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Math Schemata and the Origins of Number Representations

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Abstract

The contrast Rips et al. draw between "bottom-up" and "top-down" approaches to understanding the origin of the capacity for representing natural number is a false dichotomy. Its plausibility depends upon the sketchiness of the authors' own proposal. At least some of the proposals they characterize as bottom-up are worked-out versions of the very top-down position they advocate. Finally, they deny that the structures that these putative bottom-up proposals consider to be sources of natural number are even precursors of concepts of natural number. This denial depends upon an idiosyncratic, and mistaken, idea of what a precursor is.

Rips et al. criticize a "bottom-up" approach to the origin of the capacity for representing natural number. According to the bottom-up view that they believe characterizes most current work on the development of numerical cognition (including mine), representations of natural number are supposedly derived, by empirical induction, from representations of sets, objects, and the quantitative resources of Figure 1 of the target article (parallel individuation of small sets, analog magnitude representations of number). While agreeing that the representational systems sketched in Figure 1 exist, and underlie a variety of behaviors of infants and young children, Rips et al. deny that these (or even the explicit representational scheme they call "simple counting") are precursors to representations of natural number. Rips et al. propose an alternative "top-down" account, in which math schemas that are the equivalent of Peano's axioms are somehow directly acquired without involving the representations of Figure 1 or of simple counting.

Rips et al. wildly misconstrue my proposal. Although I hold that the schemata of Figure 1 and of simple counting are precursors to representations of natural number, these do not exhaust the innate machinery brought to bear in this achievement; my proposal does not bottom out in these structures. My position is more of a worked-out version of Rips et al.'s top-down approach than a bottom-up approach (and thus I agree with much they have to say in their target article).

My proposal depends upon a particular form of bootstrapping process (Quinian bootstrapping) that has been well studied in the literature on the history and philosophy of science (Carey, in press; Nersessian 1992; Quine 1960). Carey (2004; in press) illustrates the role of Quinian bootstrapping in the acquisition of simple counting, which results in a representational schema that goes beyond the resources of Figure 1. Simple counting is the first schema that represents even a finite subset of the natural numbers, and the bootstrapping episode that creates this schema is only one of several that eventually result in the capacity for representing natural number. A second bootstrapping episode described In Carey(in press) underlies the integration of simple counting with the analog magnitude representations of Figure 1. These do not complete the story; however, they illustrate how it works.

All episodes of Quinian bootstrapping require top-down creation of explicit placeholder structures, the symbols of which get their meaning entirely from conceptual roles within those structures. Besides the resources needed for the construction of the placeholder structures, Quinian bootstrapping involves modeling processes through which these structures are infused with mathematical meaning. For example, nothing in Figure 1 captures the child's capacity to create ordered lists of symbols. The meaningless list, "one, two, three, four…" is a placeholder structure, the meaning of which is exhausted by its conceptual role as an ordered list. Other computational resources are drawn upon in the process of creating meaning for this placeholder structure – various logical capacities, recursion, a variety of processes that model the representations of Figure 1 in terms of the placeholder structure, as well as induction.

As Rips et al. point out, their own proposal for the innate building blocks for number representation includes knowledge that is "tacit." Their proposal suffers, however, from the lack of any hint of what they might mean by this. How are the innate math schemas they presuppose represented? What are the symbols like (format, content), and what computations do they enter into? What is tacit knowledge? The schemata instantiated in Figure 1 provide answers to those questions. The actual symbols in parallel individuation represent individuals, but the system as a whole tacitly embodies arithmetical knowledge in the processes that pick out and manipulate sets, compute a one-to-one correspondence, and compute numerical equality and inequality. The actual in analog magnitude representations represent approximate cardinal values of sets of individuals, but, again, the system as a whole tacitly embodies arithmetical knowledge in the processes that compute arithmetic functions over these values, including sums and ratios. Ditto for simple counting; much of the knowledge that ensures that simple counting constitutes a representation of a finite subset of the natural numbers is tacit, captured in the counting principles characterized in Gelman and Gallistel's (1978) classic work.

Rips et al. claim that neither simple counting, nor the representational systems depicted in Figure 1, are precursors to natural number, arguing that the concept of natural number cannot be defined in terms of structures of Figure 1, nor be derived from them by empirical induction. However, the mastery of simple counting is a necessary prerequisite for the mastery of complex counting, which Rips et al. agree is likely to be a necessary part of acquiring the math schema of natural number. The mastery of simple counting draws on the resources of Figure 1 (plus others), and, in this sense, these structures are all part of the precursors to natural number. The authors point out that on the mathematical ontology they favor, the content of a mathematical symbol is given entirely by computational role (its place in the system), and on this view, simple counting and the structures in Figure 1 play no role in the mathematical concept of natural number (which is exhausted by the concept of a unique first number, the concepts of successor and predecessor, and mathematical induction). However, aspects of these latter component concepts are implicit in the computations carried out over the schemata captured in Figure 1 and in simple counting, and provide constraints in the modeling processes through which the placeholder structures created by top-down processes come to have meaning.

My proposal, like theirs, assumes that conceptual role is the main source of content for mathematical concepts. My proposal concerns how new primitive symbols are coined and how they come to have the appropriate conceptual role. Contrary to Rips et al., I believe that the content of each symbol for a positive integer is determined both by conceptual role and by the capacity to represent cardinalities of sets of actual individuals. This hypothesis makes sense of one of the most striking facts about mathematical development: Mathematical development, both historically and in ontogenesis, often occurs in the course of modeling the world. It is no accident that Newton invented the calculus and Newtonian mechanics, or that Maxwell invented the mathematics needed to field theories in the course of modeling Faraday's electromagnetic phenomena. In the end, the big mistake that Rips et al. make is methodological: they miss the fact that modeling activities can give placeholder structures meaning, even if in the end the structures involved in these modeling processes, such as the schemata of Figure 1, are part of an acquisition ladder that is not essential to the conceptual role constructed. This is what developmental precursors really are – representations that play a role in the bootstrapping process.

Before Rips et al. have offered an viable alternative to the picture they criticize, they owe us some idea of what the math schemas they advocate are like, at Marr's algorithmic level of description, and how they are acquired.

References

Carey S. Bootstrapping and the origins of concepts. Daedalus. 2004:59–68. Winter Issue.

Carey, S. The origins of concepts. Oxford University Press; (in press)

Gelman, R.; Gallistel, CR. The child's understanding of number. Harvard University Press/MIT Press; 1978.

Nersessian, N. How do scientists think? Capturing the dynamics of conceptual change in science. In: Giere, R., editor. Cognitive Models of Science. University of Minnesota Press; 1992. p. 3-44.

Quine, WVO. Word and Object. MIT Press; 1960.

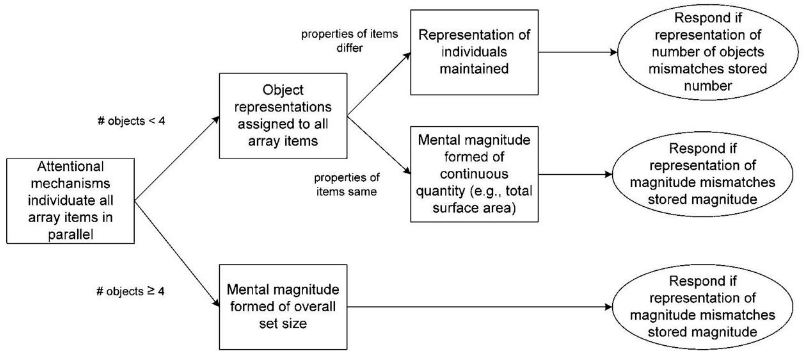

Figure 1.

A model for infants' quantitative abilities. Response rules in ovals indicate conditions under which infants look longer in addition-subtraction or habituation tasks. They are not meant to exhaust possible uses of these representations.