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Discussion

Why the verbal counting principles are constructed out of representations of small sets of individuals: A reply to Gallistel ☆

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1. Introduction

The seminal work by Gelman and Gallistel (1978) overturned the (then) Piagetian orthodoxy concerning the ontogenesis of the capacity to represent the natural numbers. Piaget (1952/1941) offered his famous studies on number conservation as evidence that children do not have a concept of number until age 6 or so. He explained this putative late emergence in terms of the absence of logical abilities required to support number representations, which he argued are not achieved until the stage of concrete operations. Gelman and Gallistel (1978) replied that any child who counted could thereby represent natural number, so long as the child followed what they called the "counting principles" (stable order, 1–1 correspondence and the cardinal principle that the last numeral reached in a count represents the cardinal value of the enumerated set). Indeed, these three counting principles guarantee that verbal numerals represent quantities that satisfy the successor function. That is, for any set whose cardinality n is represented by a given numeral, the next numeral in the list will represent cardinality $n + 1$.

Since the late 1970s, Gelman and Gallistel have systematically studied the acquisition of verbal counting in childhood as a window onto the ontogenetic sources of

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knowledge of the natural numbers (Gallistel & Gelman, 1992, 1978; Gelman & Lucariello, 2002). They have argued that the way children acquire verbal counting shows that knowledge of the counting principles is innate. The more recent formulations of the hypothesis have taken preverbal knowledge of the counting principles to be implemented in the mechanism that generates analog representations of number (e.g., Gallistel & Gelman, 1992; Gelman & Lucariello, 2002) This is the mechanism Gallistel has in mind when he refers to the ‘‘preverbal counting system.’’

Like virtually all researchers in this field, we agree with Gallistel and Gelman that the verbal numeral list deployed in a count routine is the first explicit representation of the natural numbers mastered by children growing up in numerate societies. Indeed, our project derives from the work Gelman and Gallistel initiated almost thirty years ago: we have studied the acquisition of verbal numerals and of verbal counting as a means of understanding the ontogenesis of knowledge of the natural numbers. However, we disagree with Gallistel (and Gelman) on two major points. First, we believe the evidence shows that the count list is first mastered much as children learn to recite the alphabet, that is, without attributing any significance to the order. Thus, we believe that knowledge of the counting principles is not innate, but rather constructed as a result of children′s attempt to make sense of the verbal count list. Second, although we fully agree that analog magnitudes are part of our innate cognitive resources and that they eventually provide an important part of the meaning of verbal numerals, we take our data and that of others (e.g., Condry & Spelke, in press) to convincingly show that knowledge of the verbal counting principles is not constructed out of analog magnitudes but out of representations provided by a system we call ‘‘enriched parallel individuation.’’ In rejecting a role of the analog magnitude system in the early development of knowledge of the meaning of numerals, we stand in stark opposition to Gallistel’s (and Gelman’s) theory of the acquisition of verbal counting. In what follows, we explain why we disagree with Gallistel, addressing his criticisms along the way. But first we clarify the logic of our project, for Gallistel’s comments suggest it may not have been clear.

2. The nature of our questions and the logic of our methods

In our view, there are three preverbal systems of discrete quantification, each elicited in different circumstances. Gallistel accepts one of these systems: the analog magnitude system in which the cardinal value of a set is represented by an analog symbol that is a linear or logarithmic function of the number of elements in the set. It is the other two systems and their roles in the development of numerical capacities that are the basis for the disagreements. One of these systems we call ‘‘parallel individuation’’ (Carey, 2004; Feigenson & Carey, 2003, 2005). The other we call ‘‘set based quantification’’ (Barner, Thalwitz, Wood, & Carey, 2007; Barner, Wood, Hauser, & Carey, under review). The goal of our paper was to determine which of these three systems supports the numerical meanings of verbal numerals at the point of the child’s earliest mastery of them. Our two most important results were that (1) children cannot estimate the numerical size of sets beyond 4 without counting until 6 months after they have become cardinal-principle knowers (i.e., have mastered all of the counting principles) and (2) errors of application of "one" to "four" in estimation tasks do not show the noise signature of the analog magnitude system. We were eager to read Gallistel's thoughts on these data, for we obtained them with tasks and analyses that were modeled after his excellent studies of the mapping between numerals and analog magnitudes in adults (Cordes, Gelman, Gallistel, & Whalen, 2001; Whalen, Gallistel, & Gelman, 1999). Thus, we were greatly disappointed that he did not engage any part of our findings in his commentary, especially since they undermine the hypothesis he favors.

As Gallistel remarks, the general hypothesis that frames our work is that knowledge of the verbal counting principles can be induced from mappings between individual numerals and core number representations. It is with this hypothesis in mind that we sought to determine which individual numerals are mapped to which preverbal number representations in the process of the construction of the verbal counting principles. But Gallistel misunderstands our project when he states that we equate knowledge of the meaning of a numeral with knowledge of how it maps to analog magnitudes. As we explicitly stated in our paper, the logic of our project is just the opposite. We say:

> "(...) does not knowledge of the counting principles implicate knowledge of the mappings between large numerals and analog magnitudes? Not necessarily. (...) it may be possible to know the meaning of a symbol *qua* symbol in the count list without knowing its meaning *qua* symbol mapped onto an analog magnitude. Thus, there could be a period during which children who can determine what numeral to apply to a large set of objects (e.g., 10) by counting it, cannot do so if they are prevented from counting and are thereby forced to rely on the mapping between large numerals and analog magnitudes."

It is for this very reason that we took great pains to make sure that our measure of children's knowledge of mappings between numerals and core number representations was *independent* from our measure of knowledge of verbal counting; e.g., our Fast Cards task (Experiment 1) *prevented* the use of verbal counting so that it would test knowledge of mappings between numerals and core number representations in subset-knowers *and* in cardinal principle-knowers (Subset-knowers have assigned numerical meaning to only a subset of the numerals in their count list; that is they are "one-," "two-," "three-," or "four-"knowers. Henceforth, we use "CP-knowers" as short for "cardinal principle-knowers"). We followed this experimental logic all the way to the end. Our most important result is that there are children who have knowledge of the counting principles and can count at least up to "ten" but have not mapped numerals beyond "four" onto analog magnitudes – i.e., CP non-mappers.

Gallistel is right that we did not consider Gelman's "structure-mapping" proposal (e.g., Gelman & Lucariello, 2002) in our article. We did not consider it because the proposal has never spelled out how children note a structural/functional isomorphism between preverbal counting and verbal counting without already understanding how verbal counting represents the natural numbers. We would welcome a worked out proposal, for, as we say below, we are sympathetic to Gelman's general approach to learning. Contrary to Gallistel's commentary, however, the proposal we do consider in our paper – that the acquisition of individual mappings between numerals and magnitudes plays a role in the acquisition of the numeral list counting system – *was* explicitly endorsed in Gallistel and Gelman (1992). They write: "Learning to count involves, in part, learning a mapping from the preverbal numerical magnitudes to the verbal and written symbols and the inverse mappings from these symbols to the preverbal magnitudes." (p. 42), and that "Children assimilate verbal counting because it maps onto the unconscious preverbal counting process. The count words map to the preverbal magnitudes." (p. 65). This is a very serious possibility, endorsed explicitly or implicitly by many people in the field (Dehaene, 1997, 2001; Spelke & Tsivkin, 2001; Wynn, 1992, 1995). This is why we set out to test this particular version of the hypothesis that analog magnitudes govern the acquisition of verbal counting.

One related misconception must be countered. Gallistel assumes correctly that we are testing the whether children learn the meanings of the first 3 or 4 numerals by mapping them onto the corresponding magnitudes, but suggests that we think this mapping is the only possible way these numerals could be numerically meaningful. Quite the contrary, the aim of our project was to determine whether analog magnitudes are part of the input to the acquisition of the verbal counting principles, or whether the counting principles are constructed from different representations of the meanings of the first four numerals, those delivered by the system of representation we call "enriched parallel individuation." We take our results to provide strong support for the latter hypothesis. This is the view we defend below.

3. Evidence that knowledge of the counting principles is constructed

The most damning problem for Gallistel's (and Gelman's) preverbal counting hypothesis is that the analog magnitude system is not a preverbal counting mechanism. In fact, there is no evidence for *any* preverbal counting mechanism. Evidence that non-human animals and human infants represent number is not tantamount to evidence that they count – for there are many different ways of deriving numerical information from arrays of individuals. Current evidence indicates that analog magnitudes are not created via an iterative counting process in which each individual must be counted sequentially. It takes infants no longer to create analog magnitude representations of 8 items than of 4 items (Wood & Spelke, 2005) or for adults to create analog magnitude representations of 100 items than of 50 items (Barth, Kanwisher, & Spelke, 2003). These results are best explained by models of analog magnitude representations in which the cardinal values of sets are created in parallel (e.g., Dehaene & Changeux, 1993; Verguts & Fias, 2004) or derived from computations over variables that can be measured equally quickly for sets of any size (e.g., in a two-dimensional display, total area occupied by objects and average object area; see Church and Broadbent (1990) for a relevant model of "countless" enumeration of sequences of events).

We agree with Gallistel that evidence that children understand the numerical significance of counting from the very outset of memorizing a verbal count list would in itself be evidence for a preverbal counting system. However, in accord with the conclusion that humans are not born with any form of knowledge of the counting principles, multiple measures of knowledge of the counting principles have been used in multiple cultures and have shown that, in all these cultures, at least one year elapses between the time at which children begin to recite a count list and the time at which they begin to use it as representation of the natural numbers (Le Corre, Li, Jia, & Shui, 2003; Le Corre, Van de Walle, Brannon, & Carey, 2006; Sarnecka, Kamenskaya, Ogura, Yamana, & Yudovina, in press; Schaeffer, Eggleston, & Scott, 1974). It is in light of this evidence that we asserted that children's early counting is a routine that they learn without grasping its numerical significance. It is also the evidence that has led most researchers in the field to consider the debate between nativist and constructivist views of the acquisition of verbal counting to be settled in favor of the latter (Condry & Spelke, in press; Frye, Braisby, Lowe, Maroudas, & Nicholls, 1989; Fuson, 1988; Hurford, 1987; Le Corre et al., 2006; Mix, Huttenlocher, & Levine, 2002; Schaeffer et al., 1974; Siegler, 1991; Wynn, 1992).

Gallistel takes strong exception to this characterization of the acquisition of verbal counting, and counters that his and Gelman's group of investigators have provided good evidence that children do understand counting from the outset. He also argues that the studies that failed to support his and Gelman's hypothesis did so because the tasks they used to measure knowledge of verbal counting (e.g., Wynn's (1990, 1992) Give a Number task) taxed children's fragile performance systems (e.g., knowing when to deploy counting to perform some task, controlling attention, working memory, and motor plans) to such an extent that they prevented them from expressing their knowledge of counting. Had these studies used more age-appropriate tasks (e.g., Gelman's (1993) What's on This Card task) they would have found that children's counting is numerically meaningful from the beginning.

We disagree with Gallistel's counter-arguments. First, many studies have shown that children begin to recite a count list around age 2:0 (Le Corre et al., 2006; Sarnecka et al., in press; Wynn, 1990, 1992). Most of the studies Gallistel refers to did not include children who were younger than 3 (Gelman, 1972; Zur & Gelman, 2004), and those that included 2-year-olds yielded a mixed pattern of success and failure (Gelman, 1993). Therefore, these studies fall short of providing evidence that verbal counting is numerically meaningful from the very beginning. Most importantly, Le Corre et al. (2006) explicitly tested whether children's long failure to use their count list as a representation of number should be attributed to a performance deficit rather than to their lack of knowledge of the counting principles. They tested the same children on a demanding task (i.e., Wynn's Give a Number, the task we used in Experiment 1 to distinguish "subset-knowers" from "cardinal principle-knowers") and on easier tasks (e.g., Gelman's What's on This Card, the task we used in Experiment 2). We found that children who were classified as "*n*"-knowers on the basis of Give a Number were also classified as "*n*"-knowers on the basis of What's on This Card; e.g., "one"-knowers – children who have learned an exact meaning for "one" but not for any other numeral in their count list – were classified as such on both tasks. More generally, we found that children who failed to show that they understood counting on Give a Number (i.e., children who were "subset-knowers") also:

(1) were unable to fix a set when they were told that they had given the wrong number of objects and were asked to change the number they had given to make it right; (2) almost never used the last numeral of their count to refer to the number of stickers in a set they had just counted correctly (in the What's on This Card task); and (3) agreed that a puppet that had counted five elephants out loud as it slowly put them in a bin one at a time had just put "six elephants" in the bin. The only children who did not make these mistakes were the ones who had been classified as CP-knowers on the basis of their performance on Give a Number. Given that these results show the same knowledge across tasks that make strikingly different processing demands on the child, the qualitative differences in the counting behavior of subset-knowers and cardinal principle-knowers strongly suggest that what ultimately separates these groups is knowledge of the counting principles (see Wynn (1992) for further evidence of the consistency of children's knowledge of numerals and counting across tasks that make different processing demands).

Sarnecka and Carey (under review) provide additional evidence that cardinal principle-knowers differ from subset-knowers precisely in understanding how counting implements the successor function. Only cardinal principle-knowers understand the implications of going up one item in the count list for the direction and unit of change for the cardinal value of the set represented by a given numeral. The tasks used by Sarnecka and Carey were arithmetic tasks, and so belie Gallistel's claim that arithmetic tasks elicit evidence of understanding the cardinal principle by children otherwise classified as subset-knowers.

4. Evidence that analog magnitudes are not the source of knowledge of the counting principles: The dialectic of Le Corre and Carey (2007)

Our results are problematic for the hypothesis that analog magnitudes ground learning the meanings of verbal numerals and learning to count, whether the proposal is that they do so via mappings to individual numerals or via some structural mapping. First, our data show that mappings between individual numerals and analog magnitudes are not constructed until well after children have induced the counting principles; we refer readers to our paper for the arguments that support this conclusion. Incidentally, if Gelman and Gallistel are right that the counting principles are understood earlier than indicated by the tasks that pattern with Give a number, then verbal numerals are integrated with analog magnitudes even *later* relative to learning how counting represents number than we have claimed in our paper. Thus, on either view of when children master the counting principles, the mappings of numerals to analog magnitudes cannot be their source. Second, although our study was not designed with the structure-mapping version of the preverbal counting hypothesis in mind, our data also militate against this hypothesis. It is hard to imagine any version of this proposal that would not involve children recognizing that numerals later in the count list represent greater numbers (as specified by analog magnitudes; see Wynn (1992) for a proposal to that effect). This is precisely what subset-knowers and CP non-mappers fail to demonstrate. When asked to estimate the number of individuals in a set, they do not produce larger numerals for sets of 10 than for sets of 5 (see also Condry & Spelke, in press).

In Le Corre and Carey, we suggest that a system we call ''enriched parallel individuation'' is the most likely cognitive precursor of knowledge of the verbal counting principles. Gallistel argues that there are two serious conceptual flaws in the cognitive architecture we are proposing: it does not have representations of sets, and, for that reason as well as others, it does not have numerical content. He also takes issue with our enriched version of the parallel individuation system. In order to answer Gallistel's concerns, we now describe the systems of quantification we believe are available to infants (parallel individuation and set-based quantification), and then defend the view that, allowing a relatively minor enrichment, parallel individuation provides the cognitive architecture that supports the earliest meanings of the first four numerals, and that thereby provides the conceptual planks for the construction of verbal counting.

5. Representing sets

Parallel individuation is indeed the system of representation studied by Pylyshyn and others in their work on attentional indices (Pylyshyn & Storm, 1998; Scholl & Pylyshyn, 1999) and by Kahneman, Treisman, Luck and many others in their work on object-based attention and visual short-term memory (Cowan, 2001; Kahneman, Treisman, & Gibbs, 1992; Vogel, Woodman, & Luck, 2001). In parallel individuation, the individuals in small sets are represented in working memory by a set of symbols, one symbol for each individual in the set. This system of representation has a sharp capacity limit of 3 or 4 items. Thus, the representation of a set of 3 crackers might be {cracker, cracker, cracker} or {□□□}, depending upon whether the symbols for crackers are iconic or discrete. Gallistel appears to be unaware of the work of Feigenson and her colleagues (Feigenson & Carey, 2003, 2005; Feigenson et al., 2002; Feigenson & Halberda, 2004) that shows that preverbal infants can make models of at least two sets of individuals, each subject to the set-size limit on parallel individuation, and hold both in working memory at once. This work shows that the adult literature cited above does not fully describe the system of parallel individuation shared by infants and adults.

The system we call ''set-based quantification'' distinguishes individuals from sets of multiple individuals with no limit on the size of the sets represented, and supports quantification over these sets in terms of the contrasts explicitly represented in natural language systems of quantification (e.g., *singular/plural, some, all, more*). Much less work characterizes preverbal quantification over sets. What has been done shows that under some circumstances both human infants and rhesus macaques represent a singular/plural distinction (Barner et al., 2007; Barner et al., under review). It also shows that before their second birthday, English-learning toddlers have begun to explicitly quantify over sets with determiners and quantifiers like ''some,'' and ''a.'' Contrary to Gallistel's remark when he worries about wading into deep linguistic territory, ''a'' has numerical content as well as discourse content. It introduces *one* new individual into the discourse, in contrast to *some*; it is for that reason that, in some languages the morpheme for the indefinite singular determiner is identical to the numeral for one (e.g., ''un'' in French). Therefore, mastery of ''a'' does show that set-based quantification is available right around the time when children first learn ''one.''

We agree with Gallistel's clear arguments that there are no representations of number without some capacity to attend to and index sets. There is no denying that cardinal values are a property of sets. However, we disagree that the parallel individuation *system* cannot track sets. We reject this conclusion for the same reason we would disagree that the computations in the analog magnitude system are not carried out over sets – a conclusion that also follows from the logic of Gallistel's argument. Indeed, no model of the analog magnitude system, not even Gallistel's, includes explicit symbols for sets. We reject both conclusions because the representations in these systems would be useless if they were not somehow connected to representations of sets. For example, analog magnitudes can enter into computations of numerical order (e.g., Brannon, 2002; Brannon & Terrace, 1998, 2000). For the result of these computations to have any use in the world, the analog magnitude *system* must somehow keep track of the sets associated with each magnitude in the comparison. Knowing that approximately 8 is more than approximately 5 would be useless if the organism who computed this result could not somehow connect one magnitude with, say, this set of 8 raisins over here and the other with that set of 5 raisins over there. In fact, Gallistel's very point is that, without some notion of set, these computations could not even get off the ground because the sets, among other things, determine what is to be compared to what. For this reason, evidence that infants can use parallel individuation to choose a set of three crackers over a set of two crackers (Feigenson et al., 2002) implies that the parallel individuation *system* somehow keeps track of which set is *cracker, cracker, cracker* and which is *cracker, cracker*. Likewise, evidence that infants can use set-based quantification to distinguish a collection of more than 1 cracker from 1 cracker (Barner et al., 2007) implies that the system keeps track of which set is *singular* and which is *plural*.

Thus, when we include braces in our notations for the symbols in parallel individuation, we are not imposing representations only available to symbolically sophisticated adults onto the infant mind. To be clear, neither are we taking a stand on whether the child has an explicit symbol ''{}'' with the content ''set.'' Rather, we mean only to capture the set indexing and tracking capacities needed by parallel individuation, set-based quantification *and* analog magnitudes to support the numerical capacities they have been shown to support.

6. The numerical content of parallel individuation

While implicit or explicit representations of sets are necessary to create representations of number, they are not sufficient. Thus, having explained why experimental evidence warrants the inclusion of symbols for sets in the machinery available to parallel individuation and set-based quantification, we now turn to a discussion of how it represents number.

As Gallistel (1990) pointed out in his insightful discussion of mental representations, the content of a mental symbol is not only given by its extension but also by the computations it enters into. Thus, while it is true that the parallel individuation system does not have symbols for numbers, and may not have explicit symbols for sets – it may only have explicit symbols for individuals – it has numerical content because the computations its symbols enter into include numerical ones. First, representations of individuals are sensitive to numerical identity, creating new symbols when there are spatiotemporal or featural cues that a newly encountered individual is numerically distinct from an individual already represented in the model. The net result is that there is one symbol for each individual in a represented set (so long as the set-size limit on parallel individuation is not exceeded). Thus, the models maintain 1–1 correspondence between symbols in the head and individuals in the set.

Gallistel wonders whether we are taking the computation of numerical identity as evidence that parallel individuation has numerical content simply because we are confusing "identity" and "equality." Quite the contrary, it is he who does not seem to appreciate that the notion of numerical identity can be used to construct representations of the natural numbers. For example, in first order logic, one way of expressing the proposition "there are two individuals" is "$\exists x \exists y (x \neq y) \wedge \forall z (z = x \vee z = y)$." The "$=$" and "$\neq$" in this formula refer to numerical identity. That is, the formula requires that x be numerically distinct from y (i.e., x and y are different individuals) and that any other individual be numerically identical with x or y (i.e., there are no individuals other than x and y). This formula only applies to sets of exactly two individuals so, in this sense, it is a representation of two. Therefore, the claim that the system of parallel individuation has implicit numerical content is no more confused than the claim that the language of first order logic can be used to construct representations of the natural numbers.

In addition to tracking numerical identity, the parallel individuation system supports computations of one-to-one correspondence (among other quantitative computations), allowing infants to compute whether two sets are numerically equivalent or whether one has more (Feigenson, 2005; Feigenson & Carey, 2003, 2005). These computations also contribute to the numerical content of the parallel individuation system. Notice that the verbal numerals also depend on the computation of one-to-one correspondence to function as symbols for the natural numbers. Thus, Gallistel's snide remark that one might as well assign numerical content to an internal combustion machine is off the mark in two respects. Contents are assigned only to representations, not to entities in the world. And if computations of one-to-one correspondence can play a role in imparting numerical content to the verbal numerals, then they can equally do so for symbols in parallel individuation.

7. Enriched parallel individuation

Thus far we have argued that the infant system of parallel individuation is richer than Gallistel supposes, going beyond the attentional and object tracking mechanisms studied by Pylyshyn, Treisman, Luck and others to include working memory models of multiple sets, over which a variety of quantitative computations are defined. Yet, the representations of parallel individuation are still not sufficient to support the meanings of numerals. The problem is that the representations in this system, being working memory models of small sets, are too temporary to provide word meanings. This is why we proposed "enriched parallel individuation" as a plausible amendment to the parallel individuation system.

In enriched parallel individuation children create long term memory represents of small sets: {i}, {j k}, {l, m, n} or {a, b, c, d} where j, l, b, etc. are symbols for abstract or specific individuals, gathered into sets of 1, 2, 3 and 4. What makes these representations *of number* is that each is mapped onto the relevant numeral and deployed on the basis of one-to-one correspondence; e.g., {i} is mapped onto the numeral "one" and "one" is applied to sets represented in working memory models that can be put into 1–1 correspondence with {i}. Similarly, to support the meaning of "two," children create a long term memory model – e.g. {j, k} – map this to the numeral "two" and apply it to sets in working memory that can be put in 1–1 correspondence with this long term memory model. Again, notice that what gives these representations numerical content is their computational role; Gallistel is quite correct that {j, k} is itself just a representation of a set of an individual and another individual. Although this system is enriched relative to parallel individuation because its symbols can be stored in long-term memory, it does not require the creation of new representational *content*. It only uses computational capacities shown to be part of infant parallel individuation – the capacity to create models of sets using parallel individuation and the capacity to compare sets on the basis of 1–1 correspondence.

Gallistel wonders how we motivate a limit of 4 in this scheme. As he says, there is no motivated limit on the sizes of sets held in long-term memory. One could reel off the names of her 6 siblings from a long term memory representation, in spite of showing a strict limit of 3 items in tasks measuring visual working memory capacity. Thus, the limit on the enriched parallel individuation system cannot come from limits on long-term memory. What does limit the system is the fact that the application of a numeral to some attended set requires holding the attended set in working memory, so that it can be compared against the long term memory models that determine which numeral (if any) applies to it. Since these working memory models are limited to sets of 4, enriched parallel individuation can only support the application of "one" to "four." Notice that this explains why children can only assign numerical meaning to "one" to "four" prior to acquiring the counting principles. For example, a 2-year-old who saw (and heard) his mother refer to a set of five apples as "five apples" would not be able to learn much about the meaning of "five" because he cannot create a working memory model of the set that he could then store as a model for "five" in long-term memory.

This proposal has several advantages. First, enriched parallel individuation makes use of computational devices firmly demonstrated to be in the repertoire of preverbal infants. Second, the enriched parallel individuation system provides the very representations needed to support bootstrapping proposals that have been offered by several researchers to account for how the counting principles are constructed (Carey, 2004; Hurford, 1987; Klahr & Wallace, 1976). Last, and most important, the enriched parallel individuation hypothesis provides the best explanation of our data. To repeat, we find that the only numerals that can acquire numerical meanings via mappings to enriched parallel individuation – i.e. "one" through "four" – are the very ones that are learned prior to the acquisition of verbal counting. Based on a review of historical and cross-cultural linguistic data, Hurford (1987) arrives at the same conclusion regarding the historical origins of verbal counting; i.e., he argues that the first four numerals, and *only* these numerals existed as quantifiers prior to the existence of a count list. Moreover, we also find that the noise in estimating sets of 1–4 elements, both by subset-knowers and young cardinal principle-knowers, implicates parallel individuation rather than counting or analog magnitudes as underlying the meanings of the numerals deployed in our estimation tasks.

8. How the counting principles are constructed in childhood

We would like to end by commenting that our view of the learning process that builds the counting principles is an example of what Gelman and Lucariello (2002) call "structure mapping." Indeed, according to our bootstrapping proposal, children discover a structural similarity between two very different representations of linear order: next in a list of symbols, and next in a series of sets related by +1. This supports the induction of the general principle, "for any set whose cardinal value n is represented by numeral "n," the next numeral in the list represents the cardinal value of $n + 1$." Sarnecka and Carey (under review) provide evidence that knowledge of this generalization divides subset-knowers and cardinal principle-knowers. Thus, we endorse Gelman and Lucariello's basic insight concerning the structure of the learning mechanism that supports the acquisition of verbal counting, although we differ from them in what we take its input to be.

9. Conclusion

We take our enriched parallel individuation hypothesis to survive Gallistel's criticisms, and to provide the best account of the data concerning the acquisition of verbal counting. We would welcome a worked out proposal through which analog magnitude representations could support the acquisition of counting, while also accounting for all that is known about early numeral learning. In the absence of such a proposal, the bootstrapping accounts formulated in terms of representations akin to enriched parallel individuation are the only game in town.

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