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From Map Reading to Geometric Intuitions

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The origins and development of our geometric intuitions have been debated for millennia. The present study links children's developing intuitions about the properties of planar triangles to their developing abilities to read purely geometric maps. Six-year-old children are limited when navigating by maps that depict only the sides of a triangle in an environment composed of only the triangle's corners and vice versa. Six-year-old children also incorrectly judge how the angle size of the third corner of a triangle varies with changes to the other two corners. These limitations in map reading and in judgments about triangles are attenuated, respectively, by 10 and 12 years of age. Moreover, as children get older, their map reading predicts their geometric judgments on the triangle task. Map reading thus undergoes developmental changes that parallel an emerging capacity to reason explicitly about the distance and angle relations essential to euclidean geometry.

Keywords: spatial cognition, mathematical cognition, spatial symbols, euclidean geometry

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Euclidean geometry lies at the foundation of many human achievements, but its cognitive origins remain a scientific and philosophical problem (Plato, 2012). How do we come to judge that the internal angles of a triangle sum to a constant value or that a triangle's three side lengths entail three specific corner angles (Euclid, 1990/300 B.C.E.)? To make these judgments, we must infer the ways in which distances and angles are related to one another in the same planar figure. Although such intuitions appear to develop in humans with or without formal schooling (Izard, Pica, Spelke, & Dehaene, 2011), children do not begin to grasp them until rather late: depending on the study, somewhere between 7 and 13 years of age (Gibson, Congdon, & Levine, 2015; Izard, Pica, Dehaene, Hinchey, & Spelke, 2011; Izard, Pica, Spelke, & Dehaene, 2011; Lehrer, Jenkins, & Osana, 1998; Piaget, Inhelder, & Szeminska, 1960). The protracted development of these intuitions contrasts with children's early successes on another uniquely human skill, symbolic map reading by the geometric correspondences that relate a map to an environment. By 2.5 years, children can use the relative positions of circles or lines on an overhead map to locate objects or surfaces in a room (Winkler-Rhoades, Carey, & Spelke, 2013). By 4 years, they can navigate with maps of small-scale environments (such as of a single room; Shusterman, Lee, & Spelke, 2008), although their ability to navigate with maps of larger-scale environments (such as those composed of several rooms) continues to develop thereafter, as do the underlying cognitive maps that represent the elements of such larger-scale environments (Hazen, Lockman, & Pick, 1978; Kosslyn, Pick, & Fariello, 1974). Here we investigate whether older children's map reading undergoes changes through development that predict the emergence of intuitions that align better with euclidean geometry.

The precocious map reading shown by young children relies on depictions that present only the geometric relations between a form on a map and the structure of a small-scale environment within one room. Most maps depict much larger environments and are more complex. In addition to the geometry of the environment, they convey information about absolute distance (indicated by a scale legend), direction (indicated by a compass), landmark types or terrain topology (with graphic devices such as dotted lines indicating municipal boundaries or blue forms indicating bodies of water), and specific landmarks (indicated by written words; Downs, 1985). Effective engagement with these maps is much less intuitive for children; indeed, map reading undergoes a long and challenging development, and it relies on a number of different skill (Liben & Myers, 2007; Liben, Myers, Christensen, & Bower, 2013).

Nevertheless, numerous studies have been conducted since the pioneering research of Herbert Pick, charting the development of children's cognitive maps of both large- and small-scale spaces (see Pick & Rieser, 1982). These studies have revealed young children's competence with simple, geometric maps of one or several contiguous enclosures (Callaghan & Corbit, 2015; Davies & Uttal, 2007; Huttenlocher, Newcombe, & Vasilyeva, 1999;

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Jirout & Newcombe, 2014; Shusterman et al., 2008; Uttal, 1996, 2000; Uttal & Wellman, 1989; Vasilyeva & Bowers, 2006). Young children differentiate among targets in a room using maps that indicate the targets' relative distances along a line (Huttenlocher et al., 1999; Shusterman et al., 2008) or within a shape (e.g., the farthest vs. closest corner of a triangular environment; Izard, O'Donnell, & Spelke, 2014; Shusterman et al., 2008; Vasilyeva & Bowers, 2006) or the targets' positions relative to corners of distinct angle sizes (e.g., at the smallest vs. biggest corner of a triangular environment; Izard et al., 2014; Shusterman et al., 2008; Vasilyeva & Bowers, 2006).

Such geometric map reading has been the focus of many studies of children's early symbolic spatial capacities, and it may be the earliest developing uniquely but universally human geometric ability (Dehaene, Izard, Pica, & Spelke, 2006; Spelke, Lee, & Izard, 2010). Nevertheless, when young children read geometric maps of simple 3D environments, they use the same information that guides their nonsymbolic navigation and visual form analysis, abilities that children share with other animals (see Cheng & Newcombe, 2005 and Spelke & Lee, 2012, for reviews). Infants and young children primarily use distance and directional information to guide their navigation on novel paths between known locations (e.g., Landau, Gleitman, & Spelke, 1981) and to reorient themselves in rectangular environments (Lee, Sovrano, & Spelke, 2012); they primarily use relative length and angle information to recognize and differentiate between 2D planar forms of different shapes (Izard, Pica, Dehaene et al., 2011; Schwartz, Day, & Cohen, 1979). But does children's early use of the symbolic geometry in maps relate specifically to their use of the nonsymbolic geometry for navigation and visual form analysis that humans share with other animals?

In past work, individual young children's map reading was shown to be closely tied to their abilities to use geometry in tasks of navigation and visual form analysis. Dillon, Huang, and Spelke (2013) presented the same 4-year-old children with a navigation task, a visual form analysis task, and two map tasks. In the navigation task, children had to reorient themselves in each of three uniform rectangular enclosures differing in aspect ratio (after Huang & Spelke, 2015). In the form analysis task, children had to pick out a deviant shape from an array of six shapes (after Dehaene et al., 2006; Izard, Pica, Dehaene et al., 2011). In the map task, children stood inside one of two 3D fragmented triangular enclosures—one with three side walls but gaps at the corners and one with three corner angles but gaps at the sides. They saw maps depicting the complete, connected triangle with a single dot at a side or corner, and they were asked to place a toy at the location in the enclosure that corresponded to the dot's location on the map.

In the navigation task, children reoriented by the relative distances and directions of the walls of the rectangular environment, as in past research (Lee et al., 2012). In the form analysis task, they located the deviant figure by analyzing the relative lengths and angles of the lines and corners in each figure of the displays, again in accord with past research (Dehaene et al., 2006; Izard, Pica, Dehaene et al., 2011). In the map task, children were more flexible with their use of these types of geometric information: They successfully located targets either within the enclosure presenting surfaces of distinct distances or the enclosure presenting corners of distinct angles.

In addition, however, Dillon et al. (2013) found that 4-year-old children relied on different information when reading maps in the two enclosures. When reading maps in the enclosure with sides but no corners, they were most accurate at locating targets that appeared at or opposite the side with the most distinct distance, providing evidence that they relied on the distance information guiding reorientation. When reading maps in the enclosure with corners but no sides, they were most accurate at locating targets that appeared at or opposite the corner with the most distinctive angle, providing evidence that they relied on the corner angle information guiding visual form analysis.

Regression analyses corroborated these findings: Individual children's map reading in the sides-only enclosure correlated with their success on the navigation task, even after controlling for their verbal intelligence, success in the form analysis task, and success in the corners-only enclosure. Moreover, individual children's map reading in the corners-only enclosure correlated with their success in the form analysis task, even after controlling for their verbal intelligence, success in the navigation task, and success in the sides-only enclosure. Importantly, no correlation was found between children's performance on the navigation task and the form analysis task, suggesting that these tasks tap distinct spatial abilities. Strikingly, there also was no correlation between children's map reading in the two enclosures, even though the enclosures presented the same overall shape and were accompanied by identical maps.

When adults were presented with this task in informal experiments, they extrapolated a complete triangle from the visible sides or corners of the enclosure and used that extrapolation to find the location indicated on a map. The above findings suggest, however, that 4-year-old children do not fill in the information missing from each enclosure to construct a shape description that relates side distances and corner angle sizes. If they had, then their map reading performance should have been the same across the two different environments, and individual children's performance in one environment should have predicted their performance in the other. For example, children might have located a corner target in a fragmented enclosure without corners by inferring its overall shape using the visible sides of the enclosure. Children's failure to use their sensitivity to angle information in the side-only enclosure suggests that they did not adopt this strategy.

Early map reading may originate, therefore, in the capacities for navigation and form analysis that humans share with other animals. If it does, however, then young children's map reading likely relies on shape representations that are too impoverished to support the euclidean intuitions that capture the side and corner relations of planar polygons. For example, in Euclid's side-side-side proof for congruent triangles, three side lengths of a triangle imply three specific corner angles. When reading maps in different contexts, however, young children appear to attend either to the sides or to the corners of a triangle but not to both at once, even when the map they are using presents a fully connected triangle displaying both types of information.

Further evidence for such a limit to children's early spatial capacities comes from research by Izard, Pica, Spelke, and Dehaene (2011), investigating the nature and development of geometric intuitions about the sides and corners of planar triangles. Izard, Pica, Spelke, and Dehaene (2011) tested the judgments made by people in the U.S. and France and by people with little or no formal education, living in Amazonian villages. Adults and children aged 5–13 years were presented with the bottom two corners of a triangle, described as "towns" located at the intersection of straight navigable "paths" on an entirely flat "land." They were asked to indicate the location of the third town where the top two paths met (i.e., the top corner of the implied triangle) and to use their hands or a goniometer to indicate the size of the angle at which the paths met (Izard, Pica, Spelke, & Dehaene, 2011). Accurate responses to these questions required that participants consider the angle sizes of the bottom two corners as well as the distance between them.

Adults in both cultures showed fairly accurate and equivalent performance on both location and angle judgments. This finding provides evidence that the ability to produce the location and angle information describing the missing corner does not depend on formal education. U.S. children's (aged 5–7 years) performance, however, revealed striking errors. They failed systematically when estimating the angle size of that corner (few Amazonian children were tested in this age group). Children's erroneous angle judgments ran contrary to the fundamental euclidean principle that the three interior angles of any planar triangle sum to a constant value. Izard, Pica, Spelke, and Dehaene (2011) reasoned that all of the younger children's judgments reflected a global size strategy: When the distance between the two visible corners was large, children judged that both the distance and the angle size of the third corner was large. In two groups of older children in France and in the Amazon (mean age = 10 years), the accuracy of angle judgments improved and began to accord with the euclidean principle that the sum of the interior angles of a triangle is constant and independent of the triangle's global size. Thus, the ability to consider the relations between side distances and corner angles in planar polygons in this triangle completion task may develop between 7 and 13 years of age. This study does not support strong developmental conclusions, however, both because of the small sample sizes tested (an inherent limitation to testing of the Amazonian population) and because of the relatively difficult tasks presented to the participants (especially the demands of constructing, with the hands or goniometer, an angle of a specific size).

If the geometric intuitions underlying performance in this task develop across these ages, what accounts for this development? The similar performance of adults in the two cultures eliminates some possibilities. Because the Amazonian participants had little or no formal education and no instruction in formal geometry, intuitions about the relations between triangle sides and corners evidently develop without explicit teaching. Because these participants also had a limited numerical vocabulary and minimal access to more complex maps (e.g., ones that include scale, direction, and graphic elements; see above) and measurement devices (Pica, Lemer, Izard, & Dehaene, 2004; Dehaene, Izard, Spelke, & Pica, 2008), these cultural artifacts also are not likely to underlie this development. Amazonian adults do, however, interpret and create pictures, and they are highly accurate in their use of novel, simple overhead maps of the sort used to test young children's geometric map reading (DeLoache, 2004; Dehaene et al., 2006; see above). Might adults' geometric reasoning depend, in part, on their experiences navigating by spatial symbols?

Here we explore this possibility through studies of children in our own culture. We ask whether developmental changes in the use of simple geometric maps are associated with the development of geometric intuitions about planar figures. Because such map reading emerges spontaneously, long before such intuitions, and because it initially relies on isolated distance or angle information in young children, we hypothesize that children come to use distance and angle relations in a more integrated fashion over development when they navigate by geometric maps. Moreover, we hypothesize that such integrated use of distance and angle information in this intuitive symbolic spatial task will predict children's use of distance and angle relations in a geometric intuitions task, like that of Izard, Pica, Spelke, & Dehaene (2011). Abilities to navigate by geometric maps and to reason about abstract, unseen parts of triangles may both depend on an emerging, underlying capacity to relate distance, a key property of the perceived navigable environment, to angle, a key property of perceived object shape.

Overview

In this study, we modified the map task from Dillon et al. (2013) and the triangle completion task from Izard, Pica, Spelke, & Dehaene (2011) both to simplify the latter task and to focus explicitly on children's ability to rely on distance information and angle information together when reading maps and when answering questions about planar triangles. We adapted the map task so that both the environments and the maps presented fragmented triangles either with sides but no corners or with corners but no sides. On half of the trials (congruent), the map and environment presented the same geometric features (either sides or corners); on the other half of the trials (incongruent), they presented complementary features (e.g., a map presenting just the sides of the triangle designated a location in an environment consisting of only the triangle's corners). We asked whether children could relate sides and corners of a triangle across maps and environments during map reading if the task required it. Moreover, we quantified to what extent they were able to do so by comparing their performance when maps and environments presented congruent versus incongruent information.

To assess children's geometric intuitions, we adapted Izard, Pica, Spelke, and Dehaene's (2011) triangle completion task. We presented two fragmented corners implying a triangle, described as an imaginary navigable layout. In contrast to the original task, which used a static computer-generated display, the corners were represented by manipulable objects that were altered in position and angle size by the experimenter. Rather than require that children indicate the location of the third angle of a triangle by pointing or the size of the third angle with their hands or with a goniometer, we changed the two base corners of the triangle and asked children, in separate blocks of trials, whether the implied third corner moved up, moved down, or stayed in the same place, and whether its angle size got bigger, got smaller, or stayed the same size. We examined how each kind of transformation affected children's judgments to assess how children related side distances to corner angles when making judgments about planar triangles.

In addition to these two tasks, children were given the Peabody Picture Vocabulary Test (PPVT; Dunn & Dunn, 1997), a task evaluating their receptive vocabulary, which served as a control measure of verbal ability (e.g., Hodapp & Gerken, 1999; Verdine, Golinkoff, Hirsh-Pasek, Newcombe, Filipowicz, & Chang, 2014). Children's scores on the PPVT were standardized by age to get a comparable measure for individuals across the different age groups. The map task in one environment always was followed by the map task in the other environment (though the order of the two environments was counterbalanced), and the PPVT always followed the triangle completion task. The order of these two sets of tasks was also counterbalanced across children. We looked for relations between performance on the two sets of tasks across children, controlling for PPVT scores, to investigate whether children's map reading through development specifically predicted their judgments about planar triangles.

Participants

Thirty-two 6-year-old children (12 females; mean age 6 years 7 months, range 6 years 0 months–6 years 11 months), 32 ten-year-old children (12 females; mean age 10 years 5 months, range 10 years 0 months - 10 years 11 months), and 32 twelve-year-old children (17 females; mean age 12 years 6 months, range 12 years 1 month–12 years 11 months) completed this experiment. All children were recruited by mail and by posted flyers in a middle- to upper-middle class area in the northeast United States. Most children were Caucasian, and data from all three age groups were collected during the local schools' three-month summer vacation. Four additional children were tested but excluded immediately after their testing session (i.e., prior to any analyses) because their performance indicated that they failed to understand one of the tasks or were not willing to perform it.[1]

Map Reading Task

Method

Children were tested by an experimenter who did not administer the test of triangle completion and was unaware of children's performance on that test. The experimenter presented two sets of 2D maps displaying fragmented triangles in two differently fragmented, 3D triangular enclosures (Figure 1). To focus on developmental changes in the integration of side distances and corner angles in a map-reading context, we aimed for variable levels of performance within each age group and comparable levels of performance across age groups. To this end, we presented older children with triangles with more subtle differences among their three side distances and three corner angles. Six-year-old children were presented with maps and enclosures forming a 35°–60°–85° triangle, 10-year-old children were presented with maps and enclosures forming a 40°–60°–80° triangle, and 12-year-old children were presented with maps and enclosures forming a 50°–60°–70° triangle (Figure 2). All enclosures were centered in a cylindrical room with white paneled walls, symmetrical lighting, and a concealed spring-loaded door. For all age groups, one enclosure consisted of three 25-cm high, equal-length (92 cm) flat side walls of white foam core without connecting corners (side-only enclosure), and the other consisted of three 25-cm high corners of white foam core made from equal-length (46 cm) segments without connecting walls (corner-only enclosure). The two fragmented enclosures were thus identical in the length of foam core that defined their overall shape. Three green disks, serving as target locations, were placed either at the corners of the triangle formed by the overall shape of the enclosure or at the midpoints of its sides; the disks appeared at physically present sides or corners on six trials in each enclosure and at gaps between two sides or corners on the other six trials.

The maps depicted fragmented triangles with gaps at each corner (side maps) or at the center of each side (corner maps), and a green dot indicating the target location for that trial (see Figures 1 and 2). On congruent trials, the maps presented the parts of the triangle that were present in the 3D enclosure (e.g., side maps were shown in the enclosure composed of side walls); on incongruent trials, the maps presented the complementary parts of the triangle (e.g., side maps were shown in the enclosure composed of corner angles).

For each trial, the child stood in the center of an enclosure, while the experimenter stood outside of the enclosure, held a map in front of them for the child to see, and asked the child to place a small toy on one of three green disks in the room, specified by the green dot on the map (Figure 1). Each child was tested on two 12-trial blocks, one block in the enclosure of sides and one block in the enclosure of corners. Within each enclosure, children received two six-trial blocks: one with maps presenting congruent information and one with maps presenting incongruent information. In each block, three targets were at the center of each side of the triangle, and three were at each corner. Children's facing direction relative to the enclosure varied randomly for each trial (at 0° [corresponding to their facing perpendicular to the triangle's shortest side], 60°, 120°, 180°, 240°, or 300°), and for 10- and 12-year-old children, the orientation of the maps also varied randomly relative to the child (0° [corresponding to the triangle's shortest side being presented at the bottom of the map], 90°, 180°, 270°) to discourage children from using mental rotation to align the map with the environment. In previous research (Dillon et al., 2013), young children showed no evidence of using mental rotation in this task; and our pilot research suggested that rotation of the map as well as the child rendered the map task too difficult for the young children. Map order, placement location order, facing direction, and, for the older children, map orientation order were counterbalanced across children within each block. Each child's performance was assessed as the proportion of correct responses (chance = .33) in each of the two enclosures (sides or corners) and with congruent or incongruent maps.

Before the test trials, two practice trials were presented using color rather than geometry to specify the target location: Children located either a purple or pink disk in the room after the experimenter pointed to a purple or pink dot on a piece of paper that depicted nothing else. Children were given feedback on these trials but no feedback on their performance with the side and corner maps.

Results

We found no sex differences in children's overall performance on this task—6 years: $t(30) = -1.15$, $p = .258$; 10 years: $t(30) = 0.45$, $p = .654$; 12 years: $t(30) = 0.77$, $p = .448$—and so we collapsed across sex for all subsequent analyses. To determine

[1] On the map task, one 6-year-old child and one 10-year-old child placed the toy on a disk near them without looking at the enclosure on any test trial. On the geometric intuitions task, one 6-year-old child provided the response "stays the same" for all test trials, and one 12-year-old child repeated the properties of the practice triangle for all test trials.

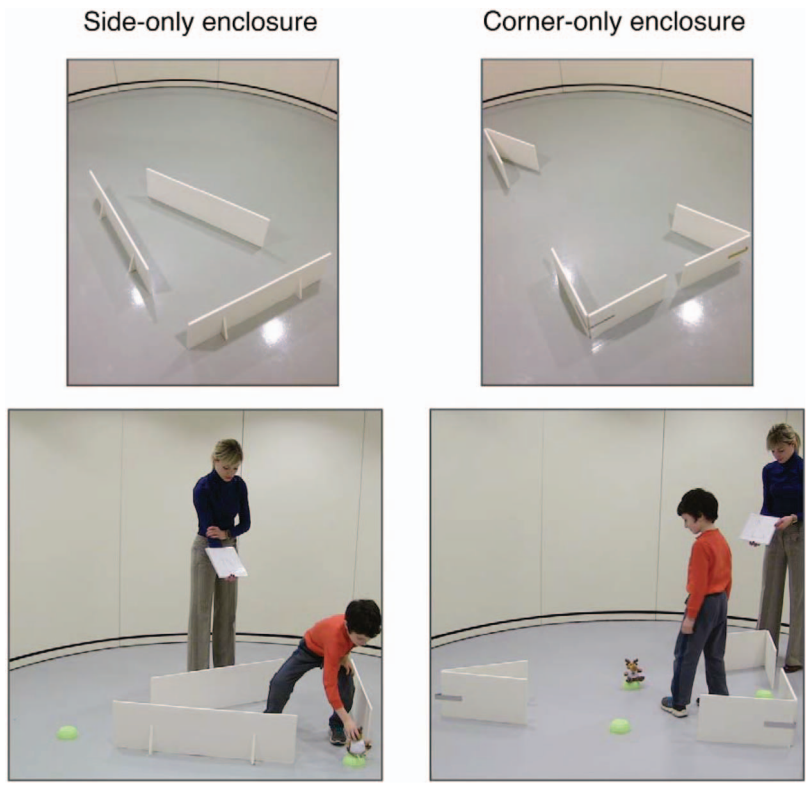

Figure 1. Top: Semi-overhead views of the side-only and corner-only enclosures forming the 35°–60°–85° triangle presented to 6-year-old children. Bottom: Eye-level views of a 6-year-old child participating in the map task. The depicted individuals provided signed consent for their likenesses to be published in this article. See the online article for the color version of this figure.

whether we had equated for the difficulty of the map task across the different ages, we next compared the consistency and accuracy of children's performance across the three ages. First, children in the three age groups performed with high and roughly equal reliability across the 24 map task trials, as measured by Cronbach's alpha (6 years: 0.787; 10 years: 0.757; 12 years: 0.864). A regression analysis including both age and verbal intelligence (PPVT scores) as predictor variables and performed across the full sample of children, revealed that PPVT scores predicted children's overall success on the map task, but age did not ($\beta = .29$; $p = .005$; $\beta = -.04$; $p = .670$, respectively). These results suggest that our difficulty manipulation (in which older children were given maps and enclosures with more subtle distance and angle relations) was effective (also see Table 1). Moreover, they suggest that more detailed analyses of performance are likely to be comparably sensitive at the different ages.

Children's performance next was analyzed separately for each age group. Six-year-old children performed above chance in both environments and with both congruent and incongruent maps (Table 1). Moreover, their performance was not related to their PPVT scores, $r(30) = 0.05$, $p = .784$. A within-participants ANOVA including enclosure (sides only or corners only), target location (at a side or at a corner of the overall shape formed by both enclosures), and congruency (maps and enclosures presenting the corresponding vs. noncorresponding parts) revealed no effect of enclosure, $F(1, 31) = 0.04$, $p = .839$, $\eta_p^2 = .00$, suggesting that children were equally successful at finding targets in the side and corner enclosures, but an effect of target location, $F(1, 31) = 5.01$, $p = .032$, $\eta_p^2 = .14$, with children performing better at side target locations. The Enclosure × Target Location interaction was not significant, $F(1, 31) = 0.24$, $p = .625$, $\eta_p^2 = .01$, indicating that children performed no better when targets appeared at a physically present side or corner than when they appeared at a gap between two sides or corners. In addition, there was a main effect of congruency, $F(1, 31) = 7.49$, $p = .010$, $\eta_p^2 = .20$: Children performed better when the map and the enclosure both presented side or corner features. Children performed above chance on the incongruent map trials, overall (Table 1), as well as on 10 of the 12 individual incongruent trials (binomial tests, two-tailed, ps $< .05$). They did not perform above chance on the two incongruent trials in which the target was located at the 60° corner (binomial tests, two-tailed, ps $> .05$; see the online supplemental material for proportion correct at each trial location for each age).

The primary analysis for each age group focused on the relation between children's performance in the two enclosures. A bivariate correlation (as in Dillon et al., 2013) revealed no relation between performance in the two enclosures across the 6-year-old children, $r(30) = .24$, $p = .192$. Like younger children in previous studies (e.g., Dillon et al., 2013), the 6-year-old children who performed well when reading maps in the enclosure presenting just sides were no more likely than other children to perform well in the enclosure presenting just corners. Both the cost children incurred in navigat-

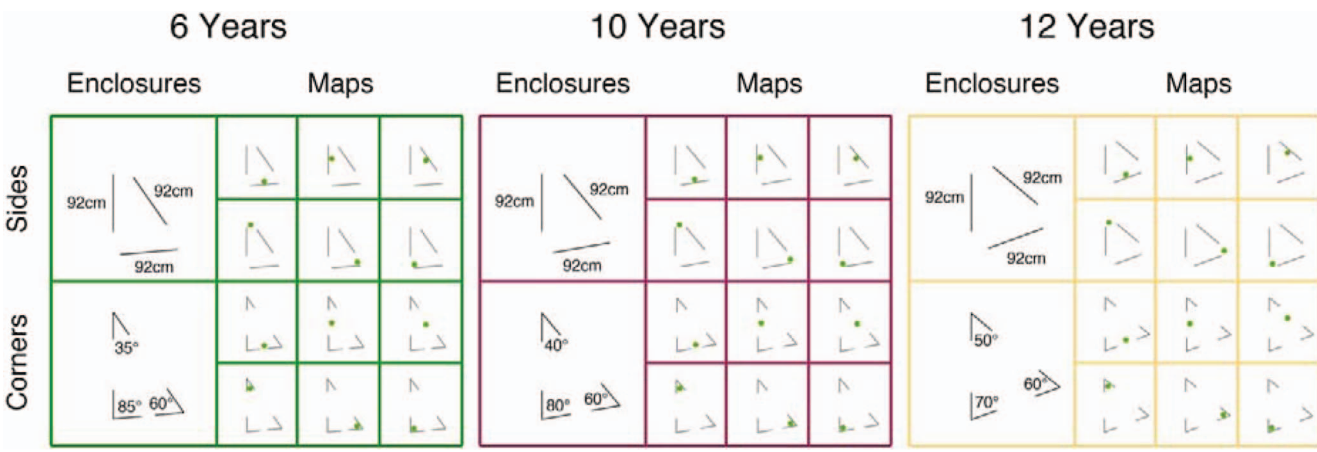

Figure 2. Overhead schematics of the 3D, side-only and corner-only enclosures presented to children at different ages (left column of each age group). Maps on square pieces of paper depicted fragmented triangles that either matched or were complementary to the fragmented triangular enclosures (right columns of each age group). In both cases, every map depicted the same triangular shape as the enclosure at a scale of 1:13. The surrounding cylindrical room in which the enclosures were placed was not included the maps. In contrast to their depiction here, maps were presented at variable orientations relative to the enclosure (see text). See the online article for the color version of this figure.

ing by incongruent maps and the lack of correlation between performance in the two enclosures suggest that children failed to relate distances and angles to navigate by maps using an overall shape description that applied to both of these fragmented triangular environments.

Ten-year-old children also performed above chance in each environment and with each type of map (Table 1), and their performance was not predicted by their PPVT scores ($r(30) = .119$, $p = .515$). In other respects, however, 10-year-old children showed a different performance profile from the 6-year-old children. The ANOVA including enclosure, target location, and congruency revealed no main effects and no interactions. In particular, 10-year-old children performed equally well with congruent and incongruent maps, $F(1, 31) = 1.20$, $p = .282$, $\eta_p^2 = .04$. In contrast to 6-year-old children, they showed no cost when side or corner information in the map did not match the information in the 3D enclosure (Table 1). Moreover, there was a significant correlation between performance in the two enclosures across children at 10 years, $r(30) = .45$, $p = .010$, and this relation persisted after controlling for children's PPVT scores ($\beta = .46$; $p = .008$). Unlike the younger children, the 10-year-old children who performed well in the environment presenting just sides also tended to perform well in the environment presenting just corners. This correlation, together with the finding that children incurred no cost in navigating by incongruent maps, provides evidence that 10-year-old children related distances and angles to navigate by maps using an overall shape description that applied to both of these fragmented triangular environments.

Twelve-year-old children performed above chance on all conditions and on all incongruent trials of the map task (Table 1; see the online supplemental material). In contrast to the younger children, 12-year-old children's map performance was related to their PPVT scores, $r(30) = 0.55$, $p < .001$. The ANOVA revealed an effect of enclosure, $F(1, 31) = 4.25$, $p = .048$, $\eta_p^2 = .12$, with children performing better in the enclosure with corners, but no effect of target location, $F(1, 31) = 0.01$, $p = .925$, $\eta_p^2 = .00$, or congruency, $F(1, 31) = 2.33$, $p = .137$, $\eta_p^2 = .07$, and no interactions. Like the 10-year-old children, the 12-year-old children showed no significant cost when side or corner information in the map did not match the information presented in the enclosure. Moreover, there was a significant bivariate correlation between their performance in the two enclosures, $r(30) = .72$, $p < .001$, and this relation persisted after controlling for children's PPVT scores ($\beta = .53$; $p < .001$). The children who performed well in the environment presenting just sides also tended to perform well in the environment presenting just corners. At 12 years, as at 10 years, children evidently related distances and angles to navigate

Table 1
Means, Significance Tests, and Effect Sizes (Cohen's d) Evaluating 6-, 10-, and 12-Year-Old Children's Performance on the Map Task (Chance = .33) in Both 3D Enclosures and With Both Map Types

Age and enclosure	Map	*M*	Significance test	Cohen's *d*
6 years				
Side	Side	.73	$t(31) = 8.67$, $p < .001$	3.11
Side	Corner	.65	$t(31) = 6.53$, $p < .001$	2.35
Corner	Side	.63	$t(31) = 5.69$, $p = .004$	2.04
Corner	Corner	.73	$t(31) = 9.43$, $p < .001$	3.39
10 years				
Side	Side	.77	$t(31) = 10.25$, $p < .001$	3.68
Side	Corner	.81	$t(31) = 12.03$, $p < .001$	4.32
Corner	Side	.69	$t(31) = 7.78$, $p < .001$	2.79
Corner	Corner	.82	$t(31) = 14.06$, $p < .001$	5.05
12 years				
Side	Side	.62	$t(31) = 5.58$, $p < .001$	2.00
Side	Corner	.56	$t(31) = 4.45$, $p < .001$	1.60
Corner	Side	.64	$t(31) = 6.27$, $p < .001$	2.25
Corner	Corner	.69	$t(31) = 6.77$, $p < .001$	2.43

by maps using an overall shape description that applied to both of these fragmented triangular environments.

Finally, we evaluated the moderating effect of age on the strength of the relation between performance in the side and corner enclosures. Age significantly moderated this relation across children ($\beta = .89$; $p = .010$). From 6 to 12 years, the shape representations that children use to read maps in triangular enclosures of sides become progressively more strongly related to those that they use to read maps in triangular enclosures of corners.

Discussion

Six-year-old children in the present study showed a significant cost when using a map that depicted only the sides (or corners) of a triangle in an environment that presented only its corners (or sides). Moreover, there was no correlation in map performance across 6-year-old children in enclosures presenting sides or corners of the same triangular shape. These results together suggest that 6-year-old children do not read maps by constructing an integrated shape representation from fragmented side or corner information. The findings accord with those of a previous experiment conducted with 4-year-old children, as described above (Dillon et al., 2013; see also Huang & Spelke, 2015), and suggest that 6-year-old children continue to recruit distinct processes for map reading when relating maps to environments in which locations are specified by the distances and directions of flat surfaces, on one hand, or by the angle sizes of corner landmarks, on the other.

One finding nevertheless tempers this suggestion: Although 6-year-old children performed less well with incongruent than congruent maps and environments, their performance in the incongruent condition was above chance. Indeed, they showed some ability to locate the correct target on most incongruent trials. There are several possible reasons for this finding. First, sensitivity to relations between side distances and corner angles may have begun to develop in some of the children. Alternatively, 6-year-old children may have succeeded on some trials by focusing their attention on the shortest side of the triangle, where there was substantial overlap between the information presented in the map and in the environment. Further research is needed to distinguish between these possibilities.

Could the limits observed in 6-year-old children's map reading be attributable to the size of the navigable space used in the map task? We believe that this explanation is unlikely to be correct for four main reasons. First, tasks suggesting an effect of enclosure size on children's navigation report that small enclosures limit young children's ability to use, for example, information defining the geometry of the locale with a landmark cue, such as a colored wall, which breaks the locale's geometric symmetry. Such effects disappear, however, by age 6 in all studies (Hermer-Vazquez, 1997; Hermer-Vazquez, Moffet, & Munkholm, 2001; Learmonth, Nadel, & Newcombe, 2002; Learmonth, Newcombe, Sheridan, & Jones, 2008), and so the 6-year-old children in the present study would not be subject to this limit. Second, although previous studies suggest that small spaces limit the kind of information that children are able to use during navigation, the present study and prior work (e.g., Dillon et al., 2013) show that children are perfectly capable of using either side distance or corner angle information to interpret maps, that is, children are just as successful in the enclosure presenting only the triangle's sides as they are in the enclosure presenting only the triangle's corners. The limitation for 6-year-old children appears to lie in their not integrating side and corner information to form a unified representation of the same overall shape. Third, in search or placement tasks such as the present one, children have shown facility in using maps or models at a variety of scales to find locations on small-scale Lego objects as well as furnished rooms considerably larger than the present environments (e.g., Dillon & Spelke, 2015; DeLoache, Miller, & Rosengren, 1997). Finally, although the results of the present task reveal limits in 6-year-old children's integration of different spatial elements in a small-scale environment, our results are strikingly similar to the geometric limitations exhibited by 6-year-old children in larger-scale spaces. In particular, when 6-year-old children are asked to produce a model of a large and complex familiar environment, they fail to integrate information about their routes through that space with the location of that space's landmarks in their generated models (Hazen et al., 1978). Indeed, even children's cognitive maps of large-scale environments at this age are "poorly integrated," insofar as they treat areas that are partially bounded as separate subspaces instead of part of one unified, multichambered environment (Kosslyn et al., 1974). Thus, 6-year-old children's limitations in the present map task are unlikely attributable to the size of the navigable space but rather descriptive of a more general limitation in their use of geometry for map reading.

By age 10, the limitations in younger children's map reading appear to diminish. The 10- and 12-year-old children suffered no cost when using side information in a map to interpret corner information in the environment, and their performance in the two differently fragmented environments correlated over and above the effect of verbal intelligence, even though such PPVT scores also predicted map performance overall. Across the three age groups, the correlation between performance in the two enclosures strengthened, suggesting that more integrated shape representations come to underlie children's map reading as children get older, through at least 12 years of age. In the General Discussion, we consider the possible nature of these shape representations.

Might older children also use these more integrated shape representations when they make judgments about the properties of planar triangles? To begin to address this question, we presented the children in the present study with questions about the location and angle size of a triangle's third corner after changes to the other two corners. We asked whether the same developmental pattern emerges in children's judgments about triangles as it does in map reading: Do older children's judgments about planar triangles show increasing evidence of more integrated representations of distance and angle?

Triangle Completion Task

Method

The triangle completion task (adapted from Izard, Pica, Spelke, & Dehaene, 2011) challenged children to make explicit verbal judgments about changes to the location and angle size of a triangle's implied third corner after changes to either the distance between its other two corners or to the angle sizes of those corners. Children were presented with a magnetic white board positioned vertically on a table by an experimenter, who did not administer the map task and was unaware of children's performance on that task. The board was described as representing part of an entirely flat "land" that extended in all directions. Two small circular magnets, positioned as the bottom two corners of a triangle, represented "towns" on this land. Four 8-cm long magnetic strips were described as "roads" that passed through the center of the towns and extended on forever without turning (Figure 3). Two of these roads were positioned 35 cm apart to form the base side of the triangle and two roads extended upward from each town at a 45° angle to form the other two sides of the triangle. The experimenter, seated next to the child and facing the same direction as the child, used a white-board marker to demonstrate how the two side roads extended on to meet at a third town, pointing out its location by drawing a dot on the spot at which the marker lines came together, and indicating its angle size by drawing a "U" shape on the interior of the angle formed there. Then, on each of four trials, a different transformation was made to the two bottom corners: their angle size decreased; their angle size increased; the distance between them increased; or the distance between them decreased (see Figure 3). Children were asked, after each of the four transformations, whether, after that change, the third town moved up, moved down, or stayed in the same place (location questions), and whether the size of the angle formed by the roads at their junction increased, decreased, or stayed the same (angle questions). Transformations always occurred in the above order, but the location and angle questions were tested in counterbalanced blocks across children. The triangle was returned to its starting position before the next transformation and question were presented.

Results

We found no sex differences in children's overall performance on this task—6 years: $t(30) = 0.75$, $p = .460$; 10 years: $t(30) = -0.77$, $p = .448$; 12 years: $t(30) = 1.74$, $p = .092$—and so we collapsed across sex for all subsequent analyses. Children's performance first was analyzed separately for each age group. For the 6-year-old children, performance on this task was not significantly correlated with their PPVT scores, although that correlation showed a positive trend ($r(30) = .348$, $p = .051$). A within-participants ANOVA including question type (whether the child was queried about the third corner's location or its angle size) and transformation (a change either to the distance between the two visible corners or to the angle sizes of those corners) revealed an effect of question type, $F(1, 31) = 96.67$, $p < .001$, $\eta_p^2 = .76$, with better performance on the location questions, and no effect of transformation, $F(1, 31) = 3.72$, $p = .063$, $\eta_p^2 = .11$, with a trend toward better performance after angle transformations. The Question Type × Transformation interaction was significant, $F(1, 31) = 4.37$, $p = .045$, $\eta_p^2 = .12$, with better performance after the angle transformations on the location questions. Six-year-old children performed significantly above chance (chance = .33; all tests two-tailed) on the location questions after changes to the distance between and the angle sizes of the bottom two corners: distance transformation: $t(31) = 4.65$, $p < .001$, Cohen's $d = 1.67$; and angle transformation: $t(31) = 10.31$, $p < .001$, Cohen's $d = 3.70$. In contrast, children performed significantly below chance on both sets of angle questions after these transformations: distance transformation: $t(31) = -2.44$, $p = .020$, Cohen's $d = 0.88$; and angle transformation: $t(31) = -3.38$, $p = .002$, Cohen's $d = 1.21$ (Figure 4).

What might account for this pattern of results? Izard, Pica, Spelke, and Dehaene (2011) suggested that young children's answers to questions about both the location and the angle size of a triangle's third corner accorded with a size strategy: Young children judge that both values increase with increases to the angle size and/or distance between the other two corners. In the present study, this strategy would yield correct answers on location questions but incorrect answers on angle questions. To test for this possibility, we first determined the reliability of children's responses by calculating the relation between their responses to the larger- and smaller-size transformations for each question type/judgment combination using a mixed linear model. This relation was significant at 6 years, b = .49; $t(40.69) = 7.28$, $p < .001$, Cohen's $d = 2.28$, indicating that 6-year-old children's responses were consistent across trials probing the same type of geometric information. Next, we considered children's performance on the angle questions alone, coding their responses as consistent or inconsistent with the size strategy. A significant proportion of 6-year-old children's judgments were consistent with the size strategy (.64 of responses, chance = .33), $t(31) = 7.10$, $p < .001$, Cohen's $d = 2.55$ (see the online supplemental material): They judged that the size of the third corner angle would get bigger after increases (or smaller after decreases) to the distance between the two visible corners or to their angle sizes. Finally, we tested

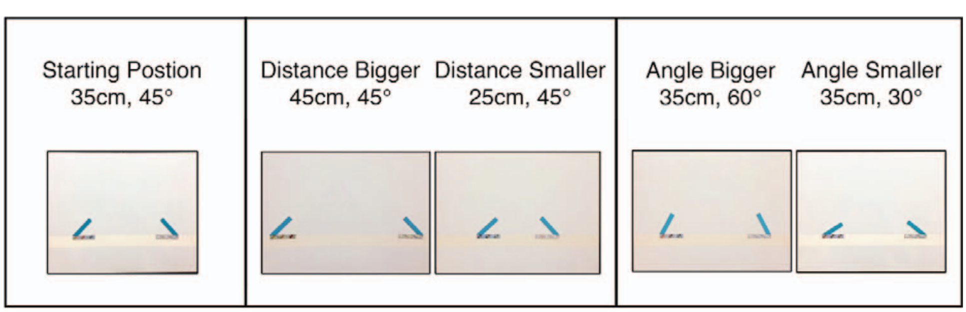

Figure 3. Display used for the triangle completion task. Before each question, children saw the starting position setup (left panel). Then, one of four transformations was made to the bottom two corners, changing either the distance between them (middle panel), or their angle sizes (right panel). Before the next transformation, the magnets were returned to their starting position. See the online article for the color version of this figure.

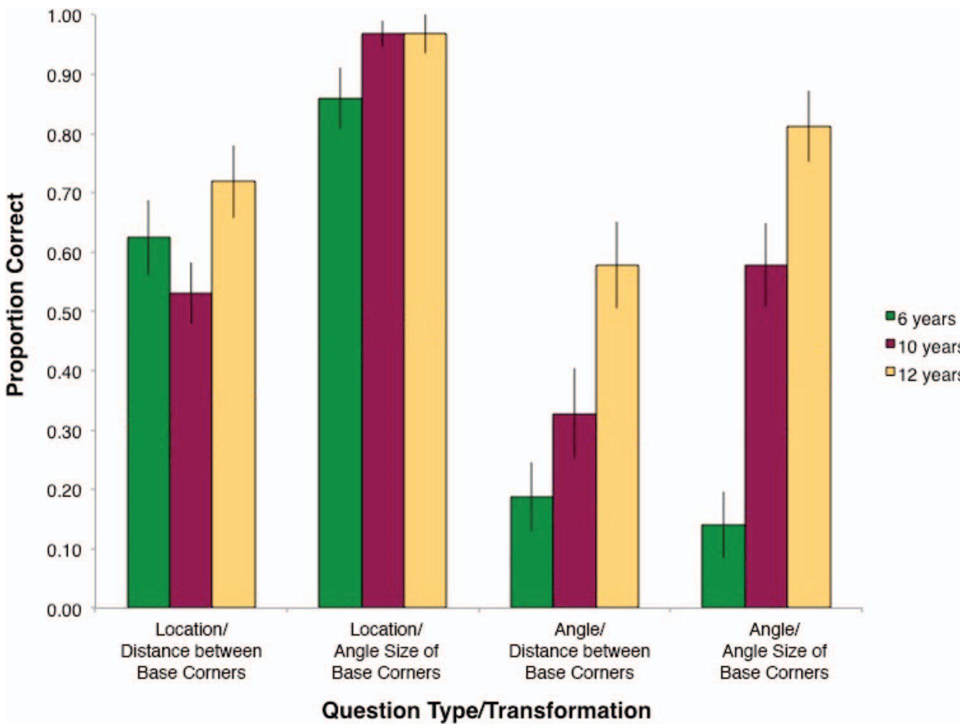

Figure 4. The proportion of children's correct responses in the triangle completion task (chance = .33). Six-year-old children responded that the third angle of a triangle gets bigger (or smaller) not only with increases (or decreases) in the distance between the bottom two corners (angle/distance between base corners bars), but also with increases (or decreases) in the sizes of the bottom two angles (angle/angle size of base corners bars). Ten-year-old children failed in the former case, but not the in latter, and 12-year-old children succeeded in both question types under both transformations. A consistent size strategy would have yielded correct responses on the location questions, but incorrect responses on the angle questions, whereas a consistent euclidean-based strategy would have yielded correct responses on both question types (see the online supplemental material). See the online article for the color version of this figure.

whether there was any correlation between successful performance on the location and angle questions across children. A positive correlation would indicate that children who were good at making judgments about the location of the triangle's third corner were also good at making judgments about its angle size, reflecting the accurate euclidean strategy for all question types. A bivariate correlation revealed, however, that 6-year-old children's successful performance on the location and angle questions was not correlated, $r(30) = -.10$, $p = .601$. Thus, these youngest children appeared to base their judgments on the size of the triangle and not the euclidean relations between side distances and corner angles.

The 10-year-old children's performance also was not predicted by their PPVT scores, $r(30) = 0.12$, $p = .364$. The within-participants ANOVA revealed effects of question type, $F(1, 31) = 26.46$, $p < .001$, $\eta_p^2 = .46$, with better performance on the location questions, and of transformation, $F(1, 31) = 45.19$, $p < .001$, $\eta_p^2 = .59$, with better performance after angle transformations. The Question Type $\times$ Transformation interaction was not significant, $F(1, 31) = 2.61$, $p = .116$, $\eta_p^2 = .08$. Like the younger children, 10-year-old children judged correctly that the distance of the third corner would increase (or decrease) with increases (or decreases) to the distance between or the angle size of the bottom two corners—distance transformation: $t(31) = 4.03$, $p < .001$, Cohen's $d = 1.45$; and angle transformation: $t(31) = 29.38$, $p < .001$, Cohen's $d = 10.55$ (Figure 4). On the angle questions, they performed above chance after angle transformations, $t(31) = 3.31$, $p = .002$, Cohen's $d = 1.19$, but not after distance transformations, $t(31) = -0.03$, $p = .980$, Cohen's $d = 0.01$. The 10-year-old children's responses on this task were reliable, as revealed by the significant relation between their responses to the same question types with transformations that increased or decreased in magnitude (b = .39; $t(128) = 4.50$, $p < .001$, Cohen's $d = 0.80$). An analysis of performance on the angle questions only revealed that 10-year-old children's responses tended to accord with the size strategy also used by younger children (.50 of responses, chance = .33), $t(31) = 3.32$, $p = .003$, Cohen's $d = 1.16$ (see the online supplemental material), although the proportion of size-based responses were significantly less in the 10-year-old children than in the 6-year-old children, $t(62) = 2.24$, $p = .029$, Cohen's $d = 0.57$. Finally, the bivariate correlation of performance on the location and angle questions across children was not significant ($r(30) = .18$, $p = .317$). Ten-year-old children thus showed an intermediate pattern of responses, with reliance on a size-based strategy, but also better intuitions about the relations between the side distances and corner angles of planar triangles.

At 12 years, children's performance was predicted by their PPVT scores, $r(31) = .42$, $p = .016$. The within-participants ANOVA revealed effects of question type ($F(1, 31) = 6.76$, $p = .014$, $\eta_p^2 = .18$), with better performance on the location questions, and of transformation ($F(1, 31) = 37.29$, $p < .001$, $\eta_p^2 = .55$), with better performance after angle transformations, and no Question Type × Transformation interaction, $F(1, 31) = 0.03$, $p = .869$, $\eta_p^2 = .00$. Children performed above chance on both location and angle questions after both types of transformations—location questions: distance transformation: $t(31) = 6.57$, $p < .001$, Cohen's $d = 2.36$; angle transformation: $t(31) = 20.44$, $p < .001$, Cohen's $d = 7.34$; angle questions: distance transformation: $t(31) = 3.32$, $p = .002$, Cohen's $d = 1.19$; and angle transformation: $t(31) = 8.27$, $p < .001$, Cohen's $d = 2.97$—and their responses were reliable, b = .50; $t(14.12) = 4.93$, $p < .001$, Cohen's $d = 2.62$. An analysis of performance on the angle questions revealed a nonsignificant trend for responses that ran contrary to a size strategy (.24 of responses, chance = .33), $t(31) = -1.73$, $p = .094$, Cohen's $d = 0.62$. Moreover, the number of size-based responses were significantly less in the 12-year-old children than in the 10-year-old children, $t(62) = 3.52$, $p = .001$, Cohen's $d = 0.89$ (see the online supplemental material). Finally, the bivariate correlation of performance on the location and angle questions across children was significant, $r(30) = .36$, $p = .045$, indicating that children who performed better on the location questions also tended to perform better on the angle questions, as would be expected by a consistent euclidean strategy. This relation did not persist after controlling for children's PPVT scores ($\beta = .27$; $p = .154$). Although some children judged erroneously that the third corner angle would get larger after increases (or smaller after decreases) in the distance between the other two corners (see the online supplemental material), these responses were outnumbered by accurate judgments about the relations between the side distances and corner angles of planar triangles.

Further analyses tested whether age and PPVT scores were associated with performance on this task across the full sample of children. A regression analysis including both variables and performed revealed that both age and PPVT scores separately predicted children's successful responding ($\beta = .49$; $p < .001$; $\beta = .27$; $p = .002$, respectively). Finally, we found that age modulated the strength of the relation between children's judgments about the location and the angle size of a triangle's third corner ($\beta = .72$; $p = .044$). From 6 to 12 years therefore, children become increasingly able to judge how the different properties of planar shapes interact with one another, in accord with basic principles of euclidean geometry.

Discussion

The triangle completion task revealed marked changes in children's judgments from 6 to 12 years of age. At 6 years, children's judgments concerning the properties of triangles were consistent but incorrect: They judged erroneously, for example, that the third angle of a triangle would get bigger with an increase to the distance between the other two corners (its size does not change) and with an increase to the angle sizes of the other two corners (it gets smaller). Their performance on both question types was consistent with a global size strategy, in which bigger triangles are bigger in all respects. This size-based strategy implies that children's intuitions fail to capture the euclidean principles that the side and corner relations in triangles are invariant over changes in absolute scale, and that the three interior angles of all planar triangles sum to a constant value.

Although 10-year-old children rarely judged that the size of the third angle of the triangle would change congruently with changes to the sizes of the other two angles, they nevertheless judged that the third angle would increase in measure after the other two angles moved farther apart. Twelve-year-old children, in contrast, tended to answer both of these questions correctly, although their errors, when they made them, were qualitatively similar to those of younger children (see the online supplemental material). These findings provide evidence for both continuity and change in children's intuitions about the properties of planar triangles.

What factors might produce this change? The oldest children's success was related to their PPVT scores, and the relation between their ability to engage correct, euclidean-based responses for both distance and angle questions was not wholly separable from this more general measure of verbal ability. Because no such relations were found in younger children and no other measures of language were used in this study, the nature of this association is not clear. It is possible that children's facility to extend their vocabulary at 12 years contributes to their developing mastery of the language of geometry. Alternatively, because the PPVT requires that children relate spoken words to 2D depictions of objects and events, it is possible that the geometric abilities underlying the discrimination of these depictions affect older children's PPVT performance. Further research is needed to distinguish between these possibilities. Whatever the contribution of language learning or verbal ability to children's geometric judgments, however, the present findings provide evidence for a marked improvement in those judgments between 6 and 12 years of age.

Might the developmental change in children's use of geometry in maps, as revealed in the present test of map reading, relate to the present changes in children's geometric judgments in the triangle completion task? The similar developmental pattern observed on these two tasks is consistent with this possibility, but this pattern provides no evidence for a more direct connection between performance on the two tasks. The next analysis tests whether children who excel at map reading also perform better when answering questions about the geometric properties of triangles, after controlling for age and verbal intelligence.

Relations Between Map Reading and Geometric Intuitions

Method

Because children's performance on both the map task and the triangle completion task was reliable at each age (as described above), regression analyses were conducted at each age to determine whether individual children's overall scores on the map reading task predicted their overall scores on the triangle completion task, after controlling for PPVT scores. A further regression analysis was conducted on the full sample of children, testing whether age affected the strength of the relation between map reading and triangle judgments. Distribution normality for these variables was confirmed by examining histograms and Q-Q plots. In addition, the variance of children's performance on the map reading task and the triangle completion task was approximately equivalent (map task: 0.04; triangle completion task: 0.04).

Results

After accounting for effects of age and verbal ability, the age-specific regression analyses showed no relation between children's map reading and their geometric intuitions at 6 years (β(map task) = .06; p = .708) and only a nonsignificant relation between performance on these tasks at 10 years (β(map task) = .30; p = .102). In contrast, the two tasks were significantly correlated across children at 12 years (β(map task) = .53; p = .010). By age 12, children's performance on the map task converged reliably with their judgments on the triangle completion task probing their geometric intuitions.

The final regression analysis examined how developmental changes might affect the relation between map reading and geometric intuitions by collapsing across groups and testing for the moderating effect of age on this relation. Across the full sample of children, age significantly affected the strength of the relation between map reading and geometric intuitions (β = .20; p = .039). Thus, the predictive power of map reading on geometric intuitions tends to grow as children get older.

Discussion

This set of regression analyses investigated whether changes in children's use of simple geometric maps relate to changes in their judgments about the properties of planar shapes. A separate analysis for each age group revealed that these two tasks were significantly related to one another in the 12-year-old group only, although there was a nonsignificant trend toward this relation at 10 years. In an analysis that collapsed across all three age groups, age moderated the strength of this relation. Thus, the representations and processes that children recruit during map reading accord more and more through development with those on which they rely when answering explicit questions about the geometric properties of triangles. Below, we ask what representations and processes might account for this growing relation.

General Discussion

The present study reveals a relation between children's map reading and their judgments about planar triangles. Moreover, detailed patterns of performance in these two tasks suggest that this relation may depend on an emerging ability to relate distances and angles within the same planar figure. Young children's map reading relies in part on geometric representations that humans share with other animals to navigate the environment and recognize the objects in it. These representations have limits, however, that are reflected in young children's map reading and possibly also in their explicit judgments about triangles. When reading maps, young children rely exclusively on side distance information in an enclosure that presents side walls but not corners, and they rely exclusively on corner angle information in an enclosure of the same overall shape that presents corner angles but not connected side walls (Dillon et al., 2013). When making judgments about the properties of shapes, moreover, children fail to judge that angle size is conserved over changes in a triangle's absolute scale (Gibson et al., 2015; Izard, Pica, Spelke, & Dehaene, 2011).

By age 10, children's map reading in fragmented environments appears to rely on the same overall shape representation implied by the different fragments. This finding suggests that children infer missing sides from an enclosure presenting only corners, and vice versa. Because the shortest side of the triangles used in the present experiment showed large overlap between the two types of fragmented figures, however, we cannot exclude the possibility that older children relied more on this overlap in some cases. Developmental changes in map reading as measured by the present study thus suggest, but do not conclusively reveal, a more integrated representation of the spatial layout and its different distances and angles in older children.

By age 12, children's judgments about the side and corner properties of planar shapes reflect some principles of euclidean geometry. Children's use of more integrated shape representations during map reading may foreshadow this achievement. Our strongest evidence for a link between performance on these two tasks is that as children get older, these tasks become more and more correlated with one another. The strategies that older children use when reading simple geometric maps depicting fragmented figures predicts their responses on a test probing the location and angle size of the third, unseen corner of a triangle. But, abilities to navigate by purely geometric maps emerge very early in human development, whereas reasoning about triangles improves greatly over middle and later childhood. Why might these two abilities show convergence through development?

The present experiments cannot definitively answer this question. Nevertheless, we suggest that map reading and reasoning about triangles both improve as children develop more abstract and integrated representations of the relation between the geometric properties of distance and direction that guide navigation, and the geometric properties of angle and relative length that guide visual form analysis.

Might maps promote the integration of these properties into a common representation? Symbols in general allow humans to represent diverse types of information efficiently and may therefore provide a medium in which different information can be held in memory, manipulated, and combined. For example, Arabic symbols allow for the manipulation of exact large magnitudes; and tree diagrams allow for the efficient representation of biological taxonomies. Maps may similarly allow for the efficient representation of diverse features of the environments that they represent. Like other spatial symbols, maps are culturally widespread (DeLoache, 2004), and they are similar to the pictures that children encounter and interpret from an early age (Bloom & Markson, 1998). When a map depicts only the sides of a triangular enclosure that is composed only of corners, and the map is presented by an apparently helpful adult as a useful source of information about that enclosure, children may reanalyze the sides on the map to recover information about the corners in the environment (see Callaghan & Corbit, 2015 for a discussion on intentionality in children's use of spatial symbols). Maps thus may encourage a use of spatial representations that integrate different elements of the environment.

Although the present findings raise this possibility, they cannot confirm it. The observed correlations between children's map reading and their judgments about triangles remain after controlling for PPVT scores, suggesting that these changes are not wholly reflective of differences in language experience, vocabulary size, or verbal ability. Nevertheless, these correlations could in part rely on other developments, including age-related changes in sensitivity to objects' visual properties (Kaldy & Blaser, 2009), to the robustness of shape perception, attention, and working memory (e.g., Gibson, 1969; Giofrè, Mammarella, & Cornoldi, 2014), to aspects of executive function that allow older children to shift more effectively from one source of information to another (Bull & Scerif, 2001), or to capacities for analogical reasoning that allow children to apply geometric relations to different types and properties of objects (Loewenstein & Gentner, 2005). Further experiments, probing correlations between developmental changes in the present tasks and changes in these capacities, could test these possibilities.

Even if the correlation between performance on the present map task and triangle completion task does depend on the combination of distinct geometric representations of distance and angle, the present findings would not reveal whether the experience of reading maps causes advances in children's geometric reasoning. Training experiments are needed to explore whether tasks that exercise map reading play any causal role in the development of knowledge of geometry, and if so, what cognitive changes underlie such effects (see Uttal et al., 2013, for a comprehensive review of spatial training studies). In the context of the present findings, it might be particularly informative to test whether training with fragmented maps and environments paired incongruently improves 10-year-old children's ability to solve the present triangle completion task more than training with the same maps and environments paired congruently.

The present findings also raise questions concerning the processes by which older children and adults reason about the properties of geometric figures. Do older children use mental simulations to complete missing parts of figures when answering questions about them, or do they apply propositional understanding, for example, that the three internal angles of a triangle sum to a constant? If success on the triangle completion task depends in part on mental simulation processes, then how do older children and adults go beyond these simulations to identify the specific aspects of shapes that enter into geometric judgments? If success depends in part on propositional knowledge, then how do children select the propositions to be applied to particular spatial arrays? Further research, applying chronometric, eye-tracking, or computational modeling approaches, could address these questions.

Finally, given that young children's map reading is related to geometric abilities from our evolutionary past, the question remains as to whether there is any detectable relation of nonsymbolic navigation and object recognition in older children's explicit judgments about planar shapes. It is possible that developmental changes in children's map reading and geometric intuitions build on the more limited geometric representations guiding navigation and object recognition. Alternatively, older children might not engage these cognitive systems when relating distances and angles in symbolic or explicit contexts. By testing for effects of training experiments, aimed at enhancing older children's navigation or object recognition on children's explicit judgments about the properties of triangles, we may determine whether early developing and evolutionarily ancient systems of representation serve as guides to the judgments that support formal geometry. If our geometric abstractions build on symbolic and nonsymbolic geometric skills that arise early in development and are used throughout our lives, then efforts to enhance those capacities through education may benefit from a pedagogy linking the formal systems children must master to the everyday acts of navigation, object recognition, and map reading in which they readily take part.

References

Bloom, P., & Markson, L. (1998). Intention and analogy in children's naming of pictorial representations. *Psychological Science, 9,* 200–204. http://dx.doi.org/10.1111/1467-9280.00038

Bull, R., & Scerif, G. (2001). Executive functioning as a predictor of children's mathematics ability: Inhibition, switching, and working memory. *Developmental Neuropsychology, 19,* 273–293. http://dx.doi.org/10.1207/S15326942DN1903_3

Callaghan, T., & Corbit, J. (2015). The development of symbolic representation. In L. S. Liben & U. Müller (Eds.), *Cognitive processes: Vol. 2. Handbook of child psychology and developmental science* (7th ed.; pp. 250–295). Hoboken, NJ: Wiley. http://dx.doi.org/10.1002/9781118963418.childpsy207

Cheng, K., & Newcombe, N. S. (2005). Is there a geometric module for spatial orientation? Squaring theory and evidence. *Psychonomic Bulletin & Review, 12,* 1–23. http://dx.doi.org/10.3758/BF03196346

Davies, C., & Uttal, D. H. (2007). Map use and the development of spatial cognition. In J. M. Plumert & J. P. Spencer (Eds.), *The emerging spatial mind* (pp. 219–247). New York, NY: Oxford University Press. http://dx.doi.org/10.1093/acprof:oso/9780195189223.003.0010

Dehaene, S., Izard, V., Pica, P., & Spelke, E. (2006). Core knowledge of geometry in an Amazonian indigene group. *Science, 311,* 381–384. http://dx.doi.org/10.1126/science.1121739

Dehaene, S., Izard, V., Spelke, E., & Pica, P. (2008). Log or linear? Distinct intuitions of the number scale in Western and Amazonian indigene cultures. *Science, 320,* 1217–1220. http://dx.doi.org/10.1126/science.1156540

DeLoache, J. S. (2004). Becoming symbol-minded. *Trends in Cognitive Sciences, 8,* 66–70. http://dx.doi.org/10.1016/j.tics.2003.12.004

DeLoache, J. S., Miller, K. F., & Rosengren, K. S. (1997). The credible shrinking room: Very young children's performance with symbolic and nonsymbolic relations. *Psychological Science, 8,* 308–313. http://dx.doi.org/10.1111/j.1467-9280.1997.tb00443.x

Dillon, M. R., Huang, Y., & Spelke, E. S. (2013). Core foundations of abstract geometry. *Proceedings of the National Academy of Sciences of the United States of America, 110,* 14191–14195. http://dx.doi.org/10.1073/pnas.1312640110

Dillon, M. R., & Spelke, E. S. (2015). Core geometry in perspective. *Developmental Science, 18,* 894–908. http://dx.doi.org/10.1111/desc.12266

Downs, R. M. (1985). The representation of space: Its development in children and in cartography. In R. Cohen (Ed.), *The development of spatial cognition* (pp. 323–345), Hillsdale, NJ: Erlbaum.

Dunn, L. M., & Dunn, L. M. (1997). *Peabody Picture Vocabulary Test-III*. Circle Pines, MN: American Guidance Service.

Euclid. (1990). *Great books of the western world: The thirteen books of Euclid's Elements. The works of Archimedes, including The Method. Introduction to Arithmetic by Nicomachus* (2 ed.). Chicago, IL: Encyclopedia Britannica. (Original work written 300 B.C.E.).

Gibson, D. J., Congdon, E. L., & Levine, S. C. (2015). The effects of word-learning biases on children's concept of angle. *Child Development, 86,* 319–326. http://dx.doi.org/10.1111/cdev.12286

Gibson, E. J. (1969). *Principles of perceptual learning and development*. New York, NY: Appleton-Century-Crofts.

Giofrè, D., Mammarella, I. C., & Cornoldi, C. (2014). The relationship among geometry, working memory, and intelligence in children. *Journal of Experimental Child Psychology, 123,* 112–128. http://dx.doi.org/10.1016/j.jecp.2014.01.002

Hazen, N. L., Lockman, J. J., & Pick, H. L. (1978). The development of children's representations of large-scale environments. *Child Development, 49,* 623–636. http://dx.doi.org/10.2307/1128229

Hermer-Vazquez, L. (1997). *Cognitive flexibility as it emerges over evolution and human development: The case of spatial reorientation*. (Unpublished doctoral dissertation). Cornell University, Ithaca, New York.

Hermer-Vazquez, L., Moffet, A., & Munkholm, P. (2001). Language, space, and the development of cognitive flexibility in humans: The case of two spatial memory tasks. *Cognition, 79,* 263–299. http://dx.doi.org/10.1016/S0010-0277(00)00120-7

Hodapp, A. F., & Gerken, K. C. (1999). Correlations between scores for Peabody Picture Vocabulary Test—III and the Wechsler Intelligence Scale for Children—III. *Psychological Reports, 84,* 1139–1142. http://dx.doi.org/10.2466/PR0.84.3.1139-1142

Huang, Y., & Spelke, E. S. (2015). Core knowledge and the emergence of symbols: The case of maps. *Journal of Cognition and Development, 16,* 81–96. http://dx.doi.org/10.1080/15248372.2013.784975

Huttenlocher, J., Newcombe, N., & Vasilyeva, M. (1999). Spatial scaling in young children. *Psychological Science, 10,* 393–398. http://dx.doi.org/10.1111/1467-9280.00175

Izard, V., O'Donnell, E., & Spelke, E. S. (2014). Reading angles in maps. *Child Development, 85,* 237–249. http://dx.doi.org/10.1111/cdev.12114

Izard, V., Pica, P., Dehaene, S., Hinchey, D., & Spelke, E. S. (2011). Geometry as a universal mental construction. In S. Dehaene & E. Brannon (Eds.), *Space, time and number in the brain: Searching for the foundations of mathematical thought* (pp. 319–332). http://dx.doi.org/10.1016/B978-0-12-385948-8.00019-0

Izard, V., Pica, P., Spelke, E. S., & Dehaene, S. (2011). Flexible intuitions of Euclidean geometry in an Amazonian indigene group. *Proceedings of the National Academy of Sciences of the United States of America, 108,* 9782–9787. http://dx.doi.org/10.1073/pnas.1016686108

Jirout, J. J., & Newcombe, N. S. (2014). Mazes and Maps: Can Young Children Find Their Way? *Mind, Brain and Education, 8,* 89–96. http://dx.doi.org/10.1111/mbe.12048

Kaldy, Z., & Blaser, E. (2009). How to compare apples and oranges: Infants' object identification tested with equally salient shape, luminance and color changes. *Infancy, 14,* 222–243. http://dx.doi.org/10.1080/15250000802707088

Kosslyn, S. M., Pick, H. L., Jr., & Fariello, G. R. (1974). Cognitive maps in children and men. *Child Development, 45,* 707–716. http://dx.doi.org/10.2307/1127837

Landau, B., Gleitman, H., & Spelke, E. (1981). Spatial knowledge and geometric representation in a child blind from birth. *Science, 213,* 1275–1278. http://dx.doi.org/10.1126/science.7268438

Learmonth, A. E., Nadel, L., & Newcombe, N. S. (2002). Children's use of landmarks: Implications for modularity theory. *Psychological Science, 13,* 337–341. http://dx.doi.org/10.1111/j.0956-7976.2002.00461.x

Learmonth, A. E., Newcombe, N. S., Sheridan, N., & Jones, M. (2008). Why size counts: Children's spatial reorientation in large and small enclosures. *Developmental Science, 11,* 414–426. http://dx.doi.org/10.1111/j.1467-7687.2008.00686.x

Lee, S. A., Sovrano, V. A., & Spelke, E. S. (2012). Navigation as a source of geometric knowledge: Young children's use of length, angle, distance, and direction in a reorientation task. *Cognition, 123,* 144–161. http://dx.doi.org/10.1016/j.cognition.2011.12.015

Lehrer, R., Jenkins, M., & Osana, H. (1998). Longitudinal study of children's reasoning about space and geometry. *Designing learning environments for developing understanding of geometry and space, 1,* 137–167.

Liben, L. S., & Myers, L. J. (2007). Children's understanding of maps: What develops? In J. Plumert & J. Spencer (Eds.), *The emerging spatial mind* (pp. 193–218). New York, NY: Oxford University Press. http://dx.doi.org/10.1093/acprof:oso/9780195189223.003.0009

Liben, L. S., Myers, L. J., Christensen, A. E., & Bower, C. A. (2013). Environmental-scale map use in middle childhood: Links to spatial skills, strategies, and gender. *Child Development, 84,* 2047–2063. http://dx.doi.org/10.1111/cdev.12090

Loewenstein, J., & Gentner, D. (2005). Relational language and the development of relational mapping. *Cognitive Psychology, 50,* 315–353. http://dx.doi.org/10.1016/j.cogpsych.2004.09.004

Piaget, J., Inhelder, B., & Szeminska, A. (1960). *The child's conception of geometry* (E. Lunzer, Trans.). New York, NY: Norton.

Pica, P., Lemer, C., Izard, V., & Dehaene, S. (2004). Exact and approximate arithmetic in an Amazonian indigene group. *Science, 306,* 499–503. http://dx.doi.org/10.1126/science.1102085

Pick, H. L., & Rieser, J. J. (1982). Children's cognitive mapping. In M. Potegal (Ed.), *Spatial abilities: Development and physiological foundation* (pp. 107–128). Cambridge, MA: Academic Press.

Plato. (2012). *Plato's* Meno (2nd. ed.; GMA Grube, Trans.). Indianapolis, IN: Hackett Publishing Company Incorporated. (Original work written 380 B.C.E.).

Schwartz, M., Day, R. H., & Cohen, L. B. (1979). Visual shape perception in early infancy. *Monographs of the Society for Research in Child Development, 44,* 1–63. http://dx.doi.org/10.2307/1165963

Shusterman, A., Lee, S. A., & Spelke, E. S. (2008). Young children's spontaneous use of geometry in maps. *Developmental Science, 11*(2), F1–F7. http://dx.doi.org/10.1111/j.1467-7687.2007.00670.x

Spelke, E. S., & Lee, S. A. (2012). Core systems of geometry in animal minds. *Philosophical Transactions of the Royal Society of London Series B, Biological Sciences, 367,* 2784–2793. http://dx.doi.org/10.1098/rstb.2012.0210

Spelke, E., Lee, S. A., & Izard, V. (2010). Beyond core knowledge: Natural geometry. *Cognitive Science, 34,* 863–884. http://dx.doi.org/10.1111/j.1551-6709.2010.01110.x

Uttal, D. H. (1996). Angles and distances: Children's and adults' reconstruction and scaling of spatial configurations. *Child Development, 67,* 2763–2779. http://dx.doi.org/10.2307/1131751

Uttal, D. H. (2000). Seeing the big picture: Map use and the development of spatial cognition. *Developmental Science, 3,* 247–264. http://dx.doi.org/10.1111/1467-7687.00119

Uttal, D. H., Meadow, N. G., Tipton, E., Hand, L. L., Alden, A. R., Warren, C., & Newcombe, N. S. (2013). The malleability of spatial skills: A meta-analysis of training studies. *Psychological Bulletin, 139,* 352–402. http://dx.doi.org/10.1037/a0028446

Uttal, D. H., & Wellman, H. M. (1989). Young children's representation of spatial information acquired from maps. *Developmental Psychology, 25,* 128–138. http://dx.doi.org/10.1037/0012-1649.25.1.128

Vasilyeva, M., & Bowers, E. (2006). Children's use of geometric information in mapping tasks. *Journal of Experimental Child Psychology, 95,* 255–277. http://dx.doi.org/10.1016/j.jecp.2006.05.001

Verdine, B. N., Golinkoff, R. M., Hirsh-Pasek, K., Newcombe, N. S., Filipowicz, A. T., & Chang, A. (2014). Deconstructing building blocks: Preschoolers' spatial assembly performance relates to early mathematical skills. *Child Development, 85,* 1062–1076. http://dx.doi.org/10.1111/cdev.12165

Winkler-Rhoades, N., Carey, S. C., & Spelke, E. S. (2013). Two-year-old children interpret abstract, purely geometric maps. *Developmental Science, 16,* 365–376. http://dx.doi.org/10.1111/desc.12038

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