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Language, gesture, and judgment: Children's paths to abstract geometry

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ABSTRACT

As infants, children are sensitive to geometry when recognizing objects or navigating through rooms; however, explicit knowledge of geometry develops slowly and may be unstable even in adults. How can geometric concepts be both so accessible and so elusive? To examine how implicit and explicit geometric concepts develop, the current study assessed, in 132 children (3–8 years old) while they played a simple geometric judgment task, three distinctive channels: children's *choices* during the game as well as the *language* and *gestures* they used to justify and accompany their choices. Results showed that, for certain geometric properties, children chose the correct card even if they could not express with words (or gestures) why they had made this choice. Furthermore, other geometric concepts were expressed and supported by gestures prior to their articulation in either choices or speech. These findings reveal that gestures and behavioral choices may reflect implicit knowledge and serve as a foundation for the development of geometric reasoning. Altogether, our results suggest that language alone might not be enough for expressing and organizing

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geometric concepts and that children pursue multiple paths to overcome its limitations, a finding with potential implications for primary education in mathematics.

Introduction

Human infants and many nonhuman animals, from primates to insects, show sensitivity to geometry as they navigate through familiar environments or recognize objects by their shapes (Cheng & Newcombe, 2005; Spelke & Lee, 2012). In contrast, abstract geometric reasoning develops slowly in children and remains fragile even in educated adults, who perform no better than adults with no education on difficult tasks of triangle completion (Izard, Pica, Spelke, & Dehaene, 2011) and overestimate what they have learned from Socratic dialogues (Goldin, Pezzatti, Battro, & Sigman, 2011). Why is geometry both so accessible to action and perception and so opaque to thought? Here we attempted to shed light on this question through studies of young children's communication about geometry by examining three distinctive channels: their decisions (choices), their speech, and their gestures.

Geometry core systems allow human infants to present a high sensitivity to the geometry of their environment—the distance, angle, shape, and sense relations among extended surfaces (Lee, Sovrano, & Spelke, 2010; Samuelson & Smith, 2005; Smith, 2009). This sensitivity to geometry appears to build on at least two distinct early developing systems supporting navigation and object recognition (Landau & Lakusta, 2009; Lee & Spelke, 2010). Potentially, by harnessing these systems, children might also develop conceptions of truly abstract geometry (Dillon, Huang, & Spelke, 2013). To learn formal geometry, children must gain explicit access to the information captured by these early developing systems. But how? In other domains, including the natural number concepts at the center of the elementary school mathematics curriculum and the mental state concepts at the center of children's intuitive psychology and abilities to learn from others, the development of language and gestures guides children to the concepts that adults find to be most useful and relevant. Nevertheless, language might be not enough to develop knowledge of geometry because the key properties of even the simplest geometric concepts—such as *point, line, angle,* and *parallels*—are not captured by ordinary language (Landau & Jackendoff, 1993; Landau, 2017). For example, although lines in geometry are one-dimensional, perfectly straight, and infinitely extended, the ordinary word *line* refers to extended bodies (e.g., clothes line, fishing line) with none of these properties (e.g., thick line, wavy line, short line). No terms of ordinary speech, moreover, refer to key properties of lines such as parallelism and perpendicularity. How then do children and adults gain access to the basic concepts of Euclidean geometry?

The inaccessibility and signature limits of geometry-based navigation and object recognition systems can still be discerned in human adults and older children (Dehaene, Izard, Pica, & Spelke, 2006). Adults struggle to understand basic geometric properties of triangles and squares despite an otherwise successful mastery of mathematics in secondary school and college (Goldin et al., 2011). Older children become aware of the simplest properties of triangles, such as the relationships between the sizes of their angles, only during adolescence (Izard et al., 2011). Nevertheless, humans transcend these early systems of geometry in many contexts. Adults combine representations of distance, direction, relative length, and angle for a wide range of purposes, including explicit geometric reasoning (Dehaene et al., 2006; Izard et al., 2011). The current research begins to ask how adults come to accomplish this feat, and why it emerges so late in children, by analyzing how younger children reason about geometry in three distinctive channels: choices, speech, and gestures.

Gestures and words can convey different, and often contradictory, information

For more than three decades, researchers have investigated the role of co-speech gestures in the development of knowledge (Goldin-Meadow & Alibali, 2013; Goldin-Meadow, Wein, & Chang, 1992; LeBaron & Streeck, 2000; McNeill, 2005; Riseborough, 1982). When spoken languages are not available, gestures can assume linguistic forms and functions, as in the case of sign languages (Goldin-Meadow, 2010, 2014). But for speakers of oral languages, gestures accompany speech and provide an additional representational format that can promote learning (Hunsicker & Goldin-Meadow, 2012; Iverson & Goldin-Meadow, 2005). The gestures accompanying speech often foreshadow the development of later concepts. For instance, presented with challenging numerical problems or in the Piagetian classical conservation of number task, children's choices and words sometimes are guided by incorrect strategies, whereas their gestures reflect the correct strategy (Church & Goldin-Meadow, 1986; Rowe & Goldin-Meadow, 2009). This mismatch between gesture and speech may manifest a state of transitional knowledge; children's gestures might be related to their zone of proximal development, reflecting a higher understanding in their implicit knowledge (Goldin-Meadow, 1986; Goldin-Meadow, Alibali, & Church, 1993).

Moreover, children improve their explicit numerical justifications and performance (choices) if they are shown appropriate gestures or are led to produce the appropriate gestures themselves (Alibali & Goldin-Meadow, 1993; Göksun, Hirsh-Pasek, & Golinkoff, 2010). The simple request to gesture while verbally explaining a math task led children to select more mature problem-solving strategies, making them more receptive to a later math lesson and consequently improving their performance (Pine, Lufkin, & Messer, 2004).

Implicit actions and choices may reflect knowledge that is not accessible to language or explicit representations

Studies combining choices with think-aloud procedures have identified dissociations throughout development between implicit and explicit mastery of abstract concepts (Broaders, Cook, Mitchell, & Goldin-Meadow, 2007; Efklides, 2008). For example, in a place-change false belief task, 2- and 3-year-old children sometimes look to the location where another person last saw an object while stating verbally and, incorrectly, that the person thinks the object is in the place where the child knows it to be (Baillargeon, Scott, & He, 2010; Ruffman, Garnham, Import, & Connolly, 2001). These behaviors have been reported in a variety of paradigms that include violation of expectation (Onishi & Baillargeon, 2005) and anticipatory looking tasks (Senju, Southgate, Snape, Leonard, & Csibra, 2011; Southgate, Senju, & Csibra, 2007). Moreover, recent results suggest that the development of implicit and explicit false belief might follow different trajectories. Whereas explicit false belief tasks might depend on language and executive functions, implicit false belief tasks might not (Grosse Wiesmann, Friederici, Singer, & Steinbeis, 2017).

Another dramatic dissociation between explicit and implicit mastery of abstract concepts occurs in cases where people respond correctly but exhibit no gestures, actions, or speech in support of that performance. In blindsight studies, people claim to lack any knowledge of a visible object's presence, but if encouraged to guess whether the object is present (verbal report) or to point where they think it might be (action/choice), they sometimes succeed (Kolb & Braun, 1995; Marshall & Halligan, 1988), although not on every trial (Morgan, Mason, & Solomon, 1997). Do young children similarly make forced-choice judgments that reflect correct geometric reasoning without being able to express the reasons for these choices explicitly when they are encouraged to guess?

Goal of the current study

This study aimed to discover how implicit and explicit geometric knowledge develops in children by simultaneously measuring three different channels—*choice, speech,* and *gestures*—during a simple geometric task. Across a range of geometric properties, the study sought the developmental path of explicit knowledge. It asked whether a single developmental pattern characterizes the development of children's understanding of all geometric properties or whether understanding of different geometric properties develops along different paths. To evaluate possible developmental pathways involved, given the cross-sectional nature of the study and based on the hypotheses listed below, we examined (a) whether geometric concepts are scaffolded through choices first and then later the ability to reflect the same knowledge in speech or gesture appears and (b) whether accurate geometric gestures emerge first and then children express their understanding of those geometric concepts in words or choices.

To this end, children were presented with an oddity task based on past research (Dehaene et al., 2006; Dillon et al., 2013). Briefly, they were asked to examine a set containing six cards, to locate the odd one that did not belong with the rest (*choice*) and to explain the reason behind their choice (through *speech* and/or *gestures*). In each of the set of cards, five of the figures were similar with respect to their geometrical properties (e.g., angle size, global shape, parallelism, left/right symmetry), whereas one figure differed on that property. After children indicated their choice, several patterns could be observed given the geometrical properties exhibited in the sets presented. Potentially, recognizing some traits may have a different trajectory than making judgments about others (Lee et al., 2010). The challenge was to be able to separate these abilities methodologically.

Our analytic strategy followed two working hypotheses:

Hypothesis 1 (*H1*). Based on research providing evidence that for preverbal cognitive systems, such as number and other domains, infants can perform correctly in a task—for example, they can discriminate between large numbers of objects but without being able to explain their reasoning in words or gestures (Carey, 2009; Kinzler & Spelke, 2007; Spelke, 2010)—we hypothesized that younger children's implicit knowledge of geometry will allow them to solve geometric problems even when they speak and act (gesture) as if they have no understanding of the geometric relationships about which they reason. Explicit knowledge of some geometric concepts will be scaffolded in implicit knowledge (through choices) prior to their articulation in either words or gestures (Fig. 1, left).

Hypothesis 2 (*H2*). Based on studies of the role of gesture in children's mathematical reasoning (see Goldin-Meadow, 2014, for a review) and previous results in mismatch between speech and gestures, showing that children can express different information in their gestures from what they conveyed in their words (Alibali & Goldin-Meadow, 1993; Perry, Church, & Goldin-Meadow, 1988), we hypothesized that children will reveal implicit knowledge of emerging geometric concepts through gestures that they fail to express in either choices or verbal justifications. Children may express their understanding of emerging geometric concepts through gestures before they express these concepts in words or use them to guide deliberate actions (choice) (Fig. 1, right).

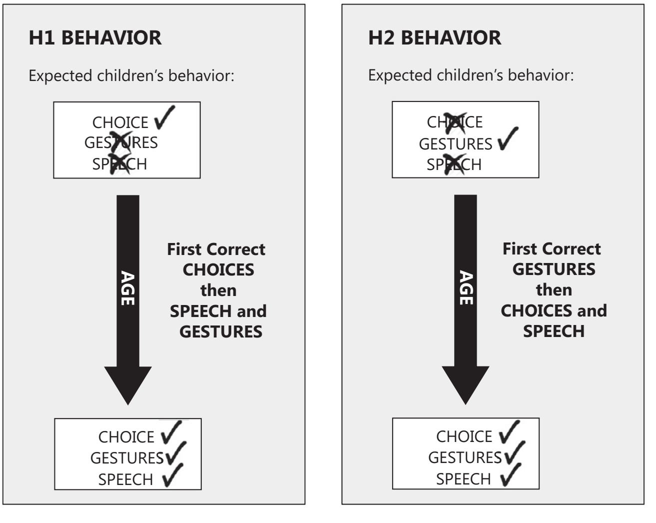

Fig. 1. Model of progressive availability of geometric concepts for choices, speech, and gestures. H1 behavior: Implicit knowledge of geometry can be expressed by choices when guessing is encouraged. H2 behavior: Implicit knowledge of geometry can be expressed through gestures even when choices and words are absent or wrong. Both behaviors will contribute to the development of abstract geometry.

Consequently, we expected two different developmental patterns: H1 behaviors guided solely by implicit knowledge will diminish with age, given that with the development of language children will provide consistent explanations and gestures for their correct responses even if they lack the precise geometric vocabulary, and even when children respond incorrectly, they will present H2 behaviors in which they express through gestures some aspects of their implicit geometric understanding. With time, suitable gestures will drive the consolidation of geometrical knowledge in both choices and speech. Therefore, with development, correct recognition of geometric concepts will be progressively available for all channels—choices, speech, and gestures. The developmental time course predicted for both hypotheses is summarized in Fig. 1.

Method

Participants

Of 132 children who participated in the current study, 109 children completed the game: preschool children aged 3 or 4 years (n = 22; 14 girls; M_{age} = 50.2 months, range = 39–59) and 5 years (n = 26; 14 girls; M_{age} = 66.7 months, range = 62–75) and primary school children in first grade (n = 29; 15 girls; M_{age} = 83.1 months, range = 77–88) and second grade (n = 32; 15 girls; M_{age} = 95.3 months, range = 90–102). The other 23 children were left out of the analysis, 1 due to a technical mistake, 14 for failure to pass the practice trials described below (8 from the 3- and 4-year-old group, 4 from the 5-year-old group, and 2 from the 7- and 8-year-old group), 3 for failing to complete the protocol, and 5 for declining to play the game.

Recruitment letters were sent to several schools in Buenos Aires, Argentina, and different appointments were made. Finally, the research was conducted in two medium high socioeconomic status schools in Buenos Aires. Children's parents or legal guardians gave signed voluntary consent. The consent form was previously approved by the ethical committee of CEMIC (Centro de Educación Médica e Investigaciones Clínicas, Protocol No. 683).

Experimental design

During the study, each child was presented with an oddity task based on past research (Dehaene et al., 2006; Dillon et al., 2013). Children were brought into a room and sat in a small chair 90° from the experimenter facing a hidden camera (Bloggie Sony TS20), which recorded the entire session. The experimenter taught the children how to play the game by first presenting practice sets (see Supplementary Fig. 1 in the online supplementary material for complete order presentation), explaining that, in order to win, children needed to find the card with the odd element within the set. Both the nature of the figures and the property uniting five of the six figures varied across trials; the correct properties to solve the game were sense or direction, topology, distance, angle or parallelism (between only two lines), and shape (form of the object defined by the external boundaries). All 20 cards used in the task can be found in Supplementary Fig. 1. There was always only one geometric property that categorically distinguished one member of the set from the others and, therefore, only one correct response. In all, three practice trials were presented to 3- and 4-year-old preschool children and four practice trials were presented to older children (5–8 years old). If children performed correctly on more than 50% of these trials (at least two of three trials for the younger children and three of four trials for the older children), the game continued with the test trials. If not, the game stopped after the practice trials and those children were not included in the study. Given that there were six different options per set (and only one correct response), chance level was at 16.66%; therefore, if performance on practice trials exceeded 50%, then children's responses were above chance level.

If children passed the practice sets, they continued playing with the test trials. The test trials consisted of 10 sets of cards for the preschool children and 16 sets of cards for the school-aged children (see Supplementary Fig. 1 for order presentation). The number of sets was reduced for the younger children because pilot testing revealed that it was hard for them to sustain attention in the 16-set game. Nevertheless, all geometric properties mentioned were assessed in both groups. For the test trials, after children's choices and before any feedback, the experimenter asked, ''And how did you know that this one [pointing at the card chosen by the children] was the odd one?'' When children finished their explanation, the experimenter asked, ''Okay, anything else?'' Next the experimenter gave informative feedback either by smiling and saying ''Very good!'' or saying ''Oops! This one is the correct card'' [while pointing at it]. The experimenter never told children about the correct properties to solve a set, nor did the experimenter discuss its geometrical properties. Once children finished playing with all the sets of cards, the experimenter said, ''Wow, you did an excellent job!'' and the session ended.

Coding

Every participant's session was recorded, and then it was coded for subsequent analysis. All three channels—choices, speech, and gestures—were analyzed through video records using a predefined list of events. These lists indexed possible gestures and terms given by children during their answers (see Supplementary Tables 1 and 2 for complete descriptions) and choices (see Supplementary Fig. 1 for correct choices and complete sets of cards). The types of events were selected before video coding according to the traits present in each set of cards. Videos were analyzed independently by two assistants with extensive training as coders who were blind to all aspects of the study. A total of five videos were coded by both assistants using the predefined list of events for both gestures and speech, and the first author first checked the coding criteria and intercoder agreement. Coders agreed on 96% of the choices.

The correct response (CR hereafter) was always only one of the six cards presented in the set arrangement, and its geometric traits made it qualitatively different from the rest. The CR is marked with a green box in Fig. 2 and is labeled as 1 using insets in Supplementary Fig. 1, but neither the green box nor the labels were present when the children played with the cards.

We considered responses to both the gestures and speech that followed the experimenter's two questions (described above) as children's responses. Note that responses were produced before the experimenter gave any feedback about the choice made, and feedback included only one of the previous statements mentioned in the ''Experimental Design'' section. Speech and gestures were coded separately. This division was necessary because speech would provide information that conveys meaning discretely, relying on codified words and grammatical interactions, whereas gestures would portray meaning in a global manner, relying on visual patterns (Goldin-Meadow & Alibali, 2013; McNeill, 2005).

Therefore, for every participant, assistants coded (a) children's card choice, (b) children's gestures performed, and (c) children's speech. Coding was based on the following guidelines. First, for *choices* to be coded, cards in each of the sets were labeled using numbers as follows: the CR always was number 1, and the remaining cards always were from 2 to 6 (see Supplementary Fig. 1 for numbers displayed in each set). It is worth mentioning that during the game with the children these numbers were never present; they were only used by assistants for coding children's choices. For this, technicians received a copy of Supplementary Fig. 1 and coded which card was chosen by each child on each trial during all sets played. Second, *gestures* elicited by children to respond to the experimenter's questions were coded using a predefined list. For gestures to be considered as isolated communicative symbols, they needed to meet certain requirements, namely that (a) a gesture needed to be directed to the experimenter who was interacting with the children, (b) the gesturer needed to be assured of the partner's attention doing the gesture, (c) the gesture could not consist of a direct manipulation of the cards used in each set but instead needed to be empty-handed either in the air or over the cards, and (d) the gestures started when children began a hand movement following the above criteria to produce an iconic representation and ended when children stopped moving their hands or began either a nonreferential gesture or an iconic representation of a different geometric trait. Using these criteria, we isolated gestures from the stream of motor behavior. Movements of the hands, either in the air or directly over the cards, that contained substantive information that listeners could extract were singled out for coding. The predefined gesture list included hand movements that indicate, portray, or represent size, shape (children could refer to different shape types such as hole, sharp, and pointy), angle, distance (between the elements on the card), position (relative display of the elements on the card), orientation (to mark right and left or symmetrically), length, and parallels (see Supplementary Table 1 for the complete list

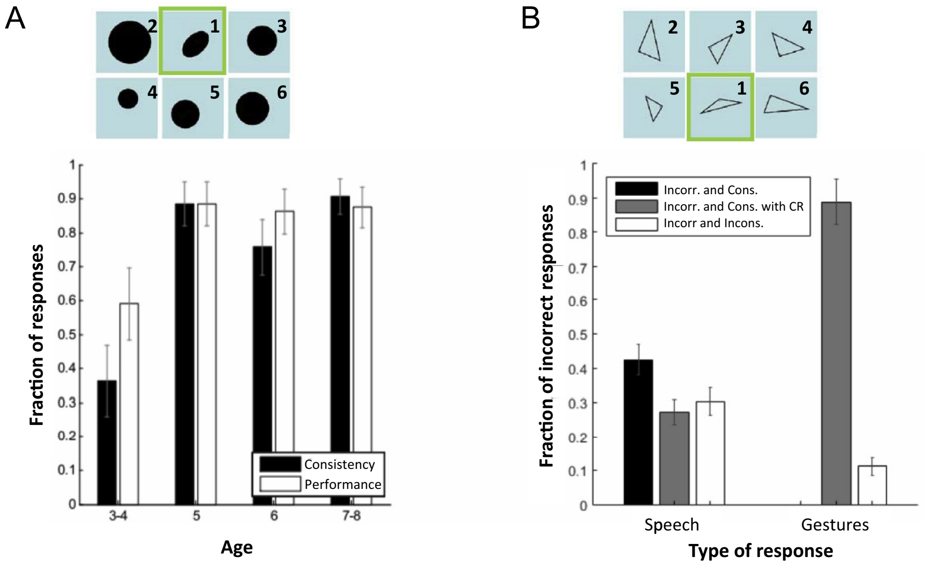

Fig. 2. Representative example of different expressions of geometric knowledge. Top: Set example images (CR: green box) for the following. (A) Example 1: Effect of age on performance and consistency. Performance (fraction of CR) and consistency (proportion of consistent justifications as a fraction of all justifications given) measured for this particular set for all age groups are shown. (B) Example 2: Fraction of incorrect responses as incorrect choice but justification consistent with the incorrect choice (Incorr. and Cons.); incorrect choice and justification consistent with the CR, not the choice made (Incorr. and Cons. with CR); and incorrect choice and justification inconsistent with children's choice or the CR (Incorr. and Incons.) (with $n_{gestural} > 0/n_{speech} > 0$) for speech and gestures. Data were normalized to the total speech and gestural responses, respectively. Standard errors are represented in the figure by the error bars attached to the columns. (For interpretation of the references to color in this figure legend and the text description of the figure, the reader is referred to the Web version of this article.)

of gestures and examples). As mentioned above, these traits were selected because of the geometrical properties present in each of the sets of cards used during the game and also based on descriptions used in previous studies (Dehaene et al., 2006; Dillon et al., 2013). For each gesture, the moment of occurrence and duration were coded. Third, for *speech,* each justification generated verbally by children to respond to the experimenter's questions was coded using a predefined list of words or terms. The list of expressions was built in the same manner as the list of gestures, based on the possible card features to which children might refer. Possible expressions accepted included not only formal geometric terms used by the children (e.g., parallel, oval, rectangle, triangle) but also concrete examples (e.g., describing an oval as "like an egg" and a triangle as "like a ramp"), and demonstrative adjectives (e.g., "like this," accompanied by a gesture) were coded as admitted responses. In this way, we attempted to code every verbal response that reflected children's geometric knowledge and not only their mastery of geometric vocabulary (see Supplementary Table 2 for the complete list of terms, including examples for each category). For each response, the moment of occurrence and duration were coded.

Consistency

The consistency of children's responses was coded for each child on each trial. To code consistency, a predefined list of events, generated by the authors for gestures and speech for each set, was used (see Supplementary Table 3 for consistency in each set and Supplementary Tables 1 and 2, for gestures and speech events and examples). This list described the children's responses (speech and gestures) that were considered consistent with the CR for each set of cards. For example, in Set 10 (see Supplementary Fig. 1 for set arrangement), the CR was the only card with an element facing toward the left; in this case, if children chose that card and pointed toward the left or right and/or gave arguments such as ‘‘This one is looking to the other side" and ‘‘All of these are facing this way and this one is facing that way" when justifying choosing that card, then the response was coded as *consistent*.

Accordingly, responses were coded as *consistent* with children's judgment if children produced at least one explanation or one gesture referring to the traits present in the card they chose. Equally, *consistent with the CR* was coded if children, independent of the card they chose, produced at least one verbal description (speech) and/or gesture referring to the distinctive trait of the CR. Finally, *inconsistent* with children's judgment was coded if children did not produce, through either speech or gestures, a reference to the traits present in the card they chose.

In addition, responses were coded combining ‘‘correctness" and ‘‘consistency" of the response given. In these cases, a response was coded as *correct* and *consistent* if children chose the CR and referred to the traits from the CR in speech and/or gestures; as *correct* and *inconsistent* if children chose the CR but gave no justification from the predefined list of events, through either speech or gestures, for that particular set; and as *incorrect* and *consistent with the CR* if children picked a card different from the CR but engaged in explanations or gestures referring to the distinctive trait of the correct card according to the predefined list. Finally, responses were coded as *incorrect* and *inconsistent* with children's response if, after choosing an incorrect card, children gave a justification (speech and/or gestures) that did not include the traits from that card.

Model

A random model was built to assess the null hypothesis that children produced gestures and speech to argue at random. For oddity games (like the one used in this study), in which the base rate of children's iconic gestures is so constrained by the nature of the task, the use of a random model is essential. This comparison model enables the discrimination between, for example, children who gesture about sense or talk about shapes on every trial during the game irrespective of the underlying correct trait of the set and children who produce gestures and explanations consistently with their choices and the specific trait present in the cards.

Accordingly, as a first step, we pooled all arguments produced by all children in all trials. Then, for each trial performed by a child, we generated 1000 ‘‘random trials" reproducing the original trial but replacing each verbal argument by an argument picked randomly from the pool of all verbal arguments. The same replacement was performed for all gestures. Because these replacements might modify the consistency of the trials, for each card set the average consistency in random trials was compared with the average consistency in actual trials.

Results

The primary goal of this research was to elucidate the development of explicit and implicit geometric reasoning. To this aim, children's performance in the oddity task (card choices), their speech, and the gestures they conveyed during the justification of their choices, prior to receiving feedback, were measured and analyzed. This methodology served to determine similarities and discrepancies in the information conveyed in these three channels and to measure the possible different developmental trajectories of explicit and implicit geometric representations.

First, consider two concrete examples that illustrate the analytic strategies used throughout the study to test the main predictions of the working hypotheses. In the following sections, these observations are quantified with statistical analyses across all trials and age groups in the study.

Representative examples

The first example illustrates the justifications that accompanied children's responses to a set in which the CR is the only oval among the circles (see Fig. 2A, green box). Consistency in children's responses can be independent of the correctness of their choice; justifications were considered consistent when either children's speech or their gestures appealed to the particular geometric property that distinguished the chosen card from the others. For instance, in this example, if a child chose Card 1 (CR) and referred to the shape of an egg during the explanation, then the response was considered consistent. According to H1, correctness and consistency will show different developmental trajectories with age. As predicted by H1, younger children showed a significant fraction of correct choices (~60%), but only half of those responses were accompanied by consistent justifications (Fig. 2A). Thus, younger children reasoned correctly about geometry and chose the correct card without being able to justify, through speech or gestures, the reasons for their choice. In contrast, older children showed comparable fractions of correct choices and consistent justifications (Fig. 2A).

The second example involves a set in which the odd card is the only one that does not contain a right triangle (Fig. 2B, green box) and finding the correct solution requires sensitivity to angle or shape. Altogether, about half of the children responded correctly on the trials (54%) (percentages for different age groups: 5-year-olds, M = 74.5%; 6-year-olds, M = 54.2%; 7- and 8-year-olds, M = 51.7%). After children made a mistake, only 40% of the explanations verbally produced were consistent with the error they had made (Fig. 2B). In contrast, as predicted by H2, nearly all the gestures produced during incorrect responses (IRs hereafter) conveyed notions of shape or angle (nearly 90% of the fraction of IRs), which reflects implicit geometric reasoning for the CR (Fig. 2B). Hence, children may convey correct geometric reasoning with gestures that they are unable to explain with words or choices.

These examples in specific sets illustrate how the data conform to the predictions of the working hypotheses. The next sections present tests of these predictions across the ensemble of all the cards and ages.

Developmental trajectories of implicit and explicit geometry representations

To test the hypothesis that younger children can make correct geometric choices even when they are unable to describe the reasoning that led to their choices (H1), the correctness of children's choices on each trial was measured together with the consistency or inconsistency of each choice with the correct geometric trait needed to solve the problem. As predicted, for the youngest group about half (40%) of the choices were correct, whereas fewer (20% of the responses, speech and/or gestures) were consistent with the correct judgments (Fig. 3A). With age, both performance and consistency increased; for the oldest children in this study, performance reached values close to 80% with very similar levels of consistency (Fig. 3A). To quantify these observations, a one-way analysis of variance (ANOVA) was performed on the difference between performance and consistency as the dependent variable, with age as the main factor, finding a significant effect of age, $F(48, 109) = 2.66$, $p = .0002$, and post hoc t tests showed that the difference between performance and consistency became smaller with age [Performance × Consistency for each age group: 3- and 4-year-olds, $t(21) = 5.785$, $p = 9.648\text{E}{-06}$; 5-year-olds, $t(25) = 1.399$, $p = .174$; 6-year-olds, $t(28) = -0.147$, $p = .884$; 7- and 8-year-olds, $t(31) = -0.072$, $p = .943$]. Between 3 and 5 years of age, a significant fraction of the children engaged in correct geometric reasoning; they selected the correct card without explaining the reasons for their choices. At 7 years of age, most children both solved these geometric problems and provided appropriate arguments (with varying levels of gestural or speech finesse) about the reasons behind their choices. An independent analysis of gestural and speech consistency for each age group revealed low gestural consistency with the CR (in 5% of the total justifications) for younger children and nearly half (40% of the total justifications) for the oldest children, whereas speech consistency changed more (from 10% to 70%) within these ages (see Supplementary Fig. 2 for a more detailed description).

The increase in consistency was accompanied by a decrease in the use of elusive strategies during justifications such as saying "I don't know," linear regression $F(1, 108) = 13.035$, $r = .109$, $p = 4.67\text{E}{-04}$ (Fig. 3B).

Altogether, these results suggest that H1 behaviors decrease with the development of language and gestures; children became able to consistently justify their choices.

Gestures express children's implicit knowledge of geometry

This section focuses on trials in which children made an error, an incorrect choice, asking whether the gestures and the speech produced during children's justifications were consistent either with chil-

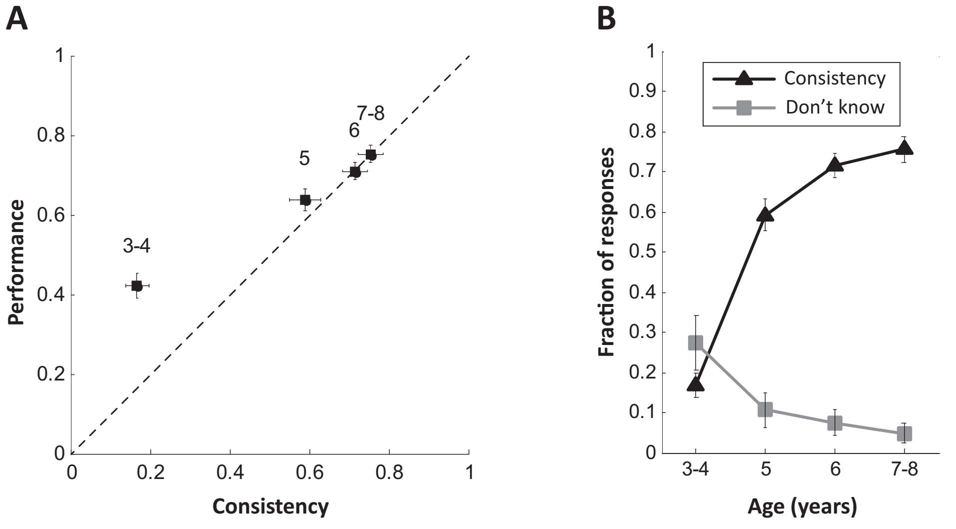

Fig. 3. Developmental changes in performance, consistency and elusive responses. (A) Performance versus consistency for all age groups. The proportion of consistent responses (as a fraction of all justifications given through speech and gestures) was measured for all sets in each age group [represented as number(s) next to each point]. (B) For each age group, the fraction of "don't know" responses was quantified and compared with the proportion of consistent responses.

dren's own IR (*consistent with IR*) or with the CR (*consistent with CR*). To this aim, the difference (Δf) between children's gestural and verbal consistency was compared with the consistency measures generated by our random model that shuffled the matching among choices, speech, and gestures, preserving the statistics of each of these variables. Positive values of Δf would indicate that children's justifications (verbal or gestural) aligned more consistently with the correct choice than would be expected by random shuffling of their aggregated responses. For instance, a child whose justifications are fully consistent with the CR will verbally express terms or will elicit gestures about shape in the example set presented in Fig. 2A. In the model, arguments will be obtained randomly across trials, and hence the level of consistency will decrease and Δf will be positive. The random model will allow the discrimination between children who gesture or speak about the same properties on every trial, independent of the traits present in the cards and irrespective of their underlying conceptual understanding, and those children who produce meaningful gesture and/or speech referring to the correct specific traits from the set.

When quantified, speech in incorrect trials was significantly more consistent with the given incorrect choice than would be expected by chance (M = 0.31, SEM = 0.08), t test, Consistency|Incorrect, $t(9)$ = 3.734, p = 4.67E−03 (Fig. 4). The vast majority of these errors correspond to sets in which children tend to speak about size and to erroneously choose a card that depicts a size outlier, whereas children's gestures convey the correct geometric property, as we describe below. Across the full sample, children did not engage in speech that was consistent with the CRs on trials in which their own response was an incorrect choice at greater than random levels (M = 0.03, SEM = 0.07), t test, Consistent With CR|Incorrect, $t(15)$ = 0.748, p = .466) (Fig. 4). Even the oldest children did not verbally justify IRs with geometrically correct terms (see Supplementary Fig. 3 for more details).

The pattern of gestures was opposite to the one observed for speech. First, and quite surprisingly, in error trials (incorrect choices) children did not make gestures that were consistent with the IRs they gave (M = 0.03, SEM = 0.05), t test for Consistency|Incorrect, $t(9)$ = 0.726, p = .486 (Fig. 4). Because the bulk of errors correspond to children erroneously choosing the cards with size outliers, this meant that children do not produce gestures about size (e.g., creating a narrow gap between the fingers to refer to something quite small) even though they talk about size when giving reasons for their judgments.

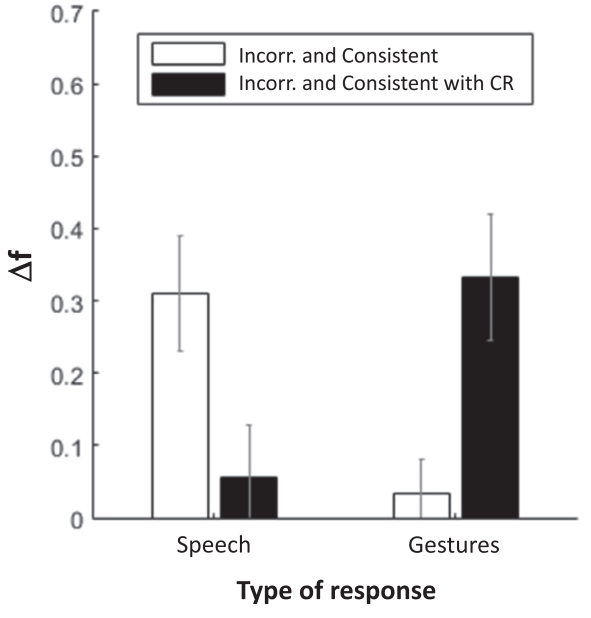

Fig. 4. Gestural and verbal consistency for incorrect responses. Differences in consistency (Δ*f*) between children's and the random model's generated responses when justifying an incorrect response are shown. Consistency with the incorrect choice and expressing correct geometric reasoning are plotted for both speech and gestures. Standard errors are represented in the figure by the error bars attached to the columns.

As predicted by H2, children revealed implicit geometric knowledge through gestures that went above and beyond what they could indicate either by their choices or by their speech. On a significant fraction of trials, gestures expressed the geometric dimensions of the CRs during incorrect choices ($M = 0.33$, $SEM = 0.09$), t test for Consistent With CR|Incorrect, $t(15) = 3{,}661$, $p = 2.316\text{E}{-03}$ (Fig. 4).

A breakdown of the analysis by age served to examine developmental changes in the discrepancy between children's choices and their justifications in both speech and gestures (see Supplementary Fig. 3 for more details). The analysis revealed a striking developmental pattern consistent with our predictions. Gestures provided evidence for sensitivity to geometry at all ages from 5 years onward; children's gestures tended to reveal correct geometric reasoning even when their choices and their speech did not. At the youngest age, sensitivity to geometry was reflected only in children's choices (H1). Nevertheless, the same trend to communicate the reasons for their choices through gestures rather than words (H2) was also present (see Supplementary Fig. 3).

Developmental trajectories of representative examples

The data presented in the previous sections show that children may reach explicit knowledge through at least two trajectories. First, given sets of cards for which the youngest children had correct performance without being able to express, through speech or gestures, the reasons for their choices, the older children are able to justify their choices with accurate consistent arguments that reveal explicit knowledge of the geometric concepts involved. For these sets, children's ability to choose the CR, their implicit knowledge, occurred before children's ability to express the reason behind their choices through language or gestures. This pattern was found to occur, in general, on trials probing sensitivity to shape. Second, for different sets of cards, young children first provide gestures that are consistent with the CR but are accompanied by incorrect choices and inconsistent language. In resonance with embodiment theories, consistent gestures appear before correct choices or consistent speech. In general, this pattern was observed on trials probing sensitivity to angle.

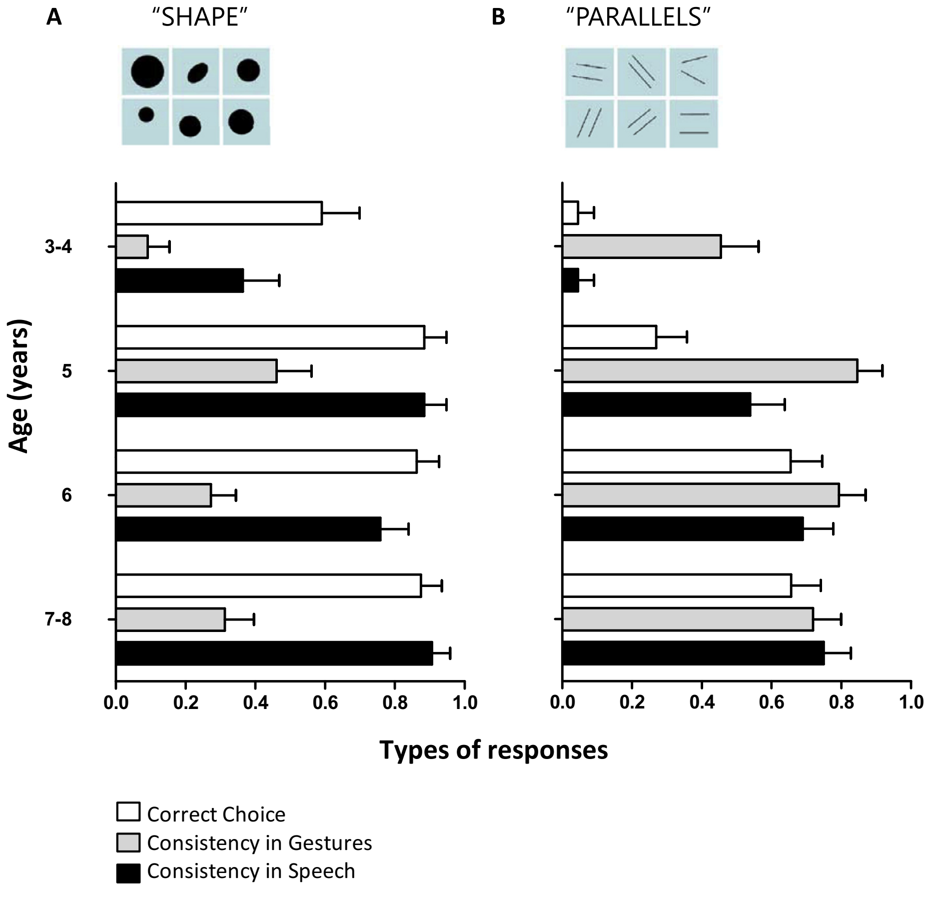

Fig. 5. Developmental trajectories of representative examples. Concrete examples of the development of H1 and H2 behaviors throughout development for two different geometric concepts are shown for shape (A) and angle (B). Bars represent correct card choices and consistency with the correct responses for gestures and speech.

Consequently, to further examine these patterns, two concrete examples served to explore whether explicit knowledge of some dimensions of geometric knowledge is scaffolded in implicit knowledge through choices, whereas implicit knowledge of other dimensions is supported in gestures. The results consistently obtained fit with the simple model, presented before and described in Fig. 1, of how different channels of geometric knowledge develop in at least two distinct trajectories (Fig. 5).

Discussion

The development of implicit and explicit knowledge of geometry was investigated during a simple geometric judgment task by measuring three channels that can provide evidence of children's knowledge: choices, speech, and gestures. The analyses sought to determine whether there were different developmental paths for the emergence of different explicit knowledge of geometric concepts, with two critical findings.

First, although the ability to solve the task prominently increases with age, younger children can solve a group of geometry problems without being able to describe in words, or gestures, the reasoning behind their choices. It appears that they may be particularly apt to succeed at this odd-one-out task on trials testing the underlying property of object shape—a key property for naming of object kinds (Landau & Jackendoff, 1993; Samuelson & Smith, 2005; Smith, 2009) but not a property that is invariant over the geometric properties, such as position, that children name. This finding accords with the evidence that object shape is a key contributor to success on the current task both for children and for adults (Lovett & Forbus, 2011). It also contributes to current understanding of the role of early emerging systems of knowledge, on the one hand, and of language, spatial symbols, and formal education, on the other, in the development of mathematical reasoning.

Preverbal systems serve as foundations for mathematical concepts, symbols, and language. There is strong evidence that systems of enumeration, such as the approximate number system (Dehaene, 2011; Halberda, Ly, Wilmer, Naiman, & Germine, 2012) and the system that supports shape-based object recognition (Landau & Lakusta, 2009; Lee & Spelke, 2010), are present in human infants and continue to function in older children and adults in diverse cultures (Feigenson, Dehaene, & Spelke, 2004). Nonetheless, language and gestures also play a role in the development of numerical and spatial reasoning (Hermer-Vasquez, Moffet, & Munkholm, 2001; Loewenstein & Gentner, 2005; Pruden, Levine, & Huttenlocher, 2011; Pyers, Shusterman, Senghas, Spelke, & Emmorey, 2010). Studies of deaf children who have not mastered a conventional sign language have found a close relation between numerical and spatial language and the ability to think about exact number and space in nonlinguistic tasks (Hyde et al., 2011; Spaepen, Coppola, Flaherty, Spelke, & Goldin-Meadow, 2013; Spaepen, Coppola, Spelke, Carey, & Goldin-Meadow, 2011). Moreover, evidence implicates gestures and spatial symbols in the development of knowledge of number and geometry (Cappelletti, Butterworth, & Kopelman, 2012; DeLoache, 1991).

Although these findings suggest that both speech and gestures have an important role in the development of spatial and numerical concepts at the foundations of arithmetic, the current findings provide an upper bound to these influences. Children may show astute geometric conceptual understanding even when they are incapable of explaining their reasoning with words or gestures, even simple ones. The demonstration that young primary school children have the capacity to organize mathematical concepts without the ability to verbalize or gesture them resonates with earlier findings showing that, in specific adult populations, numerical and geometrical abilities might not depend only on language. Capacities for advanced mathematical reasoning have been reported to survive brain injuries that cause profound aphasia (Amalric & Dehaene, 2016), and mathematical reasoning by professional mathematicians is accompanied by intense activation in brain areas involved in the core representation of number and space but little activation in areas involved in language processing (Karmiloff-Smith, 1994).

Second, the emergence of certain geometric concepts can be associated with cognitive advances in children's gestures but not with their performance or their language. Just as developmental changes in children's pointing and gaze following contribute to the development of a new understanding of communication and social cognition (Clements & Perner, 1994; Lee & Kuhlmeier, 2013), iconic gestures—hand movements that represent geometric features that distinguish one card from others—accompany children's developing understanding of geometric concepts and relations. Children often provide gestural demonstrations that indicate implicit knowledge about the correct geometric dimension even when their choices and speech reflect errors in reasoning. A wealth of research suggests that gestures can convey ideas and reveal understandings that children are not yet able to express explicitly (Church & Goldin-Meadow, 1986; DeLoache, 1991; Göksun et al., 2010; Kendon, 1988), suggesting that children sometimes develop implicit understanding earlier than their ability to perform or speak correctly in a task (Clements & Perner, 1994; Goldin-Meadow et al., 1993; Lee & Kuhlmeier, 2013). In this context, iconic gestures might contribute not only to express concepts that might not yet be ready to express in other channels but also to communicate aspects of children's cognitive states to others.

Three different channels in this study painted a rich picture of young children's developing knowledge of geometry: (a) performance in a forced choice, as in a typical school exam; (b) speech, which is often solely considered for grades in schools; and (c) gestures conveyed during the justification of children's choices. Although gestures are almost never used to grade children's knowledge, either in school or informally, they can be strongly associated with children's progressive grasp of formal geometry. Together, these results build on previous findings to further elucidate how implicit intuitions may serve as a scaffold for language and how gestures may help to scaffold choices and speech that guide the development of knowledge.

Perhaps even more important, the findings raise the question of whether the teaching of geometry and the evaluation of children's school learning should be revised to reflect the limits that performance and speech sometimes present and the power of gesture in making new geometric concepts accessible. The data presented here strongly suggest that, geometric concepts become explicit through at least two different trajectories, depending on the particular concepts involved. Even though, our results are too limited to warrant any changes in education policy today, they may motivate both the scientific and educational communities to reflect on this possibility and to test it more directly through research in preschool or elementary school settings. Such research may provide new insights into the development of new geometric concepts in children, and into the mental representations and processes by which those concepts are deployed in educated adults.

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Appendix A. Supplementary material

Supplementary data associated with this article can be found, in the online version, at https://doi.org/10.1016/j.jecp.2018.07.015.

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