, exceed both these limits. How do children progress from Wynn's stage 3 to stage 4 and work out the meanings of the terms for the larger numbers within their counting routine?

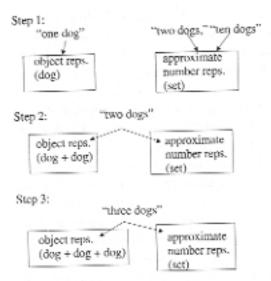


Figure 10.12
Hypothesized linkages between number words and core systems of representation at the first three steps in children's developing understanding of counting, number words, and the natural numbers

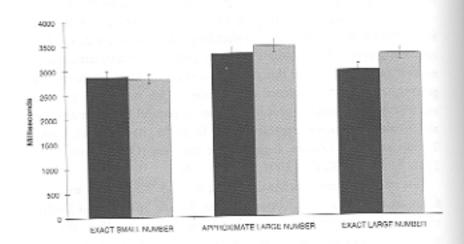
The above analysis suggests a possible answer. Once children have mapped two and three both to their system for representing individuals and to their system for representing sets, they are in a position to notice two things. First, relating the counting routine to the system of object representation reveals that the progression from two to three in the counting routine is marked by the addition of one individual to the set. Second, relating the counting routine to the system of number sense reveals that the progression from two to three is marked by an increase in the cardinal value of the set. Children may come to understand both the workings of the counting routine and the meanings of all the words it encompasses by generalizing these discoveries to all other steps in the counting routine. That is, children may achieve stage 4 when they realize that every step in the counting routine is marked by the successive addition of one individual so as to increment the cardinal value of the set of individuals. Because these representations exceed the limits of all the child's core knowledge systems, these realizations depend on elaborate conceptual combinations. Those combinations, in turn, may depend on the natural language of number words and of the counting routine.

Studies of children's learning of number words and counting therefore are consistent with the thesis that language serves as a medium for
combining core representations of numerosity and constructing natural
number concepts. To test this thesis, however, we must go beyond the
present, correlational evidence with children. One way to do this is to
ask whether the counting words of a specific natural language are causally involved in number representations in adults. Research with Sanna
Tsivkin and Gail O'Kane suggests that they are (O'Kane and Spelke
2001; Spelke and Tsivkin 2001; figure 10.13).

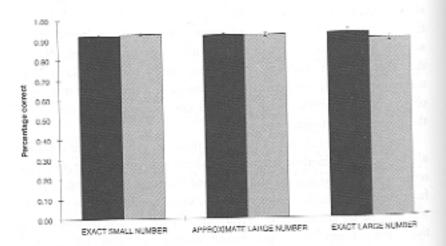
This research used a bilingual training method. Adults who were proficient in two languages (Russian and English or Spanish and English) were taught different sets of number facts. In some studies, the facts were in the domain of arithmetic: for example, adults might be taught to memorize the exact answer to a two-digit addition problem. In other studies, the facts appeared in stories and concerned the age of a character, the number of people or objects in a scene, the date at which something occurred, or some measured dimension of an object. In each study, subjects learned some facts in one of their languages and some facts in the other. In each language, a given fact could concern a large exact numerosity, a large approximate numerosity, or a small exact number of objects. Study materials were presented until subjects could retrieve all the information correctly and easily.

After learning each fact in just one language, subjects were tested on all the facts in both their languages, and the amounts of time needed to tetrieve facts in the trained and untrained language were compared. For facts about approximate numerosities or small numbers of objects, there was little or no advantage to performance in the trained language, relative to the untrained language. These findings suggest that large approximate and small exact number facts were represented independently of language for adults, as they must be for infants and nonhuman animals. In contrast, for facts about large exact numbers, there was a distinct advantage to performance in the language in which a fact was trained. This finding suggests that subjects drew on a specific natural language in

Reaction time



Accuracy



Trained language

III Untrained berguage

Figure 10.13
Performance of bilingual adults when tested for knowledge of small exact, large approximate, and large exact numbers in the language of training versus the untrained language. (After O'Kane and Spelke 2001.)

learning facts about large, exact numbers: the language in which those facts were presented.

These findings and others (see especially Dehaene et al. 1999) begin to suggest that human number representations have at least three components (see Dehaene 1997 and Spelke 2000 for more discussion). For very small numbers, these representations depend in part on what is often called a "subitizing" system (Mandler and Sheho 1982; Trick and Pylyshyn 1994): a system for representing small numbers of objects (up to four). For large approximate numerosities, number representations depend in part on a system for representing approximate numerical magnitudes (Dehaene 1997; Gallistel and Gelman 1992). For large exact numerosities, number representations depend in part on each of these systems and in part on a specific natural language.

10.4 Thought and Language

I have considered two possible answers to the question, What makes humans smart? According to the first answer, human intelligence depends on a biological endowment of species-specific, core knowledge systems. According to the second answer, human intelligence depends both on core knowledge systems that are shared by other animals and on a uniquely human combinatorial capacity that serves to conjoin these representations to create new systems of knowledge. The latter capacity, I suggest, is made possible by natural language, which provides the medium for combining the representations delivered by core knowledge systems. On the second view, therefore, human intelligence depends both on a set of core knowledge systems and on the human language faculty. Recent research on human infants, nonhuman primates, and human adults now seems to me to favor this view.

In closing, I attempt to situate this view in the context of debates over the relation of language and thought. Does this view imply that many of our concepts are learned? Does learning a natural language change the set of concepts that we can entertain? Do people who learn different languages have different conceptual repertoires? To approach these questions, I begin with one a priori objection that is commonly raised against all these possibilities.

Natural languages are learned by children who hear people talk about the things and events around them. In order for this learning to be possible, however, children must be able to conceptualize the things and events around them in the right ways: children won't, for example, learn the meaning of cow unless they can relate the utterance of the word to the presence of an object in the extension of the kind "cow." The latter representation is only possible if the child already has a workable concept of cows and a workable procedure for identifying instances of that concept. Thus, it would seem that language gives us a vehicle for expressing our concepts but doesn't provide a means to expand our concepts: we don't learn new concepts by learning a natural language.

My response to this argument is to grant it. Children learn many of the words of their language by relating those words to preexisting concepts: the concepts that are made explicit by their core knowledge systems. In particular, children learn the term left in relation to the preexisting concept "left" that is provided by their geometric system of representation. This concept, which is shared by rats, is surely independent of language, as are the child's concepts "blue" and "thing," which allow her to learn the words blue and thing. Moreover, children cannot learn, through language or any other means, any concepts that they cannot already represent. If children cannot represent the concept "left of the blue thing," as Hermer's research suggests, then they cannot learn to represent it.

Natural languages, however, have a magical property. Once a speaker has learned the terms of a language and the rules by which those terms combine, she can represent the meanings of all grammatical combinations of those terms without further learning. The compositional semantics of natural languages allows speakers to know the meanings of new wholes from the meanings of their parts. Although a child lacking the concept "left of the blue thing" cannot learn it, she does not need to. Having learned the meanings of left, blue, and thing, she knows the meaning of the expression left of the blue thing. Thanks to their compositional semantics, natural languages can expand the child's conceptual repertoire to include not just the preexisting core knowledge concepts but also any new well-formed combination of those concepts.

10.4.2 A Whorfian Research Program

If the compositionality of natural language semantics gives rise to uniquely flexible human cognition, then the thesis that language produces new concepts cannot be ruled out on logical grounds, and both this thesis and the possibilities that follow from it become open to empirical test. One much-discussed possibility that can be pursued in this context is Whorf's thesis that the members of different cultures and language groups have different repertoires of concepts. Note that no evidence or arguments in this chapter support Whorf's thesis. If the combinatorial properties of language that produce new concepts are universal across human languages, then uniquely human conceptual capacities will be universal as well. Questions about the existence of cultural differences in human conceptual capacities therefore hinge in part on questions about the origins and nature of compositional semantics. How does compositional semantics work? Is there a single, universal compositional semantics that applies to all languages, or do languages vary in their combinatorial properties? How do children develop the ability to use the compositional semantics of natural languages?

Although I cannot answer any of these questions, I close with a final suggestion. Studies of cognition in nonhuman animals and in human infants, and studies of cognitive development in human children, may shed light both on our remarkable capacity for combining word meanings into complex expressions and on our corresponding capacity to combine known concepts into new ones. Two difficult questions faced by linguists and other cognitive scientists are (1) what are the primitive building blocks of complex semantic representations? and (2) what are the basic combinatorial processes by which these building blocks are assembled? Research from the fields discussed here suggests a general approach to these questions. The building blocks of all our complex representations are the representations that are constructed from individual core knowledge systems. And the basic processes that combine them are the processes that children use in constructing their first new concepts. Studies of cognition in nonhuman animals, in human infants, and in developing children therefore may shed light on central aspects both of our uniquely human capacity for language and of our uniquely human capacity for building new systems of knowledge.

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References

Biegler, R., and Morris, R. G. M. (1993). Landmark stability is a prerequisite for spatial but not discrimination learning. Nature, 361, 631–633.

Biegler, R., and Morris, R. G. M. (1996). Landmark stability: Studies exploring whether the perceived stability of the environment influences spatial representation. The Journal of Experimental Biology, 199, 187–193.

Cartwright, B. A., and Collett, T. S. (1983). Landmark learning in bees. Journal of Comparative Physiology, 151, 521–543.

Cheng, K. (1986). A purely geometric module in the rat's spatial representation. Cognition, 23, 149–178.

Cheng, K., and Spetch, M. L. (1998). Mechanisms of landmark use in manimals and birds. In S. Healy (Ed.), Spatial representation in animals (pp. 1–17). Oxford: Oxford University Press.

Clearfield, M. W., and Mix, K. S. (1999). Number versus contour length in infants' discrimination of small visual sets. *Psychological Science*, 10, 408–411.

Collett, T. S., and Zeil, J. (1998). Places and landmarks: An arthropod perspective. In S. Healy (Ed.), Spatial representation in animals (pp. 18–53). Oxford: Oxford University Press.

Condry, K., Spelke, E. S., and Xu, F. (2000, June). From the infant's number concepts to the child's number words. Poster presented at the International Conference on Infant Studies, Brighton, UK.

Dehaene, S. (1997). The number sense: How the mind creates mathematics. Oxford: Oxford University Press.

Dehaene, S., Spelke, E., Pinel, P., Stanescu, R., and Tsivkin, S. (1999). Sources of mathematical thinking: Behavioral and brain-imaging evidence. Science, 284, 970–974.

Descartes, R. (1647). Discourse on method and essays. (Numerous translations are available.)

Dudchenko, P. A., Goodridge, J. P., Seiterle, D. A., and Taube, J. S. (1997). Effects of repeated disorientation on the acquisition of spatial tasks in rats: Dissociation between the appetitive radial arm maze and aversive water maze. Journal of Experimental Psychology: Animal Behavior Processes, 23, 194–210. Feigenson, L., Carey, S., and Hauser, M. (2002). The representations underlying infants' choice of more: Object files vs. analog magnitudes. Psychological Science, 13, 150-156.

Feigenson, L., Carey, S., and Spelke, E. S. (2002). Infants' discrimination of number vs. continuous extent. Cognitive Psychology, 44, 33-36.

Felleman, D. J., and van Essen, D. C. (1991). Distributed hierarchical processing in the primate cerebral cortex. Cerebral Cortex, 1, 1–47.

Fuson, K. C. (1988). Children's counting and concepts of member. New York: Springer-Verlag.

Gallistel, C. R. (1990). The organization of learning. Cambridge, MA: MIT Press.

Gallistel, C. R., and Gelman, R. (1992). Preverbal and verbal counting and computation. Cognition, 44, 43-74.

Gouteux, S., and Spelke, E. S. (2001). Children's use of geometry and landmarks to reorient in an open space. Cognition, 81, 119–148.

Griffin, S., and Case, R. (1996). Evaluating the breadth and depth of training effects, when central conceptual structures are taught. In R. Case and Y. Okamoto (Eds.), The role of central conceptual structures in the development of children's thought (pp. 83–102). Managraphs of the Society for Research in Child Development (Scrial No. 246).

Hauser, M., Carey, S., and Hauser, L. (2000). Spontaneous number representation in semi-free-ranging rhesus monkeys. Proceedings of the Royal Society, London, 267, 829–833.

Hauser, M., MacNeilage, P., and Ware, M. (1996). Numerical representations in primates. Proceedings of the National Academy of Sciences, 93, 1514–1517.

Hermer, L., and Spelke, E. S. (1994). A geometric process for spatial reorientation in young children. Nature, 370, 57-59.

Hermer, L., and Spelke, F. S. (1996). Modularity and development: The case of spatial reorientation. Cognition, 61, 195-232.

Hermer-Vazquez, L., Moffett, A., and Munkholm, P. (2001). Language, space, and the development of cognitive flexibility in humans: The case of two spatial memory tasks. Cognition, 79, 263–299.

Hermer-Vazquez, L., Spelke, E. S., and Katsnelson, A. (1999). Sources of flexibility in human cognition: Dual-task studies of space and language. Cognitive Psychology, 39, 3–36.

Koechlin, E., Dehaene, S., and Mehler, J. (1998). Numerical transformations in five-month-old human infants. Mathematical Cognition, 3, 89-104.

Landau, B., Spelke, E. S., and Gleitman, H. (1984). Spatial knowledge in a young blind child. Cognition, 16, 225–260.

Lea, S. E. G., Slater, A. M., and Ryan, C. M. E. (1996). Perception of object unity in chicks: A comparison with the human infant. Infant Behavior and Development, 19, 501–504. Learmonth, A. E., Nadel, L., and Newcombe, N. S. (in press). Reorientation, landmark use and size: The fate of a geometric module. Psychological Science.

Learmonth, A. E., Newcombe, N. S., and Huttenlocher, J. (in press). Toddlegs' use of metric information and landmarks to reorient. Journal of Experimental Child Psychology.

Lipton, J. S., and Spelke, E. S. (in press). Origins of number sense: Large number discrimination in human infants. Psychological Science.

Mandler, G., and Shebo, B. J. (1982). Subitizing: An analysis of its component processes. Journal of Experimental Psychology: General, 11, 1–22.

Margules, J., and Gallistel, C. R. (1988). Heading in the rat: Determination by environmental shape, Animal Learning and Behavior, 16, 404-410.

Mechner, F. (1958). Probability relations within response sequences under ratio reinforcement. Journal of the Experimental Analysis of Behavior, 1, 109–121.

O'Kane, G. C., and Spelke, E. S. (2001, November). Language dependence in representations of number, space, and time. Poster presented at the meeting of the Psychonomic Society, Orlando, FL.

Regolin, I., and Vallortigara, G. (1995). Perception of partly occluded objects by young chicks. Perception and Psychophysics, 57, 971–976.

Regolin, I., Vallortigara, G., and Zanforlin, M. (1995). Object and spatial representations in detour problems by chicks. Animal Behaviour, 49, 195–199.

Simon, T. J., Hespos, S. J., and Rochat, P. (1995). Do infants understand simple arithmetic? A replication of Wynn 1992a. Cognitive Development, 10, 253-269.

Spelke, E. S. (1990). Principles of object perception. Cognitive Science, 14, 29-56.

Spelke, E. S. (1998) Nativism, empiricism, and the origins of knowledge. Infant Behavior and Development, 21, 181–200.

Spelke, E. S. (2000). Core knowledge. American Psychologist, 55, 1233-1243.

Spelke, E. S., and Tsivkin, S. (2001). Language and number: A bilingual training study. Cognition, 78, 45–88.

Stedron, J. M., Munakata, Y., and O'Reilly, R. C. (2000, July). Spatial reorientation in young children: A case of modularity? Poster presented at the International Conference on Infant Studies, Brighton, UK.

Thelen, E. (1984). Learning to walk: Ecological demands and phylogenetic constraints. In L. P. Lipsitt and C. Rovee-Collier (Eds.), Advances in infancy research (Vol. 3). Norwood, NI: Ablex.

Trick, L., and Pylyshyn, Z. W. (1994). Why are small and large numbers enumerated differently? A limited capacity preattentive stage in vision. Psychological Review, 101, 80–102.

Van de Walle, G., Carey, S., and Prevor, M. (2001). Bases for object individuation in infancy: Evidence from manual search. Journal of Cognition and Development, 1, 249–280.

van Oeffelen, M. P., and Vos, P. G. (1982). A probabilistic model for the discrimination of visual number. Perception and Psychophysics, 32, 163-170. Wang, R. F., Hermer-Vazquez, L., and Spelke, E. S. (1999). Mechanisms of reorientation and object localization by human children: A comparison with rats. Rehavioral Neuroscience, 113, 475–485.

Wehner, R., and Srinivasan, M. V. (1981). Searching behavior of desert ants, genus Cataglyphis (Formicidae, Hymenoptera). Journal of Comparative Physiology, 142, 315–338.

Wynn, K. (1990). Children's understanding of counting. Cognition, 36, 155-193.

Wyan, K. (1992a). Addition and subtraction by human infants. Nature, 358, 749-750.

Wynn, K. (1992b). Children's acquisition of the number words and the counting system. Cognitive Psychology, 24, 220–251.

Wynn, K. (1998). Psychological foundations of number: Numerical competence in human infants. Trends in Cognitive Sciences, 2, 296–303.

Xu, F., and Spelke, E. S. (2000a, July). Large number discrimination in infants: Evidence for analog magnitude representations. Paper presented at the International Conference on Infant Studies, Brighton, UK.

Xu, F., and Spelke, E. S. (2000b). Large number discrimination in 6-month-old infants. Cognition, 74, B1–B11.